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# THE MONIST

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# THE MONIST

## A LIBERAL SOCIALISTIC PROGRAMME

THE various socialistic systems previous to Marx were naughtily characterized by him as Utopian, because they were in substance but artificial constructions, which, while fully answering to this or that ideal of equity, did not take sufficient account either of the economic structure of the society that had to be reformed or of the ruling motives and respective forces of the social classes, thus neglecting just the two fundamental historic factors which dominate with iron determinism all social transformations. These Utopian socialistic schemes consequently claimed that the attractive form of equity and social well-being in which this or that reform was presented should have the inherent magic virtue of attracting to itself the whole body of society, which in an outburst of generous enthusiasm for this ideal regime would set it up right away.

To these Utopian systems of socialism Marx and Engels opposed their own system, which they modestly described as alone truly scientific, but which, while on one hand in complete contrast to the preceding ones, on the other does not differ substantially from them. For when once a given regime—that in which all production goes to the workers—is approved as just, Marx makes himself believe that he can get the facts to prove that the present regime, from the very fact of its becoming ever more unjust, tends toward the collectivist regime, which Marx considers the only one capable of realizing his ideal of equity. The thesis produces the antithesis from which the synthesis

will be formed in the Hegelian sense. It is not the case then that Marx examines the economic evolution objectively, without preconception or preference, so as to succeed in foreseeing whither it tends, but he first caresses the idea of a given regime, exactly like all the Utopian socialists who preceded him with the only difference that he thereafter, instead of presenting it ingenuously as fitted to convince and attract to itself all mankind united in brotherhood, attempts to force the facts to prove that this regime is the one toward which economic evolution tends irresistably and fatally, not by the will of men, but by the force of things.

At the same time the Marxian socialism is quite the reverse of the previous Utopian socialism just on account of this economic fatalism, which, by making the advent of the desired regime depend on the mechanical and fatal evolution of the economic process, entirely denies *a priori* the efficacy of law in general, and of the law of property in particular, as regards changing in one direction or in another the economic process itself.

The mischief done by this economic fatalism to the action of the socialist party, though not very gravely felt as long as this party was obliged, by the feebleness of its forces, to limit itself to a merely negative action of criticism, is severely felt now that the party, with its forces so notably increased as to put it in a position to act, would require a positive programme of action.

Now the only programme that Marxism has been able to propose is that of the violent revolutionary dispossession of that little group of magnates of capitalism to which Marx predicted the capitalist class would be reduced; and by this violent revolutionary act he had the illusive hope of setting up in one day the collectivist regime, which the inevitable concentration of manufacturing concerns, driven to the utmost limit, would without fail have already estab-

lished under the frail capitalistic involucre, and all in working order.

But neither the reduction of the possessors of capital to a small group of magnates nor the concentration *pari passu* of all the manufacturing concerns, driven to such an extreme limit as to set up of itself the collectivist regime in working order, has come to pass, and the Socialist party, unable to perform that one revolutionary act indicated to them by Marx, which at the present moment would mean smashing up the whole delicate mechanism of economic production, and at the same time without any other positive programme of action, has in all countries fairly lost its bearings. This want of any clear line of policy leaves it groping in the dark, a prey to fruitless and disorderly agitation, threatened with being broken up by continual new schisms into ever more numerous sections and subsections, incapable of combating the generally revived anarchy; and all this to the immense damage of the economic production of the country, which suffers from this chronic state of objectless revolutionary agitation and to the complete discredit of the Socialist party, which proves itself incapable of any reconstructive action.

The Socialist party will not be able to get out of this state of impotence in any other way than by absolutely renouncing the mechanical economic fatalism of the Marxian school and returning to the acknowledgement of the efficacy of law, and especially of the law of property, toward modifying the course of economic phenomena.

Be it well noted that we do not intend to go back to the ingenuousness of the Utopian Socialism which thought it enough to present a proposal that bore the mark of equity in order to obtain at once the unanimous assent of all social classes; but we maintain only that when once a given social class, hitherto of very little weight as a social factor,

increases in power so as to become a preponderating political factor it can, when risen to power or participation in power, modify the laws, and especially the law of property, by means of legislation, in accordance with its clearly understood and legitimate interest, and that these modifications of the law of property, to be imposed on the other classes more or less against their will, can effectively modify in its favor the course of the economic process.

Now this preponderance, as a political factor, of the wage-earning working classes over all the classes and subclasses of the *bourgeoisie* united, is a fact already accomplished, or soon to be so, in all countries. The moment has just come therefore for the Socialist party to return to that *legal socialism* which the Marxian fatalism has wrongly disdained too much, even though the various systems certainly erred, hampered as they were by too great respect for the rights of property beyond what is strictly necessary and sufficient to stimulate to the utmost to work and saving, through excessive timidity resulting in excessive slowness and meagreness of means to be used for the desired gradual nationalization of private property.

A system there is of accelerating, much more than could any other system hitherto devised, this process of nationalization, without throwing into disorder the existent organization of production, or injuring in any way those economic activities and that virtue of thrift which today are more than ever necessary for the increase of economic production. It is that of applying to the succession duties of the State, which is raised to the dignity of co-heir, the principle of progressiveness, not only as regards the extent of the patrimony or the degree of relationship, but also the *age* of the patrimony. That is to say the total amount of the patrimony left by the deceased would be divided into so many portions according to the number of times each portion had passed from hand to

hand, by way of succession or donation or dowry, before arriving at the deceased, and the rate of participation of the State in the inheritance of these various portions of the patrimony would increase with the increase of the number of transmissions undergone by them.

The State, for example, might continue to levy on the portion due to the labor and savings of the deceased not more than what it does today with the succession duties; but on the portion inherited by the deceased from his father and due to the labor and savings of the latter, the State would levy a much larger quota, for example 50 per cent; and on the portion that had come to the deceased, through his father, from his grandfather, the original accumulator of this portion, or, more generally, that had already undergone two transmissions whatsoever as private property, the State might inherit a much larger quota, reaching even 100 per cent (which would render the division of each patrimony into three parts sufficient, as nothing could be inherited from a great-grandfather). Owing to the impossibility, however, of searching out the most distant sources of the patrimonies now existing, and also for reasons of equity, all the patrimonies existent at the moment of the promulgation of the new law might be considered as due, for example, to the extent of a third part, to the work and savings of the present proprietor, and of two-thirds as inherited by one transmission as private property; and this as regards the State succession duty to be levied at the death of all proprietors now living.

It is evident that such a reform of the rights of succession would more effectively stimulate to saving than does the present right to bequeath all. In fact, as regards one's own children, every sum saved by the heir of a given patrimony would come to have in his eyes *a value much greater, even three or four or five times greater,* than an equal sum inherited by him; while today the heir of immense posses-

sions, which he knows he can bequeath in their entirety to his children, is in no way urged to that hard work, that severe abstinence, which would be necessary to him in order to double or triple the patrimony he has inherited.

The State, co-heir to the extent of 50 per cent in the quotas of patrimonies that have already undergone one transmission by way of succession or donation or dowry, and to the extent of 100 per cent in those that have undergone two transmissions, would levy the part belonging to it, not in money but in kind: land, buildings, State and Treasury bonds, shares and debentures in limited and other companies, etc., precisely like the other heirs.

State, treasury, provincial and communal bonds, as they would become the property of the State, should be immediately destroyed. This gradual amortisement of public debts would progressively free the States and Provinces and Communes from the enormous burden of the payment of interests, which is today a heavy handicap on all the really productive economic activities. This notable diminution of the burdens on the public bodies, along with the growing incomes which these bodies would derive from the rent of lands, dwelling houses, buildings in general, as they would become nationalized, would allow of the gradual passage from a system of finance on the basis of taxes to one on the basis of rents.

The principle of share-holding by the State, or better still by the great national trade unions of manufacturing workmen, would be put into effect, in a gradual way, as gradually the shares of limited and other companies were inherited by the State.

It will be seen that this proposal would satisfy the fundamental principle of socialism, that of greater equity, and would carry out its maxim of socialization of the instruments of production and of all capitals in general, which it justly considers necessary for the complete emancipation

of the workmen. The gradual character of the process of nationalization would permit of the passage from the present to the new regime by peaceful and legal means, without disorganizing the delicate mechanism of economic production, without requiring even the improvised creation of an entirely new bureaucratic organism; it would, on the contrary, render possible the gradual transformation of some of the organs already in existence so as to fit them for the new task, as this increases in extent and importance and would give the needed time for the process of economic production to adapt itself, by increased production of necessities and diminished production of luxuries, to the new and more equitable regime. As regards the coexistence of nationalized capital with always new private capital, the profits of this latter would represent in a continually diminishing measure the exploitation of the work of others on the part of heirs who have had no share in the accumulation of their possessions, and in ever increasing measure would represent the just reward of abstinence given to one who had created new capital effectively by his savings. Moreover, from the fact that the proportion of the total of private capital to the total of that nationalized would keep diminishing, especially through strong progressivenesses of the duty as that above indicated, the proportion also between the quota of the social annual income belonging as profits to private capital and the quota belonging to labor would equally keep decreasing, in favor of the latter.

The proposal would at the same time satisfy the fundamental principles of orthodox economics and of the free-contract regime, inasmuch as it would not assign to the State any coercive function in the way of the military organization of labor, as collectivism would do, but merely that of furnishing to the workmen, through suited self-governing organs of their own, and in ever increasing measure, the means necessary for their work, thus freeing

them from their present dependence on private capitalists, and by facilitating as much as possible the association and spontaneous co-operation of labor it would aim at giving the free-contract regime a much greater development than even at present attained.

The gravest objection that can be made, and one in fact that has been made, is derived from the distrust that all economists feel towards public bodies in general, and the State especially, as regards their capacity for even the administration of those possessions which would be gradually nationalized. There can never, therefore, be too much care exercised in reducing to a minimum these administrative functions especially on the part of the State.

Thus, for example, the administration of the lands gradually nationalized should be entrusted by the State to the Provinces, or to self-governing bodies with a provincial jurisdiction, and these lands should be let either to agricultural co-operative societies or even to individual agriculturists, provided they are united in associations for the purchase of materials, for the manufacture of given products (associated dairies, wine producers, etc.), and for the sale of all the products.

The administration of town properties would be entrusted to the municipal authorities; and it is well known that when old buildings have been pulled down the house property of certain towns has been notably increased by the new buildings, especially in England, and that their administration is carried out in a satisfactory manner.

In any case there is nothing to prevent the administration of these town properties being eventually entrusted, if thought well, to these private companies, which now administer them on behalf of private proprietors.

The mines, the factories and the capital invested in them would come under the control and management of the great national trade-unions of workers of the respec-

tive branches of industry, as by degrees the State handed over to these national trade-unions, under certain conditions of State participation in the profits, the shares inherited by it. This would unite the advantages of co-operative production with those of industry on a large scale: greater interest in the work, as in co-operative production, because the workmen would enjoy the fruits of it in always fuller measure, and at the same time great manufacturing potentiality, which co-operative production has never hitherto possessed, and great discipline, which it has never succeeded in obtaining. And this because those who direct each factory, to whom would be entrusted the maintenance of the discipline, would be appointed—when the number of the respective shares nationalized would come to exceed that of the shares still in private hands—not by those only who worked in the factories, as has always been the case in the manufacturing cooperative societies, but by the whole national trade-union, whereof the workers in this or that factory would be but a small minority. Moreover, the admission of the trade-unions to the control and management of these industries—industries, be it noted, already completely organized and in full working order—would take place gradually with the increase of the number of the trade-union shares in proportion to those still in private property, so that the trade-unions, that is the respective managers, would have full time to become experienced in the control and management, which at first would continue to be entrusted to the managers appointed by the private shareholders; managers, who would probably be appointed also by the trade-union herself when she would have the majority of the shares.

There would remain but a small number of properties the administration of which would have to be carried out by the State, which would entrust it to a national institute formed for the purpose. The institute—to which would

belong also the supreme control of the administration of all the other nationalized properties, intrusted, as we have seen, to the direct management of provincial or communal bodies or of trade-unions—should, in its relation with the State, only pay over to the treasury the quotas belonging to the State of the incomes of all nationalized properties, whether those administered directly by the national institute itself, or those entrusted to provincial or communal bodies or trade-unions above mentioned; and if rendered completely self-governing and independent from the government, there is no reason why it should not be so organized that it would work as well as other similar State institutes and those of other public bodies in general, which today administer collective capital of immense amounts.

To this institute, finally, would belong also the supreme control of the National Bank of Credit to Associated Labor, which—founded by means of the capitals inherited in money—should be created for the purpose of furnishing the farmers of the nationalized lands and the trade-unions who already managed certain factories the capital necessary for working them.[1]

Although the function properly belonging to the State, or the bureaucratic function in general, be thus reduced to a minimum, we do not, however, flatter ourselves that we have yet succeeded in removing entirely from the minds of free trade economists their prepossessions as to the incapacity of the State to perform even purely administrative functions. But this objection, though certainly of a certain gravity, does not seem to be one that cannot be overcome, and for the purpose of overcoming it the thought and ener-

[1] For further details, we must refer the reader to our recent book, published by Zanichelli in Bologna. *Per una riforma socialista del diritto successorio*, which treats the question in all its bearings, and in which, besides quoting the principal criticisms urged against our answers thereto, we have also added the concrete draft of the bill that might be presented to Parliament for the effectuation of the proposal. A French edition will soon appear by the publisher, F. Rieder et C., Paris, with préface by M. Albert Thomas, the leader of the French Socialist party.

gies must be directed of all those who are convinced that no mere difficulties of a technical sort can possibly suffice to arrest the now irresistible and overwhelming onward movement of the working masses, firmly resolved on arriving at the nationalization of all the instruments of production and of all capitals in general.

To sum up: This reform in the law of succession, which would finally permit of the beginning of the much desired nationalization, by pacific and legal but at the same time rapid means, might and ought to represent, we think, that medium programme of socialistic action that is capable of reuniting the Socialist party, now completely bewildered and divided, or at least of attracting to itself the very large majority of the working and popular classes. At the same time, from the legal and gradual manner in which it would guarantee the passage from the old to the new regime, it might not encounter any very bitter resistance from the more advanced and more clear-sighted of the monied classes, who are well aware that to save our civilization from the immense ruin of a violent Bolshevik revolution and to make way for new and more flourishing prosperity, it is necessary to accept the fundamental postulates of Socialism, which, because in accordance with the supreme principles of equity, are not to be combated by any civilized society except at the peril of its own existence.

EUGENIO RIGNANO.

MILAN, ITALY.

# THE RELATION OF SPACE AND GEOMETRY TO EXPERIENCE*

## I. Geometry as a Schematization of Experience

GEOMETRY is considered by every one to rank among the most certain of sciences. One can have grave doubts, for example, as to the universal validity of any theory in biology, or even honest misgivings concerning the absolute precision of the law of the conservation of energy, but it is hard to imagine a man who is really sincere in questioning the theorem of Pythagoras, that the square on the hypotenuse of a right-angled triangle is equal in area to the sum of the two squares on the legs of the triangle. This conviction which we possess that the theorems of geometry are valid seems essentially independent of any confirmation or substantiation by experience. After we are really initiated into the processes of geometrical reasoning, our certainty of the truth of the theorem of Pythagoras cannot be augmented nor diminshed one jot nor tittle by any actual measurement of a figure illustrating the theorem, if the figure should not substantiate the theorem, so much the worse for the figure, we should say.

It is a highly significant fact, however, that this very science of geometry, which seems to keep itself so independent of experience, is one of the most useful of all sciences in our daily life of experience. The surveyor, the navigator, the carpenter, all make continual use of geom-

* This sequence of lectures was read at Harvard University in the Fall Semester of 1915.

etry in the course of their every-day pursuits, and not only do they do so, but they have an implicit confidence, which always proves to be justified, that the results of their geometrical reasonings—provided only that these are correct in a purely intrinsic, geometrical sense and are based on correctly gathered data—will lead them to perfectly correct conclusions with regard to the world of things experienced with which they deal in their daily lives. The surveyor knows that if his observations are correct, and if he has committed no error of geometry in his computations, the map which he has designed in accordance with a few elementary geometrical laws will be a good map of the region it represents. We have thus the interesting spectacle of a science which seems to scorn experience as its basis, yet furnishes results of the utmost empirical application and value. The question at once occurs to us: How does this happen?

Several theories of the nature of geometry have been devised to bridge this gap. Let us first consider Kant's discussion of geometry. I do not propose to consider here the whole of Kant's treatment of this topic, but only a certain aspect of it—that aspect, namely, which is expressed in the following passage:[1] "Geometry is a science which determines the properties of space synthetically, and yet *a priori*. What, then, must be our representation of space, in order that such a cognition of it may be possible? It must be originally intuition. . . . But this intuition must be found in the mind *a priori*, that is, before any perception of objects, consequently must be pure, not empirical, intuition. For geometrical principles are always apodeictic, that is, united with the consciousness of their necessity, as, 'Space has three dimensions.' But propositions of this kind cannot be empirical judgments, nor conclusions from them."

[1] *Critique of Pure Reason*, Transcendental Aesthetic, §3, Meiklejohn's translation.

That is, Kant says not only that geometry is known *a priori*, but also that our whole original knowledge of *space*, the subject-matter of geometry, is *a priori*, and he regards these two assertions as practically tantamount to one another. It seems to the casual observer, however, as though spatial properties could also be given to us *a posteriori*, in experience. It seems as if the straightness of a stick or its length were known quite as empirically as its color or its hardness. Whatever we may say about space, there is no question possible with regard to the statement that spatial qualities are capable of being experienced. Now, it is not with space in any ulterior sense, but with *spatial qualities* that geometry, as used by the surveyor or the navigator, deals. It is not lines in any purely abstract meaning of the term, but the hair-lines in his telescope, or the path of a light-ray, that concern him, and he knows that if his measurements of the lengths, straightness, angles, etc., of these are correct, his computations will also be correct, provided only that he has made proper use of geometrical reasoning. It is such lines as these that form part of his space—and yet he feels the need of no experiment to substantiate the result of his geometrical reasoning. The *a priori* certainty which Kant attributes to geometry is one which is utterly irrelevant to its applications in our life; the abyss between his space, to which geometry applies, and the concrete spatial properties of concrete things, remains unbridged in his system, notwithstanding the fact that he calls space the form of our external experience since the apriority of the geometry which we apply must be the apriority of an empirical intuition, not that of a pure intuition. The geometry which he discusses is one which applies to an entirely non-empirical realm, and which he nowhere brings into touch with those fields of experience in which our every-day geometry plays so great a role.

One of the chief motives which leads Kant to this somewhat incomplete if not positively unsatisfactory treatment of geometry, as one can readily see from this paragraph which we have quoted, is that he considers the apriority of geometry impossible unless our knowledge of its subject-matter is also *a priori.* It is clear, then, that *if we can consistently hold that it is possible for geometry to be a priori, and yet to have an empirical subject-matter,* one strong argument in favor of Kant's view of space has vanished, and we are able to formulate a theory of the relation of the non-empirical science of geometry to the objects of our experience as surveyors or navigators, etc., which is more consonant with the views of our every-day common sense than that of Kant. It is this view of the relation between experience and geometry— the view, namely, that geometry, though *a priori,* deals with an empirical subject-matter— which I intend to suggest as a possibility in what follows.

Before I go on, however, to my discussion of this theory, I wish to devote a little time and attention to a third theory, different both from that of Kant and from that which forms the thesis of this course of lectures. This theory is that of Ernst Mach, as expounded in his little book, *Space and Geometry.*[2] Professor Mach's views form the precise antithesis of those of Kant, both with respect to space and to geometry. As to space, he says:[3]

"If for Kant space is not a 'concept,' but a 'pure (mere?) intuition a priori,' modern inquiries on the other hand are inclined to regard space as a concept, and in addition as a concept which has been derived from experience. We cannot intuit our system of space-sensations *per se;* but we may neglect sensations of objects as something subsidiary; and if we overlook what we have done, the notion may easily arise that we are actually concerned

[2] Translated by T. J. McCormack, Open Court Publishing Company, Chicago.

[3] *Op. cit.,* p. 34.

with a pure intuition. If our sensations of space are independent of the quality of the stimuli which go to produce them, then we may make predications concerning the former independently of external or physical experience. It is the imperishable merit of Kant to have called attention to this point. But this basis is unquestionably inadequate to the complete development of a geometry, inasmuch as concepts, and in addition thereto concepts derived from experience, are also requisite to this purpose."

Mach claims, in other words, that space is essentially a system of *space-sensations* or *space--experiences*, which seems to take the form of a "pure intuition" merely because in our geometrical considerations we confine our attention to one particular phase of the objects with which we are concerned, and neglect all those aspects of our experiences which, though they are necessarily present, are not spatial in their nature. According to him, he says, we are enabled thereby to consider the interrelations of the spatial aspects of our experience with entire disregard of what the other sides of our experience may be. Nevertheless, he holds, space is given to us in a completely empirical manner. Or, as Mach puts it in another book of his,[4] "Space and time are well-ordered systems of sets of sensations."

It seems obvious to the common-sense of us all that Mach is at bottom correct in this statement, for space is somehow or other, we all should say, a system of experiences. Everything looks promising, therefore, for a satisfactory account of the sources and nature of our geometrical certainty. Let us see what the explanation of this is which Mach offers us. He expounds his view as follows:[5]

"The knowledge that the angle-sum of the plane triangle is equal to a *determinate quantity* has thus been reached

[4] *The Science of Mechanics*, translated by T. J. McCormack, Open Court Publishing Co., p. 506.

[5] *Space and Geometry*, p. 58.

by experience, not otherwise than the law of the lever or Boyle and Mariotte's law of gases. It is true that neither the unaided eye nor measurements with the most delicate instruments can demonstrate *absolutely* that the sum of the angles of a plane triangle is *exactly* equal to two right angles. But the case is precisely the same with the law of the lever and with Boyle's law. All these theorems are therefore idealized and schematized experiences: for real measurements will always show slight deviations from them. But whereas the law of gases has been proved by further experimentation to be approximate only and to stand in need of modification when the facts are to be represented with great exactness, the law of the lever and the theorem regarding the angle-sum of a triangle have remained in as exact accord with the facts as the inevitable errors of experimenting would lead us to expect; and the same statement may be made of all the consequences that have been based on these two laws as preliminary assumptions."

This result—namely, that Mach regards the certainty of geometry as of empirical origin, and simply due to the fact that our experiments with lines and angles, etc., by means of paper-folding and similar methods have always substantiated our geometrical predictions as well as could be expected when we take into consideration the inherent inaccuracies of the experiments—this result, I say, is by no means satisfactory. Nobody would ever think of testing the theorems of Pythagoras by means of a foot rule or a protractor; the only things which would be tested by such an attempt and which would have to be rejected in case of a non-verification of the theorem would be the foot rule or the protractor. However useful paper-folding and similar pursuits may be in leading our interest toward things geometrical and in giving us the first dawning ideas about what it is with which geometry concerns itself, geom-

etry deals *directly* with points, lines, planes and angles, and not, except in some periphrastic sense, with such gross topics as folded bits of paper, rules, and micrometers. Whatever the edge of a piece of paper may do or be, a *line* is the shortest distance between two points, does not cut any other line in more than one point, and has all the other properties which are attributed to lines in a text-book of geometry. If a crease in a piece of paper fails to have these properties, why—it simply is not a line. However useful geometry may be in the theory of paper-folding or navigation or astronomy, *prima facie* geometry is *not* the study of paper-folding nor of navigation nor of astronomy, and the accuracy or correctness of any part of any of these studies may be impeached without involving as a corollary the impeachment of any portion of geometry or theorem belonging to it. The geometry of which Mach talks is simply not the geometry of the mathematician; Mach solves the problem of space and geometry to his own satisfaction by flatly ignoring the non-experimental nature of geometry, just as Kant solves it by not entering into a discussion of that empirical character which actually pertains to space. Both positions are unnatural; what is the natural alternative which avoids the objections besetting each of them?

I have already stated that the view which I maintain in this course of lectures is that geometry, though *a priori*, deals with an empirical subject-matter. How is this, however, possible? How can our study of a subject which is known in a manner open to all the uncertainties and inaccuracies which beset empirical knowledge in all its manifestations—namely, space—be possessed of an *a priori* and purely intrinsic certainty, not rooted at all in experience? The answer to this question is by no means as difficult as it might seem at first sight. It will be noted that Mach does not make geometry deal with raw, undi-

gested experience, but, as he says, "All these theorems are . . . idealized and schematized experiences."

Now, the study of an idealized or schematized experience differs from that of a raw or crude experience in that it has to take account of two distinct factors—the experience, and the mode of schematization employed. To illustrate how this is the case, suppose that I am considering a set of statistical tables of the death-rate of Boston from year to year. I may regard these tables from several different standpoints. I may be interested, for example, in the seasonal fluctuations of the death-rate. In this case the table of statistics gives me information which could not have been predicted with more than approximate accuracy and certainty, and which is completely dependent upon concrete experience. On the other hand, I may be primarily interested in the method of tabulating statistical data which is used in these tables; in this case, when I have once grasped the principle underlying the method, I am quite as well able to predict anything you please in the next year's tables which concerns details that are dependent solely on the method of tabulation employed as I am to yield the same information concerning this year's tables or concerning last year's tables. The method of tabulation employed may and should be made as suggestive as possible of the actual empirical laws of the death-rate of Boston and as useful as possible in the handling of the data tabulated, but once it has been chosen, it is entirely independent of the particular empirical properties of these data, and remains essentially incapable of substantiation or of contradiction by them. Thus, though the study of his tables from the standpoint of the form of tabulation employed is of immense practical use to the statistician for the handling of his empirical material, once that form is definitively fixed, it is really an *a priori* science, notwith-

standing the fact that the data expressed in the tables are themselves known *a posteriori.*

It is possible to regard geometry in a way quite parallel to a set of statistical tables—though I do not mean to suggest that statistics play any part whatsoever in geometrical reasoning. We may regard a point, for instance, not as a direct object of experience, but as a certain arrangement or collection of objects of experience, in a manner which I shall explain in detail in the subsequent lectures of the present course. A point of this sort will, in general, depend for its actual properties on the concrete natures of the experiences of which it is constructed, but it will also have certain properties which, unlike its other attributes, are independent of the concrete natures of these particular experiences, and are predictable on the basis of a knowledge merely of the principle in accordance with which the points of our space have been synthetized from our experience. These latter properties of points are studied in geometry, while those which are dependent on concrete experiences belong rather to physics or to the other natural sciences. Thus space, which is made up of points, lines, etc., constitutes a kind of tabulation of the experiences of our outer senses; yet geometry, which has space as its subject-matter, since it depends on the method of tabulation alone, as I claim in this course of lectures, is an *a priori,* not an experimental, science. This is the view for the possibility of which I am here pleading.

My view might be stated as follows: Geometry is the science of a *form* into which we cast our spatial experiences. I shall not express my view in this manner, for I wish to keep it clearly distinct from two other views which might with equal justice lay claim to this mode of expression. These views are that of Kant, upon which we have already touched, on the one hand, and the view of those mathematicians, on the other, who hold that the only spe-

cies of geometry which can possess *a priori* certainty is that geometry which concerns itself, not with the actual points and lines of the world in which we live, but with the *laws* in accordance with which a great many of the properties of these points and lines can be deduced from a small number of properties which they seem to possess, or at any rate seem to possess approximately.

Let us first see wherein our view differs from that of Kant. Kant says that geometry is the synthetical science *a priori* of the form of the external sense, whereas we say that geometry deals with the intrinsic properties of a schematism into which we cast our external experiences; wherein lies the real difference between these two very similar views, and what is its significance? The difference is this: Kant regards geometry as the study of a schematism imposed on the world by our external senses themselves, before any act of experience, and utterly independently of any such act. On the other hand, we maintain that geometry deals with an experience schematized after it has come into existence, and with concrete practical ends in view, even though this schematism may be permanent once it has come into existence and been accepted by us. As a consequence, Kant is unable, as we have previously indicated, to explain how it is that we are able to apply geometry to experience *in a certain concrete and definite manner,* as it is applied by the sailor and the surveyor, or at least he fails to give any hint of how this application is to take place, for the schematism which constitutes the subject-matter of geometry is made, he tells us, before and without reference to the concrete experiences of the surveyor and the sailor, by the essential nature of the outer senses, themselves, and would be the same were there no such particular experiences as those of the sailor or the surveyor. We, on the other hand, are able to maintain that the schematism of geometry is useful for the sur-

veyor and the sailor just because it is designed with the purposes of the surveyor and the sailor in view. This is still true even though that schematism remains just what it is forever, once it has been selected. For example, we choose the schematism "line" in such a manner that some particular line of geometry will be determined as unambiguously as possible in a certain easily recognizable manner by every ruler edge or plumb-line or line of vision in our actual experience. Then, while the lines we have chosen in our schematism may have a host of interesting and valuable properties which are determined by the schematism alone, we may make certain of our geometrical objects standing hostage, as it were, for the physical objects mentioned above, and make our reasonings and experiments refer to these lines rather than to the physical objects themselves, so that our reasonings and experiments may be facilitated by the manifold transformations and systematizations suggested by pure geometry. Since our geometrical lines, though constructions and schemata, are constructions and schemata made on the basis of concrete experiences, we are able to recognize empirically this correspondence between geometrical lines and certain physical entities to which we have just referred, and hence make the former take the place of the latter in the formulation of scientific laws. This cannot be done on the basis of Kant's theory—and this is its fatal defect—because space, according to him, though the form of our external experience, is completely prior to any concrete experience, and hence no correspondence between certain spatial entities and certain physical entities can be recognized empirically, if we accept his theory of the matter.

So much for Kant; let us now consider the pros and cons of the view of those who hold that the only sort of geometry which can possess *a priori* certainty is that geometry which concerns itself, not with the actual points and

lines of the world in which we live, but with the *laws* in accordance with which a great many of the properties of these points and lines can be deduced from a few laws which we observe that these points and lines possess, or very nearly possess. This view, that is, says that the real subject-matter of geometry is the formal deduction of its theorems from its axioms, which are not self-evident statements concerning the space in which we live, but mere hypotheses which may perchance be satisfied by an infinity of systems, and it claims further that geometry is not at all concerned with the question whether these axioms and theorems apply to any particular objects or constructions in the world of sense. This latter application, it maintains, must be determined by experience alone, and depends on experience for its validity. Now, it is perfectly true that there is a legitimate non-empirical science, which has as good a claim to the name of geometry as the discipline which we are discussing here, which is concerned with the deduction of the theorems of Euclid from the axioms of Euclidean geometry. I doubt, however, whether this mere abstract logical deduction constitutes the whole of what we ordinarily call geometry, or even the whole of that part of geometry which can lay claims to *a priori* certainty. There certainly appear to be such things as lines, which are more than mere blank spaces in the scheme of symbolism or of logical deduction by means of which the appropriate theorems are obtained from any set of truths which can be put into the form expressed in Euclid's axioms. It seems as if these lines must, from the very necessity of their nature, satisfy the laws of Euclidean geometry, while certain particular lines bear an intimate association with such concrete empirically known things as straight edges and light-rays. This association seems to be presupposed in our every-day life when we say, "This is more nearly a true line than that," as if the true line

were a sort of a criterion with which we could empirically compare certain empirical objects. This two-faced aspect of geometry, which is *a priori*, yet deals with an empirical subject-matter, is not explained by those who hold the view we criticize, and is explained on our view.

We hold, then, that geometry is an *a priori* science, which deals with a certain schematization of experience, which we may call space, in so far as its properties depend on the method of schematization alone. This schematization has a superficial appearance quite different from that of the experiences of which it is composed before they are schematized. Experience presents us only with objects that have extension, while a point has no extension. Experience never gives us a perfectly straight line, nor a precise circle, nor an absolutely accurate sphere. All these things, however, form topics dealt with in geometry. Now, we have claimed in this paper that geometry is a schematization of experience, not in the sense that it is a kind of approximate copy of experience with all the roughnesses left out, but in the sense that it is formed from experience by the application of some principle, just as a table of statistics represents the facts it concerns in accordance with a certain principle of tabulation. Just as, notwithstanding the fact that a table of statistics does not resemble the matters tabulated, a statement about the former is but a periphrasis for a statement about the latter, so a geometrical proposition is really concerned with experience, notwithstanding the fact that its direct subject-matter has an appearance differing in many respects from that of experience.

After all this talk of geometry as a method of tabulation, many of you will want to see a concrete example of this sort of tabulation, taken from the field of geometry. It is rather difficult, however, to exhibit such an example in the limited portion of this lecture which remains. I

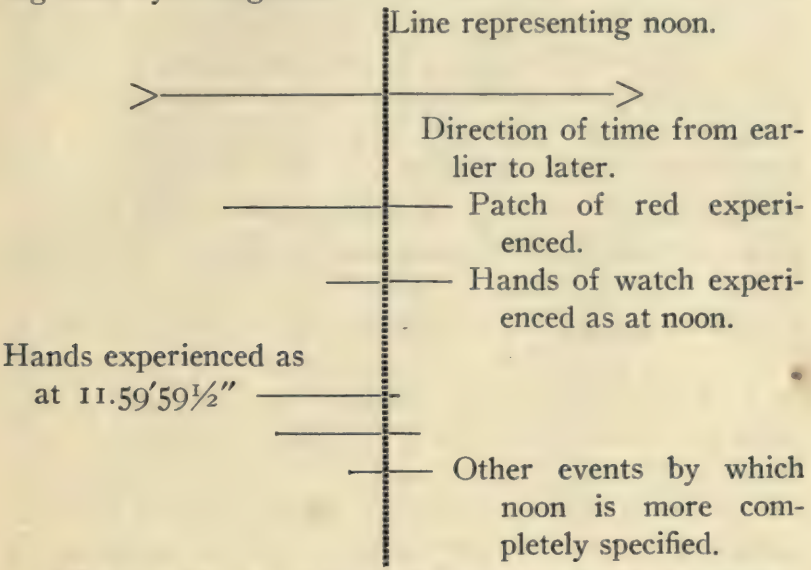
can, nevertheless, give you an example of a similar tabulation employed in a field very analogous to geometry—the study of the formal properties of time.

There are certain laws which we always unquestioningly accept as valid concerning time in quite the same spirit that we hold geometry to be *a priori*. We believe, for example, that time is composed of instants which are timeless, that no two instants are contemporaneous, that of two distinct instants, one must precede the other, and that if the instant *a* precedes the instant *b*, and the instant *b* precedes the instant *c*, that *a* precedes *c*. We consider these statements as quite as truly *a priori* as the theorem of Pythagoras, and regard the former and the latter as quite analogous with one another. The events which we experience, however, always occupy time, an event may neither precede nor follow another, and so on indefinitely. How are we able, the question is, to regard instants as tabulations of events of such a sort that we can be sure, from a knowledge of the method of tabulation alone, without any concrete empirical knowledge of events, that instants will have the formal properties we have attributed to them? I shall give such a method of tabulation in the following paragraphs, though I cannot spare the time to show, as is the case, that it is a consequence of the method itself that instants have the formal properties we have attributed to them.

Experienced events are said to happen at certain instants: what do we mean by such statements? When I say that I see this patch of red here at, say, noon, what do I mean? My first meaning is, perhaps, that I have taken out my watch, looked at it, and have seen both the hands

pointing at the figure XII, and that I have experienced this as simultaneous with my experience of the patch of red. But if I look into the matter more thoroughly, I find that this is not all I mean to assert when I say that I see this patch of red at noon. The time during which I see the hands of my watch in a certain position is always of a duration not zero. If the watch had suddenly passed out of existence while I was looking at it, I should still have continued to have seen it for a fraction of a second, during which noon would have passed beyond recall. Now, I can approach more nearly to a precise formulation of what I meant by the proposition that I saw the patch of red at noon if I name still other events which were experienced as simultaneous with the position of the hands of the watch, but which did not endure in experience for the whole period that the hands of the watch were experienced to remain in their position. For example, I can say, perhaps, that this patch was not only experienced as simultaneous with the position of the hands indicating noon, but that the experience of their indicating 11.59′59½″ had not yet died out while I saw the patch. By noting more and more events, each experienced as simultaneous with the patch, and each experienced as simultaneous with each other (for they all, we should say, are experienced as being at noon), we can finally arrive at the specification of a given instant without duration at which the patch was seen, though all the events used in fixing this instant may have consumed time, and have been possessed of all the other gross properties characteristic of *experienced* events, and the relation of simultaneity among them may have been given in experience.

Perhaps I can best illustrate our method of determining noon by a diagram.


Line representing noon.
Direction of time from earlier to later.
Patch of red experienced.
Hands of watch experienced as at noon.
Hands experienced as at 11.59′59½″
Other events by which noon is more completely specified.

We are able to continue this process further and further by the adjunction of more and more events to the set by which we determine noon. Every such event will be experienced as simultaneous with every other. Finally we shall come to a stage where no more new terms can be found which we can adjoin to our set—that is, there will be no other events which will be experienced as simultaneous with *all* the members of our set. In such a case, we shall have given as complete a determination of noon as is possible on the basis of experiences. The patch of red, which we wish to say is seen at noon, is one of these.

But what *is* noon, which seems to form the subject-matter of the proposition, "This patch of red is seen at noon," which we have been considering? We wish to interpret noon as a sort of tabulation of experience: the answer we give shall therefore read, "Noon is the whole class of events, each of which is experienced as simultaneous with each other, which contains every event experienced as

simultaneous with all its constituent events, by means of which we have dated our patch of red." This definition may seem to be circular, for it may seem that the class of events in question could not be specified except with reference to a pre-existing notion of noon. This criticism is, however, invalid, for it may be shown that if instants and events have the formal properties and interrelations that we universally attribute to them, every instant, such as noon, will determine uniquely and be uniquely determined by some set of events of which every two members are experienced as simultaneous, and which is such that it contains every event which is experienced as simultaneous with all its members. The definition of these latter entities involves no circularity, for it depends merely upon a previous acquaintance with events, the relation of simultaneity and a few elementary logical notions such as that of a collection, and not at all upon any specific acquaintance with noon, or with any other instant.

Now, the relation of simultaneity among events, whether as such it can be experienced or not, is certainly far closer to experience than an instant. In all this work, the point we are making is not that the terms and relations with which we start and which we take to represent experience are immediately given—we do not even assume that there are immediately given terms and relations—but that, if I may use such a phrase, they are closer to givenness, that they are less elaborated, that they are the results of a lesser degree of sophistication than the ordinary notions of science. Whether the "experience" with which we started in this lecture is itself already schematized or not does not concern us here; it is enough that space and time mark a degree of schematism greater in intricacy than what we here call experience.

To sum up: we have contrasted the aloofness of geometry from empirical verification with its tremendous value

when applied to experience, and have noted the problem which this situation creates. We have discussed Kant's views on the relation between geometry and experience, and have seen that his statement that geometry deals with space, which is given prior to any concrete experience whatever, is hard to reconcile with our empirical recognition of geometrical forms. We have seen that a view which should hold that geometry, though *a priori*, deals with an empirically known subject-matter, would avoid this particular difficulty. Then, taking up Mach's standpoint, it became clear to us that his view, that geometrical certainty is of experimental and empirical origin, is in direct conflict with the practice of all mankind in matters geometrical, and that we all should hold that any geometrical experiment was a test rather of the instruments of measurement used than of the geometrical theorem involved. We noticed the suggestiveness of Mach's notion of geometry as the science of a schematized experience, but saw that in the study of statistical tables, for instance, certain aspects of the study of a schematized experience may be independent of the matter schematized, and depend only on the form of the schematism. We held geometry to be of a similar nature. We observed that our view lent itself to the formulation, "Geometry is the science of a *form* into which we cast our spatial experiences," but we observed that such a formulation would also cover Kant's view that geometry is the study of the form of the external sense, and the view that geometry is merely concerned with the deduction of geometrical theorems from geometrical axioms, so that the certainty which we usually attribute to geometry is entirely dependent on the fact that it is a science of abstract deduction. Kant's view, we found, differs from our view in the fact that it makes the form of the external sense prior to all experience, and consequently cannot explain the empirical identification of spatial entities, while we hold that the

schematism with which geometry deals is imposed only after and on the basis of concrete experience. The view that geometry is only concerned with a certain deductive chain did not explain why we act as if geometry were the *a priori* study of a certain concrete system which we can apply to experience as a criterion of straightness or of circularity or of any similar geometrical property. We saw that geometrical propositions, though they seem to deal with such entities as points, lines, etc., are mere paraphrases for propositions about experience in some more direct sense. We finally gave an example of the sort of schematization or tabulation of which geometry makes use, taking this example from *Our Knowledge of the External World as a Field for Scientific Method in Philosophy*, by Mr. Bertrand Russell.

The remaining lectures will be devoted to a more or less tentative discussion of the details of the methods of tabulation and schematization used in geometry. They will very often involve the use of simple geometrical reasoning, but only to prove that the methods of tabulation we here employ yield results similar to those yielded by the methods of schematization which we must tactily use in building up the entities of our every-day geometry.

## II. The Point as a Tabulation of Solids

In our last lecture we put forward the view that geometry is concerned with the study of a certain tabulation or arrangement of experience, in so far as this arrangement is determined, not by the nature of the material arranged, but by some already fixed principle of arrangement. As an example of what such a tabulation or arrangement would be like, we gave a brief discussion of the definition of instants as arrangements of simultaneous events. In this and the following lectures we shall attempt, in a similar manner, to define the subject-matter of every-day geometry as a system of tabulations of things which can be experienced and their relations—that is, to exhibit the methods of schematism employed in geometry. In our last lecture we made the further claim that the ordinary theorems of Euclidean geometry could be regarded as consequences solely of the methods of tabulation and arrangement employed in geometry. In the ensuing portion of this course we shall try to show, as far as we are able, how points and the relations between points may be so defined as complexes of objects which can be experienced and of their empirically knowable relations that, though space will be dependent on sense, the geometrical properties of space will be independent of sense, and will follow solely from the schematism by which space is obtained from sense. We shall aim to show that, just as a cube does not depend for its cubical properties on the material from which it is made, just as a wooden, a stone and an iron cube all have eight apices, twelve edges and six faces, so a geometry, although its propositions may have relevance to the actual world in which we live, has a validity independent of the particular

nature of the world to which it applies. We maintain that geometry has this universal validity, not only in the sense in which it says that *if* any system satisfies a certain set of premises geometry is applicable to it, but also in the sense in which it asserts categorically that geometrical theorems must apply to the entities which we define as points, whatever the concrete nature of the world in terms of which they are defined may be.

The first task which we have before us is the determination of the fundamental spatial experiences in terms of which our subsequent schematizations and definitions are to be made. The first essential condition which these fundamental experiences must satisfy is that they should be genuine experiences. This excludes at once the possibility that they should deal with such essentially non-experienceable entities as points without magnitude or curves without thickness and so forth. It demands that the fundamental spatial experiences should concern such things as visible patches of color or tangible solids. This necessary condition which these experiences must satisfy leaves us still a great possibility of ambiguity as to their nature. As we are humanly unable to do what is perhaps the most natural thing in this situation and make our method of schematization apply to all experiences which we should ordinarily claim to have a spatial import, on account of the immense technical difficulties such a task would involve, we are obliged to introduce a certain degree of arbitrariness and artificiality into the selection of the fundamental experiences from which we shall build up our geometry. Whether the experience of the solid be primitive in experience or not, this much is certain, that it belongs to a much lower stratum of schematization and synthesis of experience than such unextended things as points, lines and other geometrical entities, and that things or solids are the last word in primitiveness and immediate givenness

for the man unsophisticated by psychology. Since our discussion in these lectures is only tentative anyway, and since solids offer a very convenient starting-point for the development of a schematism leading to geometry, we shall regard our primitive spatial experience as one dealing with solids. We have not yet completely specified the nature of our primitive spatial experience, however, as it is possible that there are many different kinds of facts concerning solids which can be experienced. One of the simplest to handle—although possibly not one of the simplest in the order of experience—of these facts is the fact that a certain solid is observed to contain a part in common with another solid. We shall, therefore, select an experience of the intersection or overlapping of two solids as the fundamental spatial experience. Two solids, we shall say, are experienced as intersecting or overlapping or having a part in common with one another if they both seem to contain some solid or if one seems to contain the other or if they seem to come into contact.

The experience of the overlapping of solids is not, however, as it stands, a sufficient point of departure for a schematization which is to lead to geometry. We wish to be able to define a straight line as a sort of a tabulation of solids. Now, if all we know about solids is the relations of overlapping that hold among them, we will be unable to discriminate between a straight line and a tortuous one. The whole of space could be kneaded like a lump of putty without changing a solid into anything else or altering the relations of overlapping which solids bear to one another, but by such a transformation you could deform a straight line into a curve as tortuous as you please. It is obvious, then, that if we are to be able to define straight lines in terms of the experience of the intersection of solids, we must put some kind of a limitation on the kind of solids considered. We shall put upon them the limitation that

they are to be *convex*. Now, a convex solid is one such that any two points which belong to it can be connected by a piece of a line which nowhere passes outside of it. Thus a solid sphere is convex, a cylinder is convex, a cone is convex, and a cube is convex, while a solid in the shape of an hour-glass is not convex, a doughnut is not convex, a bowl is not convex, and no figure which is hollow is convex. As a matter of fact, convexity is synonymous with the absence of hollowness in any sense, and since hollowness can roughly be judged by the eye and the finger without reference to straight lines, convexity may also be determined by a more or less direct reference to experience. We know what it means to say that a bowl is hollow and that a billiard-ball is convex long before we ever think of correlating these properties of solids with the definition of convexity just given. We can, therefore, make our fundamental experience that of the intersection of *convex* solids, and be sure that it is near to genuine experience. Further, it may be shown by a simple bit of geometry that if the world were, say, made of clay, and were so squeezed out of shape that all convex solids and their relations of overlapping should remain unchanged, every straight line would remain straight. Consequently, once the set of all convex solids in space has been identified, the set of all lines in space is determined, and it seems very probable, to say the least, that lines can be defined in terms of convex solids.

The sort of fact from the schematization of which we shall obtain space is, "This convex solid is *experienced* to intersect that one," and not simply, "This convex solid intersects that one." The formal properties of experienced intersection and of actual intersection are probably, however, closely analogous in most respects. Each solid may be regarded as having, outside of its physical extension, a sort of *aura*, of definite extent, such that two solids are experienced as overlapping when, and only when, the solids

formed out of each by adjoining to it its aura actually intersect. For example, two spheres a hundredth of an inch apart may seem to be in contact, as far as our unaided senses can tell: then we shall say that the aura of each extends at least one two-hundredth of an inch beyond its physical extension. The difference between the relation of apparent or experienced intersection among convex solids and that of their actual physical intersection is to all intents and purposes, then, a difference in the solids chosen as intersecting rather than in the formal properties of the relation of intersection itself, for if we replace convex solids by convex solids plus their auræ, we can interpret the apparent intersection of the former as the actual intersection of the latter.

We are now in a position to define our points—that is, to exhibit them as tabulations of convex solids. We shall define our points as collections or aggregates of solids. This may seem curious to many of you. "What!" you may think, "Is not a point small and a solid large? Is not a class of solids even larger than a solid? Then how can a point be a class of solids? How can the part be greater than the whole? How can points be made of solids, as you say, and solids also be made of points, as the mathematician says?" Now, all these questions result from a confusion of the relation of a member of a collection to the collection of which it is a member with the relation of an object filling a given space to an object filling a space including that which the first object fills. One tends to think, for example, that because Harvard University is a class of men, Harvard University fills more space than a single man. But, when one comes to think of this example more thoroughly, one sees that *in the sense in which a man fills a certain space,* it is nonsense to talk about Harvard University as filling any space. It is only in a metonymous sense that Harvard University can be said to fill the space occu-

pied by all its members. Harvard University has only such properties as belong to different logical dimensions from those of its members. In fact, it is a general proposition of logic that no collection can have any properties that can in precisely the same sense be significantly asserted—or denied, for that matter—of any of its members. Thus, Harvard University, although it has certain intimate connections with certain portions of space, cannot be said to occupy any space at all in the sense in which I now occupy, the space vertically above this platform, and in an analogous way, in the sense in which a solid can occupy space, a class of solids cannot occupy space, and in the sense in which a class of solids can occupy space, a solid cannot occupy space. It is, therefore, nonsense to speak of a class of solids as either smaller or larger than a solid. Hence we do not, in defining a point as a class of solids, make the part larger than the whole, for the point and the solid are rendered by such a definition incomparable as to magnitude.

The second paradoxical feature of our definition of a point—that we define a point as a collection of solids, whereas in ordinary geometry, a solid is regarded as a class of points—is eliminated still more easily. A solid, in the sense in which points are classes of solids, is an entirely different thing from a solid, in the sense in which a solid is a class of points. They are no more identical than the collection of clubs to which John Smith belongs is identical with John Smith himself. The only thing that entitles us to call both solids is that the world in which we live is probably so organized that corresponding to each solid in our first sense there is a class of points uniquely determined by it and representing no other solid than it, which we may call "the same solid as it," just as it might be that in some town one could identify every man by the list of clubs to which he belongs, and could say, whenever one should

see a list of clubs to which some man belongs, "That's John Smith," or "That's William Jones," or whoever else it might be.

Our definition of points in terms of solids is to be justified, as are all definitions in this kind of work, by its fruits. We shall so define a point that if the things we call convex solids are really the convex solids of an ordinary Euclidean space, the things we call points will correspond in a certain determinate manner to the points of ordinary Euclidean space; the things we shall later call lines will have all the nice properties that lines should have; and finally, the whole space we shall obtain as the end of our discussion will have all the attributes that pertain to our every-day space. On the basis of this first definition of points and of lines we shall give a second and finally a third definition of points and of lines which will, on the one hand, make each point of the first sort determine a single point of the second or third sort and each line of the first sort determine a line of the second or third sort in such a manner that the geometrical properties of a figure made up of points and lines of the first sort will be substantially unchanged if each point and line of the figure be replaced by the analogous point or line of the second or third sort—which will, I repeat, do all this *if the points and lines of our original system form a set satisfying the axioms of ordinary geometry*, or, to put it in a more elementary manner, if two lines in our first sense have a point in common when and only when two decent and well-behaved lines ought to have a point in common. On the other hand, we shall so frame our definitions of points and of lines of our third kind that, however irregular the formal properties of the points and lines of our first sort may be, however often lines that should intersect, did our original system obey the laws of geometry, fail to intersect, or lines that should fail to intersect do

intersect, our lines and points of the third sort must, so long as logic is logic, have all the properties appertaining to lines and to points in a Euclidean geometry. Furthermore, we shall develop a theory of measurement in this third space that we finally attain which will be consonant, on the one hand, with our usual ideas of the operations performed in actual physical measurement, and which, on the other, will be in perfect harmony with the laws of measurement laid down in ordinary Euclidean geometry. All this is done on the basis of our original definition of a point, and constitutes an ample justification for it.

After this rather long-winded apology for the definition of a point as a class of solids, let us state this definition in precise terms. *A point is a collection of convex solids such that* (1) *any two convex solids belonging to it are experienced as intersecting, and* (2) *if a convex solid is experienced as intersecting EVERY member of such a collection, it can only be itself a member of the collection.* We saw previously that the relation of experienced intersection among convex solids reduces itself to the relation of actual intersection among other solids—namely, those formed out of convex solids by adjoining their auræ to them, or as we shall hereafter call them, *a-solids.* Our definition is therefore practically equivalent to one which should read as follows: a point is a class of a-solids such that (1) any two members of the set intersect, and (2) any a-solid that intersects every member of the class must itself be a member of the class. Now, what does this mean?

Let us consider the class of all the a-solids which, as we should say in our every-day life, contain a given point $x$ on their surface or in their interior. In the first place, every two members of this set intersect, for earlier in this lecture we have taken the term intersection to cover contact or tangency, and two figures with a point in common

either intersect bodily if the point in question lies in the interior of one of them, or come into contact with one another if the point lies on the surface of each of them. In the second place, it may readily be shown that if an a-solid intersects every a-solid that contains $x$, it must itself contain $x$. This proof depends upon the fact that if an a-solid does not contain a given point, another a-solid can be found which contains the point, but does not intersect the first a-solid. Taking this principle for granted—its truth can very easily be established on the hypothesis that the aural layer of a convex solid is of a uniform thickness throughout space, or on many similar hypotheses which do not assume so much—the desired consequence follows in this way: if an a-solid intersects every a-solid that contains $x$, but does not itself contain $x$, we get a contradiction, for by the principle which we have just enunciated, there must be a second a-solid, not intersecting our first a-solid, but containing $x$, while, by hypothesis, this is impossible. Consequently, if an a-solid intersects every a-solid that contains $x$, it must itself contain $x$. We have thus shown that a collection of all the a-solids which, as we should ordinarily put it, contain some point, satisfies both the criteria which a class of a-solids must fulfil to be a point by our definition, since any two members of it intersect, and any a-solid which intersects all its members belongs to it.

To give a completely satisfactory justification of my definition of a point as a class of a-solids whereof any two intersect and which are such that any a-solid intersecting every member of the set belongs to the set, it is not enough to show, as I have just shown, that every collection of all the a-solids containing some point, which may be said to represent or even to be that point, is a point in accordance with our definition; we must also show that no other collections of a-solids are points in accordance with our defini-

tion. We must show that if, on the one hand, a collection of a-solids does not exhaust those which, as we should ordinarily state it, contain some point in common, or if, on the other, there is no point common to all its members, the collection of a-solids in question fails to satisfy one or both of the two criteria which determine whether a given collection of a-solids is or is not a point in accordance with our definition of a point. Now, it is easy enough to show that if all the members of a collection *a* of a-solids contain a given point, but do not exhaust the collection of the a-solids which contain the point, there are other a-solids—i. e., the other a-solids containing the point in question—which do not belong to the collection *a*, but intersect every member of *a*, so that *a* is not a point in accordance with our definition. It is not easy to show, however, that if a collection of a-solids is of such a nature that there is no point, to use ordinary geometrical language, which all its members contain in common, this collection of solids fails to satisfy at least one of the two criteria both of which a collection of a-solids must satisfy if we are to call it a point in accordance with the definition we have given. In fact, I have not yet succeeded in proving this theorem, and I have nowhere seen any proof given for it, yet I am convinced that it is true and that it can be proved. I am convinced of this because, notwithstanding a considerable amount of effort, I have been unable to discover a single collection of a-solids, except the collection of all the a-solids that contain some given point, which satisfies both of the two conditions which all the things that are points by our definition must satisfy. Therefore, notwithstanding the gap in my chain of reasoning, I shall go on from this point as if I had proved that our definition of a point is perfectly adequate, and that the only collections of a-solids which satisfy our definition of a point are such as are made up from all the a-solids which,

as we should naturally put it, contain some point. If we suppose that this is proved, *provided that our experience of the relation of intersection among convex solids is to receive the geometrical interpretation in terms of a-solids which we have given it,* since our first definition of points in terms of the experience of the intersection of convex solids will then be practically equivalent to our second definition of a point in terms of a-solids, our points in our first sense, though defined in terms of an experience, will well deserve the name of points.

Our next task is to define what is perhaps the next most fundamental notion in geometry—the notion of a line—in terms of our experience of the intersection of convex solids. It will be remembered that convex solids stand in a very close relation to straight lines, for a convex solid is one that contains the whole of a bit of any straight line whose ends lie inside it. Now, this fact enables us to define a bit of a straight line in terms of our experience of the intersection of convex solids as follows. We have just seen how a point may be regarded as a class of a-solids which is what we should ordinarily call the class of all the a-solids containing that point. An assumption which we shall make at this point is that all a-solids are convex and that we can thus regard a point as a class of all of a certain kind of convex solids which contain a given point. This assumption is extremely natural. It is a consequence of the other assumption which we suggested previously, to the effect that the aural layer of a convex solid is of uniform thickness throughout space, but does not presuppose the latter assumption. From the hypothesis we have stated we can readily draw the conclusion that if $a$ and $b$ are any two points *qua* classes of a-solids, then every a-solid which forms a member both of $a$ and of $b$ contains, in ordinary geometrical phraseology, the whole piece of a straight line intercepted between $a$ and $b$. That this is true follows from

the fact that *a* and *b* are points inside any a-solid which belongs to them both, since a member of a point is an a-solid which contains it. Consequently, since an a-solid is convex, any a-solid which belongs both to *a* and to *b* contains the whole linear segment or bit of line between them, and consequently every point on this segment. Therefore, every point on this segment possesses as a member any a-solid within which *a* and *b* lie. This is another application of the principle that the members of a point are the a-solids which spatially contain it. It is thus a necessary condition if *c* is to lie on the linear segment between *a* and *b* that all those a-solids which belong both to *a* and to *b* should also belong to *c*. That this condition is also sufficient may be proved on the hypotheses that the thickness of the aural layer of all a-solids is constant and that an a-solid can be transported to any part of space, and yet remain an a-solid. Both these hypotheses are very probably true—at least within that part of space whereof we have any experience at all. The deduction of the sufficiency of our condition from these hypotheses, though easy, is a little too intricate for us to give here.

We have, then, given a necessary and sufficient condition that one point, *qua* class of a-solids, should lie on the bit of line between two other points of the sort. Let us reinterpret this statement in terms of points consisting, not of a-solids, but of general convex solids. If three points, *a*, *b* and *c*, consisting of a-solids, are so arranged that *c* lies on the linear segment between *a* and *b*, and if *a′*, *b′*, and *c′* are, respectively, the points consisting of general convex solids corresponding to *a*, *b* and *c*, then it will be natural for us to say that *c′* is between *a′* and *b′* and on the line determined by them. That is, *c′* will be between *a′* and *b′* when and only when *c* contains all the a-solids common to *a* and to *b*. Now, *a* contains a given a-solid as a member when and only when *a′* contains the convex solid

from which this a-solid is formed by the adjunction of its aura, and a similar relation subsists between $b$ and $b'$, and between $c$ and $c'$. Therefore, $c$ contains all the a-solids common to $a$ and $b$ when and only when $c'$ contains all the convex solids common to $a'$ and to $b'$. Consequently $c'$ lies on the linear segment between $a'$ and $b'$ when and only when $c'$ contains all the members common to $a'$ and to $b'$. Now, we have not yet defined linear segments or any such things, and this property of $a'$, $b'$ and $c'$, when $c'$ contains the common part of $a'$ and $b'$, is defined in purely logical terms introducing only such notions as those of part and class, involving no concrete geometrical notion, except such, of course, as are involved already in the notion of a point, which we have already defined in terms of our experience of the intersection of convex solids. We may therefore *define* a point $c'$ to lie between two others, $a'$ and $b'$, when and only when $c'$ contains the common portion of $a'$ and $b'$, and we shall be sure, on the one hand, that if our experience of the intersection of convex solids has the properties that are to be expected of it, this relation of betweenness will not have been misnamed, and, on the other, that this definition involves no notions other than that of our experience of the intersection of convex solids and certain general logical notions.

I wish now to define the notions of segment, end-point and line, in terms of the relation of betweenness just defined, and hence ultimately in terms of our experience of the intersection of convex solids. If $a$ and $b$ are distinct points, the class of all the points $c$ which are such that $c$ is between $a$ and $b$ constitutes the linear segment $ab$, and $a$ and $b$ are its end-points. The *line* $ab$ is the class of all points belonging to linear segments which have at least two points in common with the linear segment $ab$. The agreement of all of these notions with the conventional

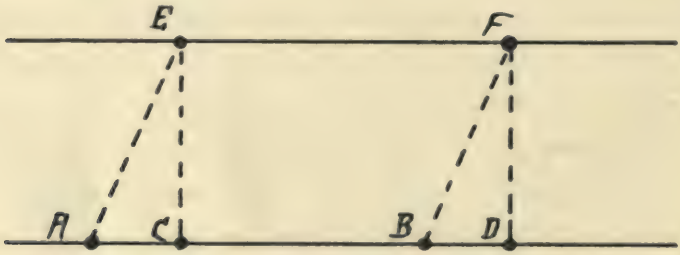
notions of segments, end-points and lines, subject to a certain reservation which we shall make in the next two lectures, will be obvious on a brief reflection. The adequacy of our definition of a line will be apparent if we reflect that any two linear segments which have two points in common are segments on the same line, while if $x$ is any point on a line $l$, and $s$ is any segment on $l$. a segment $t$ can be discovered which will contain $x$ and have at least two points in common with $s$.

To sum up what we have said in this lecture, we first defined a point as a class of convex solids, whereof any two are experienced to intersect, and which is further such that it contains as members all those convex solids which are experienced as intersecting all its members. We justified this definition of a point and showed that the entities which are thus defined as points are such things as one could naturally call points, providing that our experience of the intersection of convex solids has such formal properties as one would naturally attribute to it, since under this hypothesis each of our points will be a collection of all the convex solids which are experienced as containing some point, and may, since the notion of a point is only now defined for the first time, be identified with the latter point, which they are experienced as containing. We have defined a point $a$ as between a point $b$ and a point $c$ when $a$ contains the common part of $b$ and $c$. From this definition alone we have derived definitions of a linear segment, of the end-points of a linear segment, and of a line. All these definitions have been made solely in terms of the experience which we have chosen as fundamental—that of the intersection of convex solids.

The work in this lecture is based on that of Dr. A. N. Whitehead and Mr. Bertrand Russell on space and time, as given in Mr. Russell's *Scientific Method in Philosophy*, Chapter IV. The definitions of betweenness and of a line are borrowed from Prof. Huntington's article in the *Mathematische Annalen* for 1912, but go back to the work of Kempe and Prof. Royce.

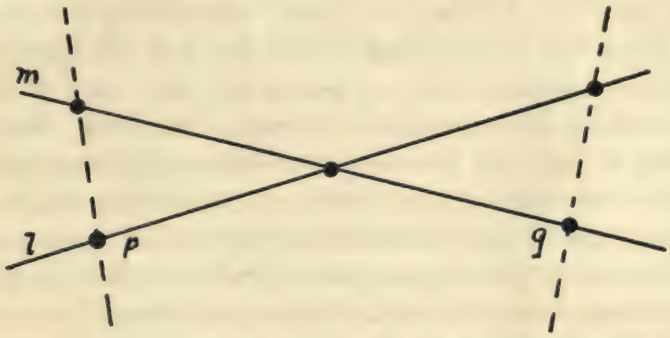
## III. The Extension of Space Beyond the Bounds of Experience

In our last lecture, you will remember, we arrived at the definition of a point as a class of convex solids, and of a line as a class of points, in terms of our experience of the intersection of convex solids. These definitions, however, and, indeed, any definitions that start directly from our experience of the intersection of convex solids, must suffer from certain rather obvious defects. We intend to use our definitions of lines and of points to set up a theory of spatial measurement. To do this, we shall make much use of the construction of parallelograms: for example, we shall define the distance AB on a given line as equal to the distance CD on the same line if it is possible to construct a linear segment or piece of a straight line EF parallel to AB in such a manner that AE is parallel to BF and EC is parallel to FD. The following diagram will represent such a situation.




This demands that we are in possession of a definition of parallelism. In Euclidean geometry, to say that two lines are parallel is equivalent to saying that they lie in the same plane and do not intersect one another. We may

define two lines as being in the same plane if and only if they both have a point in common with each of a pair of intersecting lines $l$ and $m$, and do not pass through the point of intersection of $l$ and $m$; thus in the following diagram, $p$ and $q$ are in the same plane, or, as mathematicians say, are coplanar.




We might, therefore, define two lines as parallel if they are coplanar and do not intersect, without introducing any new fundamental notions into our system. But, as we have said, there are certain defects inherent in the definitions of lines and of planes that we have already given and these defects make such a definition of parallelism undesirable. These defects are due to the fact that our experience of the intersection of convex solids does not record the intersection of convex solids at the uttermost confines of space; beyond a certain extreme distance, whatever it may be, the intersection of convex solids is not experienced. As a matter of fact, I can hardly be said to have any experience of the intersection of convex solids except in the immediate neighborhood of my own body. As a consequence, any lines which would naturally be said to meet at a point lying outside the very limited region within which convex solids are experienced to intersect would, in accordance with the definition which we are considering, be parallel lines, for, by our definition of a point, a point is a collec-

tion of convex solids of which every two are experienced as intersecting, and which contains every convex solid experienced as intersecting all its members, and consequently there can be no points outside the region of the apparent intersection of convex solids, so that any two coplanar lines which fail to have a point in common within this region must fail to have any point whatsoever in common, and must, by the definition which we are considering, be parallel. Indeed, since our points are only such points as lie within a certain region of space, the lines which we defined in our last lecture are merely such parts of lines as lie within this region. Not only would this whole condition of affairs not be consonant with our natural notions of lines and of parallelism, but it would further fail to give parallelism even the most important formal properties which it possesses in ordinary geometry. For instance, it would be impossible to use the parallelogram construction as a criterion of the equality of two different linear segments on the same straight line, as a very simple geometrical construction, for it may easily be shown that our definition will make all distances along a given line equal. Hence, our definition of parallelism is at fault, and we must look around for a new one.

In what direction are we to look for this new definition of parallelism? The defect of the definition that we have just rejected—that two coplanar lines without a common point are parallel—is due to the fact that two lines which ought to have a point in common do not always have a point in common, if we define points and lines in the manner indicated in the last chapter. If we are able, then, to introduce new definitions of a point and of a line such that every two lines which ought, as ordinary, common-sense, decently-behaved lines, to have a point in common, will have a point in common, we shall have brought the problem of finding a definition of the paral-

lelism of two lines a great deal nearer solution. This new definition of a point and this new definition of a line should, if possible, be made in terms of our previous definitions of a point and a line alone, without introducing any new notions. The question we now ask is, can new definitions of a point and of a line, which make two lines contain points in common just when they ought to, be made in terms of our previous definitions of a point and of a line?

Now, we have seen that our previous definition of points gives us only the points within a certain region of space. Let us assume, for simplicity's sake—for, though this assumption is almost certainly false, it is not, as such, essential to our further argument, and it enables one to picture in his mind's eye what I have to say much better than any other hypothesis—that our points, in the sense in which points have been already defined, are *all* the points in the interior of some closed convex solid, and that our straight lines are consequently all the segments of straight lines intercepted by the surface of this solid. How are we able to recognize in terms of the points and lines inside this solid the entities, whatever they may be, that we should naturally call points outside of the solid? The problem is closely analogous to that of the recognition that a certain set of astronomical observations from various points on the surface of the earth all are observations of a single star, even though that star is utterly inaccessible to us. The problem to which I allude is not that of recognizing the star as the same star from observations at different *times;* it is the far simpler one of discovering that many observations made at the same time pertain to a single object. Let us suppose, for example, that we have four observers, $a$, $b$, $c$, and $d$, all looking at a star or planet $x$ from different points on the surface of the earth. How

do the observers *a*, *b*, *c* and *d* know that it is a single star or planet at which they are looking?

The whole and complete answer to this question would involve considerations which are irrelevant here; it is obvious that one of the things that our four astronomers must know, however, is that they are all looking at the same *place*—that, in other words the axes of their telescopes converge on one point, or if the object at which they are all looking is sufficiently far away to be considered, for all practical optical purposes, as at an infinite distance, they must know that the axes of their telescopes point, to all intents, in one direction, or to put it otherwise, that they are all parallel. This knowledge, moreover, must be attained and is attained independently of any direct knowledge the astronomers have concerning the point to which all the axes of their telescopes converge, for this point is exceedingly remote from them, and is known by them in no other way than by these very observations concerning which we are now trying to find out why it is that the astronomers regard them as observations of a single point. When the astronomer says that at such-and-such an instant this point of space has this or that property—as for example that of being occupied by a planet—all that he means or has a right to assert must concern the observations in which the telescope is directed towards this point, for if the observations should remain the same, but the whole remainder of the universe should be changed in any manner whatsoever, the astronomer would still be entitled to make the same assertion concerning this point as formerly. The knowledge of the convergence to $x$ of the optical axes of the telescopes at *a*, *b*, *c*, and *d* is attained by a measurement of the angles which the lines between *a*, *b*, *c*, *d*, and $x$ make with one another and a measurement of the distances of *a*, *b*, *c*, and *d* from one another. These observations do not require any direct knowledge of $x$, but only of the posi-

tions of the telescopes at *a*, *b*, *c*, and *d*, for if we know the latitude and longitude of *a*, *b*, *c*, and *d*, if we know the compass-bearing of each telescope—that is, whether it is pointing east or west or southeast or north-northeast-by-north, etc.,—and if we know the slope of each of the telescopes, we know all the angles which any two lines connecting two of the points *a*, *b*, *c*, *d*, and *x* make with one another, and the mutual distances of the points *a*, *b*, *c*, and *d*. What we realy talk about, then, when we discuss the position of the planet is the aggregate of the positions of the telescopes by which it is observed.

Another thing to notice is that if we know that the telescopes at *a*, *b*, and *c* are all directed to one point, and that the telescopes at *b*, *c*, and *d*, are all directed to one point, we know that the telescopes at *a*, *b*, *c*, and *d* are all directed to one and the same point. This is rendered obvious by a simple diagram. The significance of this fact will appear if we consider that if the telescopes at *a* and at *b* are directed at one point, and the telescopes at *b* and at *c* are both directed at one point, all three telescopes need not be directed at any single point. This also is shown by a diagram.

We are now able to return to the discussion of our real problem—the problem, namely, how we are to recognize points outside that convex region of space within which all the points that we have already defined are located in terms of the points and portions of lines lying inside this region. The portions of lines lying inside our convex region—i. e., the class of all the lines that we defined in our last lecture—are the exact analogues of the telescopes or the astronomers whom we have just discussed. Just as the astronomer's statements about the position of a star really concern the positions of certain telescopes, so propositions which seem to deal with points lying beyond the bounds of our experience really concern certain collec-

tions of our lines: namely, with such as are made up of all those lines that "point at" some point lying outside our region. As far as we are concerned, such collections of lines, since they correspond uniquely to the points at which they are directed or from which they spread out, may be regarded as *constituting* these points. This situation can easily be rendered obvious by a diagram.

This definition gives rise to many problems. In the first place, how is it possible to get along in a system in which some points—those within the region of space directly accessible to experience—are the elements of which lines are classes, while other points in space ZZ those not in that region directly accessible to experience—are classes of these self-same lines? In the second place, is it possible to define the property which a class of lines has when every member of the class is directed towards some given point beyond the bounds of experience in terms of that experience which we have taken as primitive—in terms, namely, of our experience of the intersection of convex solids—without the introduction of any new experience or concept not derivable from that experience of intersection? If such a definition is possible, how are we to proceed to discover it? In the third place, how are we to tell when three or more of our new points are situated on a single line, and how are we to define such a line? These three problems will form the chief subject-matter of the remainder of this lecture and of the following lecture.

Let us take them up in the order just indicated. How, we asked, is it possible to get along in a system in which some points—those within the region directly accessible to our experience—are the elements of which lines are made up as classes, while the remainder of the points of space are classes of lines? The answer is—it is not possible, and we do not intend to try to do so in this paper. Not only is it highly inconvenient and unnatural for one

point in a system of geometry to be an aggregate of aggregates of other points, but there are good philosophical reasons—indicated by Mr. Russell in that part of the *Principia Mathematica* which deals with the Theory of Types, but too complicated and foreign to the subject-matter of this course of lectures for us to discuss here—there are good philosophical reasons, I say, for holding that no assertion which can be made significantly concerning a given entity, say $x$, can also be made significantly concerning a collection of collections which has some member of which $x$ is in turn a member. Therefore, since a line, in the sense defined in our last lecture, is a class of the points which we then defined, it is impossible for one to assert any proposition significantly concerning these points, on the one hand, and also concerning the classes of lines that we intend in the future to call points, on the other. Now it is, to say the least, extremely awkward to have to phrase every proposition that concerns itself with points in one manner when it concerns itself with the points inside a given region and in an entirely different manner when it deals with the points outside this region. We shall consequently define the points inside the region directly accessible to experience as well as those outside it as classes of the lines that we defined in our last lecture, and we shall term all points *qua* classes of straight lines *generalized points,* in order that we may not confuse them with the points defined in our last lecture. Just as we agreed to regard each of the generalized points lying beyond the bounds of our direct experience as the class of all the lines which, as we should say in every-day language, are directed towards the point, so we shall agree to regard those generalized points lying within the region directly accessible to our experience as a class of all the lines which we should usually consider to pass through some point inside this region. If such a class of lines happens to be the class of all the lines which con-

tain in common a point, in the sense defined in our last lecture, then if $x$ be the point which they all have in common and $a$ be the class of lines, we shall say that $a$ is the generalized point corresponding to $x$, but it must be clearly understood that *$a$ is not $x$*.

We are now in a position to deal with our second question: is it possible to define the property which a class of lines has when every member of it, as we should usually put it, is directed towards a given point, in terms of our experience of the intersection of convex solids? It should be noticed that this is the crucial question of this entire lecture, and that our reduction of generalized points to classes of lines having some property which we should usually call "passing through a given point," but which, as a matter of fact, we wish to define without reference to any point through which the lines are supposed to pass, is in the unpleasant situation of Mahomet's coffin until we find a way of identifying this property. In the analogous instance of the astronomers and the star or planet, a collection of telescopes all pointing at a certain point in space is, as we saw, distinguished from a collection of telescopes not all pointing at any one point in space by the fact that when all the telescopes are directed towards a single point certain trigonometrical formulae connecting the latitudes, longitudes, geographical directions, and slopes of the several telescopes hold good which do not hold good when the telescopes are not all aimed at any one point. Such a method of determining whether or not all the lines of a given collection are directed to a single point is inapplicable to the case where we are to define the generalized points of space solely in terms of its points and lines in the sense of our last lecture, for we have as yet no definition of an angle or of a distance nor of a slope: all that we have defined up to this point is the set of all the points and linear segments that lie within a given region. Our problem

hence reduces itself to that of the determination of such classes of lines as are made up from all the lines that pass through a given region—the region, namely, within which we experience the intersection of convex solids—and some chosen point inside or outside of this region, in terms of the intersection-relations of those portions of lines lying inside the region.

We can, however, narrow our problem still further, and indicate the method of its solution with still greater definiteness, if we remember a certain fact about straight lines which we pointed out when we were discussing the case where several observers are looking at a single star. It will be remembered that we showed that if the axes of the telescopes *a, b,* and *c* converge to a single point and the axes of the telescopes *b, c,* and *d* likewise converge to a single point then the axes of all the four telescopes *a, b, c,* and *d* all converge to the *same* point. As we may easily show by a diagram, we may generalize this statement and say that, given any collection of telescopes, if there are two among them, say *a* and *b,* such that if *x* be any member of the collection of telescopes the axes of *a, b,* and *x* all converge to one point, then all the axes of the telescopes of the collection converge to a single point. The converse of this statement is even more obviously true. We can thus define a collection of telescopes as one, the axes of all of whose members converge to some one point provided that it contains two members the axes of which intersect and that the collection is made up of all telescopes which form, taken together with these two, a triad of telescopes whose axes converge to a single point, and in addition of these two telescopes themselves. If, that is, we are in possession of a criterion of the convergence of a triad of telescope-axes, we are able to define the collections of all telescope-axes converging to some point or other. In exactly the same manner we are able to define certain classes of the

lines we discussed in our last lecture as classes of all the lines which, we should ordinarily say, pass through some point or other, whether that point is or is not within that region which is directly accessible to our experience, or in other words, as generalized points. If we have a criterion which enables us to discover when any three lines converge to any point whatsoever in space, for the property of line-triads which reads, "If two line-triads each of which is made up of three lines converging to a point possess two lines in common, all four lines making up the two triads pass through some single point," is not confined in its application to the axes of telescopes but applies equally well to all kinds of lines. Therefore, if we are already in the possession of a definition of a convergent triad of lines, we may define a generalized point as the class of all lines, in the sense in which we defined lines in our last lecture, which either are one of two given lines, say $l$ and $m$, or form together with $l$ and $m$ triads in which the three members of the triad stand to one another in the relation which is ordinarily denominated 'all passing through the same point,' provided only that $l$ and $m$ are distinct intersecting lines—that is, distinct lines which form two of the members of some triad of lines which all would naturally be said to pass through some point. If, then, we are able to give a definition of the relation among three portions of lines lying inside a given region of space which we should naturally call, that of all being directed towards some one point and which the mathematician terms the relation of concurrence, which involves only such notions as we can define in terms of the points and lines of our last lecture, we are in a position to define the class of all generalized points in space, wherever they may be situated. One of the notions which it is permitted for us to use in the definition of the concurrence of three lines is that of the relation which two of the lines of our last lecture bear to one another

when they possess in common one of the points of our last lecture as a member, for this notion can be defined in terms of points, lines, and notions of pure logic alone, and consequently ultimately in terms of the experience of the intersection of convex solids and of in addition only such notions as belong to pure logic and not to concrete experience.

This demand will be satisfied if we give an adequate definition of the concurrence of any three lines which do not all lie in a single plane, for we can define in terms of this relation the concurrence of any three lines whatever, whether they are concurrent or not, in the following manner: the lines *a, b,* and *c* are said to be concurrent if *d* and *e* are two lines such that each of the three lines *a, b,* and *c* forms with *d* and *e* a triad whose members are concurrent *and do not lie in the same plane.* Now, it is an extremely easy task to give a definition of a relation between three lines not all in the same plane, which, though it is slightly more general than the relation of concurrence, includes the latter as a special case, and is only *slightly* more general than it. This new relation is that which holds among three coplanar lines when they are either all concurrent or all parallel. We shall introduce into the definition of this no notion which we have not already defined in terms of the experience that we have taken as fundamental—the experience, namely, of the intersection of convex solids. We can define a plane, readily enough, as the class of all those points, in the sense in which we have already defined points, which lie on any line which has two distinct points in common with some given pair of lines that themselves have a point in common, but do not coincide. As the lines that we have already defined really represent those segments of the lines of ordinary geometry intercepted by the surface of that region of space within which convex solids appear to intersect, our planes, as they are now defined, will actu-

ally represent the planar areas intercepted by the surface of this region, provided that it is possible to draw from any point of such an area a line cutting any two given linear segments intercepted by the surface of the area in two distinct points. That this is possible we may readily show to be the case under the hypothesis, which we have every reason to believe satisfied, that the region of space accessible to our experience is convex. Now, it is a familiar theorem of elementary solid geometry that if $p$, $q$, and $r$ be any three distinct planes of which no two are parallel and which do not all possess any line in common, then the intersection of $p$ and $q$, the intersection of $q$ and $r$, and the intersection of $r$ and $p$ will form a triad of concurrent or parallel lines. The proof of this theorem is simple, and the situation it represents is illustrated by the corner of a room, where the walls and floor represent $p$, $q$, and $r$, and the three edges of the room that meet at the corner are the lines of intersection of pairs of the planes $p$, $q$, and $r$. The case where the three lines are parallel is represented by the three faces and the three edges of a triangular prism. From these examples, it is further easy to guess the truth of the converse theorem of that which we have just stated: three lines not all in one plane are concurrent or parallel *only* when they are the three lines of intersection of pairs of the planes belonging to a certain triad. If we apply these theorems to the lines of our last lecture, under the hypothesis that these represent the linear segments intercepted by the surface of a certain convex region, we shall obtain the result that three of the lines of our last lecture that do not all lie in one plane are concurrent, wherever the point of their concurrence may be situated, or parallel *when and only when they are coplanar by pairs.* Since the relation of coplanarity among the lines of our last lecture has been already defined and the concurrence or parallelism of three lines not all coplanar has not yet been defined in terms of

our experience of the intersection of convex solids, we may regard the equivalence expressed in our last sentence as a *definition* of the concurrence or parallelism of three lines not all coplanar.

We may now go on and say that three of our lines, $l$, $m$, and $n$ are concurrent or parallel, whether they are all coplanar or not, when and only when they are all distinct and there are two of our lines, $a$ and $b$, let us say, which are such that $a$, $b$, and $l$, $a$, $b$, and $m$, and $a$, $b$, and $n$ form three triads, respectively, each made up of three concurrent or parallel lines that are not all coplanar, in the manner that we just defined in the last paragraph. This definition *resembles* the definition of the concurrence of three lines, whether they are all in the same plane or not, which we suggested earlier in this lecture, but it differs from the latter in that it defines the *concurrence or parallelism* of any three lines, and not their simple concurrence. To prove the adequacy of this definition is a simple matter, and reduces itself to the proof that if $a$ and $b$ intersect, and $l$, $m$, and $n$ each form with $a$ and $b$ a triad of concurrent lines, $l$, $m$, and $n$ are concurrent, and that further if $a$ and $b$ are parallel, and $l$, $m$, and $n$ each form with $a$ and $b$ a triad of parallel lines, then $l$, $m$, and $n$ are all parallel to one another. These two propositions are obvious on inspection. We thus see that our definition of parallel or concurrent triads of lines covers those triads, and only those triads, of the lines of our last lecture that we should naturally call concurrent or parallel triads.

To sum up what we have said in this lecture: we saw that the definitions of our last lecture yield us only those points and linear segments within a certain limited region of space. We found it necessary, therefore, to search for a definition of all the points and lines of space in terms of those lying inside this region, and found our problem analogous to that of the astronomer in the location of a

planet by its parallax. We learned from this example that if we were in possession of a definition of the concurrence of three lines, we could define a point anew as a class of concurrent lines of the sort defined in our last lecture, and thus obtain a system of points extending throughout space. We searched for such a definition of the concurrence of three lines, but found instead a definition of the concurrence or parallelism of three of the lines of our last lecture, involving no concrete notion other than that of our experience of the intersection of convex solids. The problems that remain before us in the next lecture are first, that of observing what effects the difference between the relation of concurrence, for which we sought, and that of concurrence or parallelism, which we obtained, will involve with respect to our new points and their definition, and secondly, that of the definition of the lines that connect our new points.

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## RELIGION AND THEOLOGY

RELIGION is still confused with theology, although the latter is only one aspect of the religious attitude and explanatory theory. Religion, psychologically speaking, is an attitude. Philosophically or sociologically, it is an explanation, or a justification, of something which the subscriber wishes greatly to have explained or justified. Its social function is that of the control of the world in which its devotees live, or of those parts of their environment with which they are most vitally concerned. These two aspects, the affective attitude and the philosophic explanation, are indispensable to all religious experience and belief and are found together in all religious doctrine and dogma, just as creed and the reverential attitude are always associated. If the thing to be explained is a mystery which has greatly puzzled men, or which they profess not to be able to comprehend by means of the reason, or which they cannot easily justify from current causal and moral philosophies, the explanation and justification appear all the more wonderful to the believers and are greatly prized. In such cases the attitudinal side is highly developed, the believer's emotions often rising to the pitch of adoration or awe. A religion which makes such a strong appeal as to become practically exclusively accepted within the region to which it has spread is likely to be based philosophically on a very great and absorbing mystery, on the one hand, and to make a tremendous appeal to the emotions in the

forms of reverence, gratitude, love and adoration, especially for the personal objects or factors involved in the explanatory theory, on the other. Thus, the great historic religions have usually been concerned with the explanation of the whence and whither of man, a solution of his destiny as cast along the simpler and more cosmic lines, while often neglecting the terrestrial and complicated immediate aspects of his destiny, regarding them merely as incident to the other more cosmical processes. The personalities conceived of as directing this destiny have been the objects of the strongest attitudes of love and hate, gratitude and fear, and other emotions, according as the dispensation of destiny was regarded as favorable or unfavorable to the recipient.

A theological religion is one in which personalities, especially gods and goddesses, play leading roles. The mystery or problem is there, as in all religion, but the explanation is closely tied up with the personalities of the beings who are supposed to dominate the cosmic or terrestrial situation. In fact, since man at first knew neither physics nor chemistry, biology nor psychology and sociology, but explained all phenomena in terms of personalities or personal causation, in the early stages of religion the gods themselves were the explanations. Primitive, and even barbarian and early civilized man did not often seek to go beyond personality into the organization of forces and factors behind these. In fact, he reversed the scientific method of explaining social phenomena, regarding the personality in the situation as the cause of other phenomena rather than as the product of them. Environment was constituted for him practically entirely of personality or quasi-personalities and not of the elaborate complexity of physical, biological and psycho-social factors and combinations of these, as we are now learning to conceive it. All early religions were therefore theologies or pre-theolo-

gies, in which causation and process were thought of as personal or by analogy of personality activity.

Only with the coming of a more objective method of analysis of natural and social forces has religion come to be divested increasingly of the theological and to take on the metaphysical and scientific aspects and forms. The scientist of today is inclined to explain the theological personalities as ideal objectifications of man's desires and sense of the right and proper organization and balance of things, always stated in terms of personal relationship and causation, because he knew no other formula. He clothed his divinities in the perfect forms of those attributes which he himself possessed in imperfect measure. What he should have desired to be, the gods were—omniscent, omnipotent, immortal, sometimes all-good, always all-cunning, omnipresent, with infinite powers of transmutability and of visibility and invisibility at will. They possessed his own emotions and form, or those of the animals he loved, feared, or respected, but always in greater perfection than he had them, or at least in more perfect adaptation to the purposes or ideas which they were supposed to serve. Thus the gods have been regarded as the anthropomorphic creations of idealistic and striving, self-conscious, and more or less socially conscious, man.

As man's analysis of his external world proceeded to the point at which he began to be able to formulate abstract statements of physical forces, to see climatic, geographical and biological factors at work, his tendency to state all phenomena in terms of superior personality manipulation, by analogy of his own immediate experiences, slackened. As the prototypes of physics and chemistry appeared and the concept of natural law developed theological personalities began to fade. A metaphysics began to take the place of theology in the minds of the learned, and this was in turn succeeded by a developing body of scientific

concepts. Causation became increasingly abstract and less and less personal. The gods were reduced, in the thinking of the philosophers, from the role of directors of the cosmical and human social processes to that of being the agents of natural law, which was now supreme.[1] Among the scientists of today there is some danger that theological personalities may become merely mythological, disappearing altogether from their thinking as active causal concepts.[2] The scientific and logical methods of abstraction have apparently triumphed in the world of the intellectuals, if not in the minds of the masses, and physics and chemistry and bio-physics and bio-chemistry and the mechanics of the physico-geographic and the psycho-social environments have become regnant as explanatory concepts instead.

But religion has not ceased to exist with the decline of the gods and the theologies which explain them. Although the popular tendency is to regard religion as exclusively theological and to disregard any non-theological definition of religion, the movement above described, away from personality over to abstract explanation, may be observed in all the traditional religions which have adherents in the western civilized nations. The tendency is particularly to be observed in the various forms of Christianity and Judaism. We have not lost our interest in the same old mysteries of life and mind and matter, the whence and the whither, the origin and the destiny of man; but we have learned to explain these mysteries and to justify the processes—evolutionary or otherwise—more and more in terms of scientific processes and concepts. Also we have become increasingly interested in the present aspects of man's destiny, with the result that religions have become less cosmical and more social at the same time that they

[1] The philosopher Grotius made the statement that God could not act contrary to natural law, but must act in conformity with it.

[2] See Leuba, J. H., *The Belief in God and Immortality.*

have become more scientific. The concept of heaven and the heavenly society becomes less definite and that of a better human society on this earth grows to take its place. Social ethics tends to grow at the expense of ritualistic observance based on a theological past. Some have even prophesied that the great historic religions, subjected to the transforming influence of modern scientific concepts, must either lose their theological character and become humanized and socialized, or perish, giving place to newer religions which embody the explanatory theories and the idealistic devotion to them which conform to our own age.

The evolution of the objects of worship shows very clearly the growth in religion from a theological to a scientific content. There seems to be good psychological argument for Professor Giddings' contention that the first object of worship in nature was not fetisch, spirit or god, but the "Great Dreadful." Early man, just emerging from the prehuman existence, or even throughout the stages of savagery, could not have defined personality in any very tangible terms. He felt it rather than described it. Other men must have seemed rather vague and indeterminate to him, as indeed did his own personality or self. We who are accustomed to rather sophisticated analyses of self, employing as aids to the process a considerable equipment of psychological terminology, comparatively easily distinguish ourselves from others on the sensory side, at least in adulthood. But even we confuse others with ourselves when we undertake to interpret them, constantly reading our own personality traits, attitudes and beliefs into them. Primitive man, possessing few verbal aids to discrimination, must, as the evidence indicates, have had infinitely more difficulty. The very young child suffers from the same limitations in technique, only gradually learning through many trials to distinguish person from person and others from himself. Both the child and the

primitive man fear the dark and those aspects of nature which they have not learned to account for in such terminology and through such processes of analysis as will remove the mystery from them. If the civilized child does not fear the "Great Dreadful," or nature, when he grows up, as does the primitive man, and as he himself fears the dark, it is because he has by that time acquired a method of accounting for things in analytical terminology which the primitive man did not possess, and also because his economic and technic position generally in the world has so greatly improved over that of the primitive man's that nature is to him more beneficent and less voracious and terrifying.

Such a vague characterization of external things, lumping them all more or less together and regarding them as fearful or harm-doing, is a sort of beginning of personality analysis. Closely allied to it is the singling out of certain objects, which are supposed to possess general or specific powers to produce certain results or ends. These objects, in a more sophisticated stage of development, are called fetishes. They are used as means to the control of some aspects of the environment. While such objects have not personality in the sense in which human beings possess it, their powers are evidently interpreted or imputed on the analogy of the observed or imputed powers of human beings. There is no other method of interpreting them known to the primitive man. Furthermore, they are supposed to possess not only powers, but also attitudes, which change from time to time and which can be modified by certain ascertained procedures, in much the same way as human attitudes are ascertained and modified. Such objects-with-power are not yet abstracted into the category of physical objects, that is, divested of personality. That comes with insight and completeness at a later stage. Also, such objects-with-power partake of the same vagueness

and instability from which all personality suffers in the stages before there is a technique for its analysis and a terminology for its description. But such a method of explaining magical or non-human control over the environment does represent a real advance in the definition of personality over the preceding method upon which it is superimposed and which it begins to supersede. It represents a distinct advance in definition and localization of causation and activity traits, even though these may be merely imputed and hypothetical ones. There is less vagueness and generality, more that can be definitely accounted for, in the causal and control process with such an explanation. We can imagine that the primitive man who had reduced his world control process from the "Great Dreadful" to fetishism must have felt more secure and comfortable than the one who had not done so, because he could "put his finger on things," as it were. And one of the functions of religion is to make the world a comfortable place for its inhabitants to live in.

The spirit, as an instrument of magical control over the environment, is but an extension of the principle of the object-with-power. It represents the beginning of the tendency to distinguish between physical or inanimate objects and the animate objects or objects with personality. The spirit tends to associate itself with some inanimate object which it uses as a dwelling place or refuge and which it controls. This concept of a distinction between a sheltering object and the sheltered spirit was undoubtedly helped out by the dream of experience of early man. He believed that his spirit had the power to leave his body, which remained quiescent in the place where it had gone to sleep, and go off about its business. It might go hunting, travel across the mountains, visit the departed dead in the spirit world, engage in warfare, or undertake many other activities. This observed dualism of his own nature might easily be

imputed to other objects also, on the accepted basis of regarding them by analogy with human or personality beings. The whole process of imputation is, of course, quite generally familiar. The spirit of the inanimate object seems to be much more freely dissociated from its "shelter" than that of the human being, possibly because the former is much less active. Also this object may be used as a shelter for many different spirits, although the spirits seem to prefer their regular and habitual dwelling places. Human bodies may also harbor alien spirits, although normally they serve as the homes of one particular spirit.

With this development of the spirit as the embodiment of the personality, the definition of personality is greatly extended in detail and completeness. With the multiplication of spirits, especially of types of spirits, the distinctions of personality become easier to perceive and enumerate. The spirit itself is, of course, at first but poorly developed in personality attributes. It is little more than the object-with-power, the power having become detachable as spirit. But the spirits grow in richness of content as the number of things they can do multiplies. From the spirits in their well-developed stages we make the transition to the gods. A god is a spirit embodied in human or animal form, although not necessarily in flesh and blood. The god usually possesses the power of making itself invisible when it desires, of transmutation into other forms than its habitual one, and of very rapid, if not instantaneous, movement. It also, of course, possesses the power of magic, or of acting by fiat, in a very high degree. The god is the almost perfect embodiment of magical power. Gods, like spirits, are domiciled, at least in the earlier stages of their development. They have their favorite dwelling places and places of dalliance, and in these places they may be found ordinarily. But their range of movement is very great. The god is a much more complete person-

ality than the spirit. By the time the god has developed out of the spirit, human personality analysis and characterization have also developed greatly. They have approached in definiteness and richness somewhat to the condition we have today, although there is, of course, no definite psychological technique for that analysis of which we are capable. This high degree of human personality analysis, carried over by imputation to the spirits, made it possible for a rich galaxy of gods to develop in the image of man.

This is the stage of polytheism in the worship of religious objects, and strictly speaking it is the first full-fledged stage of theology. But in reality there is no essential difference between the stages of religious development in which the gods play a part and those earlier ones in which spirits and objects-with-power are the agencies of magical control, except the differences in degree of personality development. The essential distinguishing characteristic of the theological stage of worship is personality, and there is a degree of personality in all of these types of objects from the "Great Dreadful" to the gods themselves. Only in the later stages of religious development does personality begin to give way before the abstract natural forces which tend to resolve themselves into physics, chemistry and bio-physics and bio-chemistry and the various permutations and functional organizations of these.

The gods of the polytheistic stage are enriched personalizations of the spirits, their content growing with the growth of personality in general. But they are, of course, always personifications, and in the main of two types of objects, the natural physical forces and attributes and of the human emotions and traits. Thus, in the Greek theology, Jupiter is the thunderer, the god of lightening, of rain, of the majesty of the heavens. Vulcan is the personification of the volcano and the underground fires, a black-

smith at his forge, and a worker in metals. Neptune is the god of the sea, of the rolling, galloping waves; Ceres of productiveness; Apollo of the sun in the heavens, the torch bearer of the day. The passions are equally well represented. Venus is goddess of the tender passion, of love and voluptuousness and of beauty, as soft and white as the foam of the sea from which she sprang. Mars, whom even the ancients made the companion of sexual love, is the thundering god of war, cruel, fierce, powerful, delighting in the torch, more effective in the fight than powerful in intellect. The muses, the fates, and numerous other divinities, stand out as the representatives and patrons of the arts, or of some common or crucial experience in life. Early peoples, not having abstract symbols, with which to express their interpretations of human nature and social processes or even of physical nature and processes, symbolized them concretely in personifications, finding as far as possible, analogies in human action and human personality with which to express their ideas and attitudes.

Of course, we must not expect to find the polytheistic pantheons made up of merely simple personalities representing simple traits when we discover them formed at the threshold of history. Nor should we think of each god or goddess as always having his or her own well-marked-off sphere. There are, as a matter of fact, all sorts of syntheses and overlappings, conflicting jurisdictions and multiple and divided personalities, so to speak, among the gods. Diana is at least a two-in-one goddess, representing both the chase and chastity, and also closely allied to that subtle and dangerous goddess, Luna. Minerva is not only the goddess of wisdom, but she is the titular defender of the Acropolis, where she stands Mars-like with sword in hand and her shield upon her arm. Mars is both fighter and master-of-lust. Jupiter seems to have absorbed half

a dozen personalities, more or less. This multiplication of personality within the same divinity probably arose in the main from syntheses and absorptions of different gods into one. Thus, a group of divinities inhabiting the same region or representing different attributes of the same general aspect of nature or human character were apparently consolidated into one powerful deity. This latter procedure was apparently the one which gave rise to the composite character of Jupiter. Perhaps the procedure of synthesis explains the two diverse characteristics of Minerva. In other cases still, gods were imported and assimilated to those they most resembled in the adopted or adopting country. In this way a divinity, at first fairly simple, might become quite complex and even contradictory in his personality. Another source still of this complexity of character of the gods was undoubtedly a natural growth due to the gradual expansion and multiplication of personality traits in the worshippers and the accumulation of legends about the persons of the gods, building into them the traits and values which the stories attributed to them. If the gods were social fictions, equally truly may it be said that they were in large part the product of the habit of men in their leisure moments of indulging in fiction as a method of amusing themselves.

This tendency toward the synthetic fusion or synthesis of the gods proceeds until monotheism is approached, in which, theoretically at least, the valuable elements of personality and attributes of power of all the gods are merged in one supreme and all-powerful and all-wise divinity. No people as a whole ever accomplished this fusion entirely, although it is possible that some sects within a people have attained to the complete concept of monotheism and the worship of a single god. What usually results is the emergence of a powerful and dominating synthetic personality in a class by himself, with a number of subsidiary

tiers in the divine hierarchy, such as the devil (who, indeed, sometimes appears successfully to defy God), the archangels and angels, good and evil spirits, saints, etc. The highest ideal conceptions of the most approved personality values are attributed to this supreme god, in addition to perfection of magical or fiat powers, omniscience, omnipresence, power of invisibility, of annihilating space and time, transmutability, and all the other attributes which belong to the conceptions of magic rather than of science —truly what each individual would like to possess for himself.

One of the greatest aids to the development of monotheism is undoubtedly the achievement of social unity among a people. As they progress from the tribal to the national stage of economic and social development, as their cultures become merged and amalgamated, when they can think largely with a single mind, the diversity and multiplicity of gods tend to disappear and the concept of divinity acquires a unity which is comparable to that of the national mind. The monotheistic conception of divinity having once been established with a fair degree of perfection, it in turn serves as a focus for the drawing together of divergent strands of thought and ideals in the nation. Especially, if the concept of the monotheistic divinity is a dogmatic and coercive one, it is likely to attach itself to all forms of psychic expression and to draw them into close conformity, thus in the end developing a highly self-conscious and characteristic culture. Such was that of the ancient Jews, and such is that of all countries which have a dogmatic national religion, whether it be theological or non-theological. The dominant German religion in the last forty or fifty years was only incidentally theological, but was primarily that of national aggrandizement—a politico-socio-economic religion—and it exhibited as many signs of intolerance and coercion in behalf of its

"Kultur" as did the theological or monotheistic religion of the Jews and other peoples in like stages of development.

Pantheism is essentially depersonalized monotheism. All the ideal attributes of the personal god remains, such as omnipresence, omnipotence, and the rest, but they are reduced to essence instead of being super-human qualities. The god has now passed out of the human or anthropomorphic stage and has become a divinity of principle. Pantheism represents the coming of the metaphysical stage of religion, and since it grows out of the theological stage it carries over some of the theological attributes into the metaphysical stage. Pantheism appeared first among the philosophers, those who had begun to speculate about the nature of matter and force and motion and the qualities of objects and had learned to trace them back of the personality concept. It came when the analysis of physical nature reached the point where fiat and will gave way to principle or the concept of uniformity in causation. This concept of uniformity is the basis of metaphysics and a metaphysical interpretation of the world is the basis of pantheism. This is also the basis of the scientific method, but the metaphysical interpretation falls short of the scientific, because it lodges the cause of phenomena in certain essence qualities in the objects themselves. It subjects motion and force to a sort of mechanical analysis, but leaves the mystery of the theological personality in the thing, although it is transformed in nature within the object and uniformized as among objects. Thus, uniformity and inevitability are substituted for the fiat will and whimsical unaccountability in the physical world. This substitution was inevitable, once the phenomena of the physical world had been sufficiently analyzed to establish the regularity of their occurrence. Metaphysics banishes spirits and gods or merely makes them ornamental or, at most, a great *deus ex machina*, which sets the thing in

motion and then does not interfere with it again. As Grotius said, God cannot act contrary to natural law. So pantheism neglects God and substitutes Natural Law, which now becomes a depersonalized entity or cause, with all the attributes of divinity except personality, and which pervades the universe as an essence inhering in all space and matter. Modern vitalism is a belated form of this metaphysical concession to the theological persistence in the thinking of man about the events surrounding him.

Pantheism began as the religion of the philosophers who had proceeded beyond the theologico-personality concepts of causation in their analysis of physical nature and the universe. For a very long period of time it did not get into the popular thinking at all, for the masses of the people did not advance beyond theological and personality concepts in their causal thinking, at least with respect to the more ultimate phenomena of the world and universe. However, it may be noted that they began to think in quasi-mechanistic terminology regarding purely local and immediate phenomena before they did of the more distant and less easily observable phenomena. But with the spread of physical analysis and of mechanistic concepts, which has become rather marked in recent times, the masses of the people have absorbed enough of this method of thinking to arrive at a sort of metaphysics of matter and of life and to depersonalize their concept of God. The growth of astronomy, as well as of the other physical sciences, has contributed largely to this. The constant searching of space and the breaking up of matter into its elements has left no place for God as person, in any nearby part of the universe, at least. But there may still be room for God, with His old immediacy and other attributes, as essence, pervading all space and matter. The average man does not think out his change of view with so much logical clarity as this statement would seem to imply, but this is about

the sort of conclusions he arrives at, working more or less subconciously and incoherently.[3]

If the pantheism of the theists never did become the universal belief of the masses, much less has the a-theism of the scientific interpretationists in religion. This method of answering the question as to the nature and meaning of the universe was developed primarily in the nineteenth century after the rise of modern physical science. For the mysterious essences in nature and matter were substituted, through a process of scientific analysis and synthesis, categories of mass, motion and energy. These were expressed in definite quantitative mathematical, instead of vague mystical, terms. As a result of this substitution the laws and principles of science were made to replace the old concept of natural law, and science grew at the expense of metaphysics. Since the type of religion always follows and conforms to the method of interpreting phenomena, there now succeeded, in the thinking of the scientific philosophers, to the old pantheism which had grown out of a metaphysical interpretation of phenomena, a scientific interpretation of the universe and of life, couched in terms of scientific law and principle and formulae.

This new scientific interpretation of the universe was given by the theological and metaphysical interpretationists in religion the term "atheism," sometimes varied into "agnostic" and "infidel," to suit the mood or bias of the apologist or critic. Indeed, it was atheistic, since, as LaPlace said to Napoleon concerning his *Celestial Mechanics,* its treatment had no need of the hypothesis of God or the gods. Personality and personal causation in cosmic matters, and in the larger and more abiding aspects of

[3] In testing out my elementary students, mainly freshmen and sophomores, as to their concept of God, I was surprised to find that the majority of them did not think of an anthropomorphic god, at least with regard to form, at all, but had the concept of an all-pervading essence. This is indeed a great change in the popular attitude since the time when the Dorè engravings represented God as a robust, benign patriarch of fifty or sixty in human form.

terrestrial affairs, drop out and mathematical statements of the correlations of matter, motion and energy take their place. The sphere of personality causation is narrowed down to human and social phenomena; and there begin to be those who predict that a further analysis of phenomena and the development of scientific method will remove even these spheres from a pure personality interpretation.[4] Herbert Spencer was one of the first to grasp fully and set forth systematically this new viewpoint of interpretation. Following the publication of his *First Principles* in 1862 many contributions were made in this same field of philosophic interpretation and the controversy between this viewpoint and that of the theologians and the metaphysicians waxed warm for more than half a century. Comte and the Positivists had approached this viewpoint in a more metaphysical and somewhat sentimental religious scheme even before Spencer's essay appeared, and the idea had been gaining volume and clarity since the time of the French enlightenment of the eighteenth century.

The function of religion, whether theological, metaphysical or scientific in its method of interpretation, has always been to control the environment directly or to provide a theory of environmental control. The great historical religions, which have absorbed the effective attention of the masses of mankind during historic times, have endeavored to expand this theory of control and adjustment to cover the whole of space and time and to look upon man and his social organization as the product, although chief terrestrial objective, of this cosmic process. The chief questions asked and for which answers were sought in the nature of the cosmic religious order was the whence and whither of man and his duties relative thereto and to the author or authors of this destiny while

[4] For a discussion of this subject, see Ellwood, *"Objectivism in Sociology," American Journal of Sociology*, XXII, 289-307; and Bernard, *"The Objective Viewpoint in Sociology," Ibid*, XXV, 298-325.

he sojourned on the earth. The different great religions made different answers to these questions, but none was uninterested in them. Before the great historic religions developed, the problems of control were more immediate and local and man's religious interpretations less philosophical. This is the period in which control by what is called magic was dominant, but the method of magic continues still to be, at least in the larger affairs of man—those which seem to be out of his immediate reach and purview—the professed method of control, even in the professedly theological stages of religious development, that is, in the polytheistic and the monotheistic stages, and to a large degree in the pantheistic stage. Magic disappears, or tends to disappear, as the method of control, only as man's analysis and abstraction of the physical forces and processes of nature give him a capacity for refined and progressive adjustment to his environment, which he lacked in the stages of the dominance of magic.

When the system of religious control reaches the stage of development at which it becomes a theory of control instead of merely a technique of control, it serves as a source from which the individual may draw the materials for a philosophy of life for his guidance, although the average individual is likely to be guided in his conduct more by his subconscious valuations and impressions than by his consciously arrived-at determinations. A system of social ethics, at first mainly ritualistic and later more or less rational, grows up within this general system of religious philosophy to take care of the every-day human contacts. As the scientific aspect of religion develops at the expense of the theological or magical, the philosophic explanatory system develops into a correlation of the sciences, and the social ethics is attached more firmly to the sciences contributing to this philosophic system and is transformed into a sort of social philosophy. Also as the

content or objects of attention in religion are transformed from personalities or divinities into scientific principles or formulae and systems, the strong emotional attitude connected with the former is likely to be weakened before the progressive intellectualization of the latter. Dogmatism, so often the accompaniment of theologies, is much mitigated in the scientific religions; change of idea and content is accomplished much more easily, with the result that explanations of environment are brought and kept much more nearly up-to-date.

So noticeable is all this that many people would not speak of religion after the metaphysical stage is reached, at least as a general system of explanatory philosophy, but would prefer to give it simply the name of philosophy or science. To them there does not appear to be enough of the affective element to justify the term religion. But there is always a strong affective attitude towards the content of any general philosophic or scientific explanatory system which is used to orient one's thinking, or action, toward the major problems of life. People are not lacking in devotion to that general synthesis which they often call just "science" and which they use for such guidance. "Science" has its perfervid partisans as well as "religion," or, better, as theological religion or some metaphysical cult. If there is not as strong dogmatism about science it is primarily because the objects of affective attention are abstractions instead of concrete idealized personalities and they are arrived at by intellectual analysis instead of emotional synthesis.

Ex-president Eliot was quoted some years ago as stating that the religion of the future would be the outgrowth of and moralization of modern science. Many people have already developed science into a religion in this way. The age-old question of "whence" is answered for them in the facts of geology and biology, including the data and prin-

ciples of embryology, heredity, etc. They resolve the problem of "whither" by appealing to the facts of chemistry and the related sciences. Their problems of present adjustment to and control of their terrestrial and social life are cared for through the various mental and social sciences. They work up for themselves a synthetic collection of the principles and data in these various sciences which best meet their needs of explanation and adaptation and control and constitute them a more or less absolute category of truth, for which various degrees of reverence are felt and expressed. Some of the ethical culture societies professedly base their principles and teachings upon such an organization of scientific principles. Beyond a few such sporadic manifestations, the "religion of science" has not established a "church," unless the institutions for research and the dissemination of scientific knowledge could be called such. These means for the popularization of scientific truth, especially in its relation to human life and society—and all religion centers around human and social uses, at least in its affective content—will become increasingly common.

When we turn to the secondary religions, those that group themselves around some particular problem of adjustment or control, rather than the great systems of explanatory religious philosophy, we find these same principles and tendencies exemplified. The old belief that human welfare can be fostered by magic is mainly disappearing and the idea that God interferes in the process of events to promote the individual's success here or to prevent disaster elsewhere is also no longer generally held, at least by the better educated people. It may be said, therefore, that the theological interpretation of the secondary human adjustments, the theories of social and community welfare and of individual perfectability, are no longer dominantly theological. They are usually meta-

physical or scientific. As examples of the metaphysical secondary religions may be cited the various "New Thought" theories of individual psychic adjustment and Christian Science, both of which have attained much vogue and even threaten the dominance of the old theological religions. The problem or "mystery" in both these cases is how the individual can achieve an efficient functional adjustment to life. The solution to which the adherents are devoted, which constitutes the body of their religious belief or doctrine, is found in the principles of auto-suggestion and the teachings of Christian Science. The solution is primarily subjective, these religions not being social religions as such are ordinarily understood. They substitute for the old theological subjective adjustment made through prayer a new one of auto-suggestion and a modified form of psycho-therapy. Their explanations and theories are mainly metaphysical, but they utilize much scientific data and technique.

Other secondary religions which have come more fully under the control of scientific method, and which at the same time show a trend more largely from the subjective and individualistic motivation over to the social, are such social programs and cults as woman's suffrage, the conservation of natural resources for the future improvement of civilization, single tax, socialism, anarchism, classicism, cubism, and the like. In each case there is a problem of adjustment or control which seems a vital, perhaps the most vital, problem to those who are attempting to solve it. The proposed solution which they accept constitutes a religious doctrine, although it may be also a more or less approved or exploded theory in social science. Thus, the belief in monarchy or in socialism, to take examples at random, as the proper methods of organizing and controling human society for certain expressed ends becomes a religion in the minds of its convinced and enthusiastic

supporters. If one cares to look for analogies to the great historic religions they are not lacking. In the case of socialism the problem which presents itself to its enthusiasts is the establishment on earth of a social order which will bring justice and happiness to every one. The body of doctrines or beliefs consists of the economico-social theories of state ownership of the general and large scale service utilities, regulated distribution, surplus value, etc. The ultimate goal, which may perhaps be compared to the Christian heaven, is the "Co-operative Commonwealth," which formerly was a true religious slogan among the socialists. The socialist bible, among the orthodox, is or was until recently Karl Marx' *Das Kapital.* Spargo tells of a poor working man who every Sunday walked seven miles, each way, across the city to spend the day with a friend, religiously spelling out the words and sentences of this work, most of which he did not understand.[5] At one time the socialist societies undertook to edit and introduce an extensive Sunday School literature based on socialist works, and communist groups have been accustomed to meet regularly Sunday mornings to read and expound the writings of Lenine and his associates.

What is true of socialism is in greater or less degree true of all the modern issues and programs. They take on a religious significance. Social work becomes to many a true religion to which devout workers consecrate their lives. The Salvation Army is an example of a mixture of the humanitarian motive of salvaging human derelicts and traditional Christian dogma. Here a new secondary social religion is fused with a great historical theological religion. But in the social work of the associated charities and the various welfare and protective leagues the theological element is almost wholly replaced by the principles and technique of social science. No theological doctrines are taught

[5] See Spargo, *The Marx He Knew,* for a good example of the religious attitude in Socialism.

by the secular societies of this type and none is insisted upon in the recipient of relief or protection or guidance. The object is social and individual improvement through a better adjustment of the individual to the social organism. Thus the principles of the social sciences, and of all science in fact, become the theoretical or explanatory content of the new scientific secondary religions, just as they tend to be for the newly evolving great explanatory philosophic system which many men are building up to supplement or replace the historic systems of religious explanatory philosophy.

The affective element may be much stronger in the secondary religions than in the great primary explanatory religious systems, especially when based on modern science. Relationships and concepts are likely to be less abstract in the secondary religions. Control of environment tends to be more directly control of human personality. The human ends, the results for concrete living, for enjoyment and suffering, are likely to be much more easily visible. Thus the enthusiastic advocates of universal suffrage, of eugenics, of Bolshevism, of Americanization, are likely to speak with a zeal and passion to which the more or less academic advocate of the religion of science as the ultimate solution of all human problems is largely foreign. Dogmatism is inseparable from the minor religions, even when they are based on scientific rather than metaphysical and theological interpretations. It is perhaps most marked in the secondary religions based on theological interpretation. Most of the slaughterings and persecutions recorded in the history of Europe as being connected with the exercise of the Christian religion were not the product of contests over abstract theological dogmas or theories in themselves, but took place with reference to the application of these to very concrete problems of social adjustment, such as political and economic rights, theories

of government, or class distinctions. The bloody conflicts came usually over matters within the purview of secondary religions, seeking for a justification in the general theory of Christianity. And so it is today, except that the secondary religions now appeal more largely to scientific data and principles for their justification.

Even a great historical religion may undergo such a change as has been outlined here. The recent growth of Christianity has been from the theological and metaphysical toward the scientific explanatory system in content. It should not be forgotten that Christianity developed out of the old Hebraic religion largely as a social protest movement, or as an application of fundamental religious philosophy to some of the minor religious values of the time, such as the superiority of human functional values to formalism and ritual, the plea for economic and social justice, a struggle against an office-holding hierarchy, for an ethical as against a traditional religion, etc. The founder of the new religion lost his life in pursuing these ends. Thus Christianity was never merely theological dogma in point of content, as indeed no great religion has ever been, but more than most of the great historic religions it justified human and social values by its appeals to theological sanctions, and sometimes approached very closely to an appeal to purely human values or sanctions.

During the late Roman times and in the Middle Ages Christianity absorbed much metaphysical material and had its character largely transformed by it. The first large influx of the metaphysical, at least on the social side, was probably from contact with Stoicism and the Roman legal philosophy. But the greatest accessions of this sort were from the philosophies of Plato and Aristotle, the decrees of the various church councils and the writings and teachings of the church fathers. St. Augustine did a great deal to infuse neo-Platonism into the fabric of Christianity

on its philosophic side, thus laying a metaphysical basis for religious guidance of conduct for those who could assimilate the more abstract ideals. Throughout the Middle Ages the teachings of Aristotle often rivaled the Bible in influencing the writings and teachings of the great church philosophers, such as Saint Thomas Aquinas and Albertus Magnus. Especially on the ethical and political sides of Christianity did Aristotle's philosophic principles and his quasi-social science have great influence. The impress of the metaphyscal philosophy is also pronounced in the decrees of the church councils. Many of the more general questions settled there, as well as those disputed by the schoolmen, were primarily metaphysical in character. There were more disputes and decrees regarding essences, the nature of substances and the qualities of virtues and vices, the concepts of soul and sin, the spiritual jurisdiction of church and priest, and such matters, than there were about the nature and conduct and commandments of the gods. The mediaeval Roman Catholic church and its doctrine had become primarily metaphysical and only secondarily theological. The same fact is to be noted in connection with the Protestant Church and its discussions and decrees. Luther, Melanchton, Henry VIII, Calvin, the English theologians, the council which produced the famous thirty-nine articles, are concerned primarily with metaphysical concepts. God and the heavenly hosts are enveloped with a shroud of metaphysical verbiage and come into the clear light of day no more, except through the words of the popular preachers and the people themselves. The reason for this marked transformation of Christianity from a theological to a metaphysical religion was to be found in the fact that the prevailing methods of thinking had now become metaphysical, the theological concepts having gone into the background.

The present tendency in Christianity, at least among the leaders, is to bring it on over from the metaphysical stage to that of the dominance of the social sciences. Neither the metaphysical nor the theological elements of Christianity have disappeared from it, but probably remain more important than the scientific. This is especially true, if we consult the formal side of the religion. The official doctrines and confessions of faith, the traditional dogmas, the treatises on systematic Christian theology, are made up primarily of metaphysical and theological discussions and statements. But these elements play a constantly diminishing role in the newer Christianity. Its most vital lines of growth are in the direction of a social religious theory and a practical social service, both of which are based on the social sciences. This assertion is well attested by the fact that all the leading denominations of the Christian Church, the Central Conference of American Rabbis, and the Federated Council of the Churches of Christ, a body acting for the leading Protestant denominations have all made advanced declarations on the leading social questions of the day. These declarations embody the foremost pertinent principles of the social sciences as their primary substance and recognize the metaphysical and the theological traditions of the churches only incidentally or by way of courtesy. To be sure, these declarations have not the dignity of formal creeds, but they have much more weight with the intelligent membership than have the traditional creeds.

Also the denominations of the Christian Church have launched out on numerous social service enterprises of a great variety of types, ranging from simple programs of relief, through constructive programs of education and

social reform, to scientific investigations of social conditions as a basis for their other types of social service. These various undertakings, such as the rural survey work of the Presbyterian Church, the constructive information service of the Methodist Church, the investigational work of the Interchurch World Movement in connection with the steel industry and their program for a general national survey of social and religious conditions, are familiar to all students of social affairs. In fact, it is asserted both by friends and opponents of the tendency that the Christian Church of today is moving in the direction of becoming a great social service and instructional agency, abandoning for the most part its theological and metaphysical predilections, or at least relegating them to the category of the aesthetic and ritualistic. Certainly, through its embodiment of the principles of the social and other sciences, the church is coming under the domination of the scientific method and content. Not only does it no longer explain disease as the effect of evil spirits or earthquakes (after John Wesley) as the act of God punishing the wicked, but it expresses its religious values primarily in terms of the findings of the social sciences. The principles of sociology, psychology, economics and political science come to be embodied in the fabric of Christianity as it changes its character.

Some hold that this transformation marks the decay of religion before the growth of science. Such a view suffers from the fallacy that religion is synonomous with theology. There is no likelihood that religion will disappear, for it is the fundamental valuation process which man makes of his world and of his adjustments to it expressing the terminology and technique of thinking of the

time. In fact, many of the leading churchmen of the day firmly believe that the only way in which the great historic religions, including Christianity, can survive is to have their content and method transformed from the old theological and metaphysical concepts to the new scientific ones in which men now think. If such a transformation should not occur the historic religions might be lost, but not religion itself. In its new form it would take shelter with the various humanitarian movements and in social ethics. The traditional religions would remain, if at all, as aesthetic and cultural survivals.

The transformation of Christianity also illustrates one other tendency in religious evolution, that of the great historic religious philosophic explanatory systems to embody the approved secondary religions within their general content. At all times the newly arising secondary adjustment problems—the great problems of human origin and destiny being the primary problems—have been taken over with their accepted solutions into the implicational content, if not into the actual theory and doctrine, of the great religion of the age. Thus Christianity has always adopted or fostered certain social institutions and theories. It has, for example, made itself the champion of the monogamic family, of the state, of private property, and at times of slavery, from the time of the early church fathers to the present. Those secondary religions which are not generally accepted by the people are also rejected by the dominant historic religion, which is conservative. With the development of a more definitely scientific phase of religious content, perhaps we may expect to find all of the social programs sanctioned by the social sciences

approved by the dominant major religion, whether it be Christianity or some new synthetic religion based on science alone and neglecting Christian tradition.[6]

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[6] It is interesting to note that this championship of minor social religions by a dominant religion may cause apparently sound social practices or theories to be repudiated by analogy or through sympathy when a protecting religion comes into disfavor. Thus the recent dislike for Christianity on the part of certain types of radicals has often been extended to include a condemnation of monogamy and much of the accepted fabric of ethics, not because social science condemned them, but because Christianity championed them and was closely identified with them.

# EINSTEIN'S THEORY OF RELATIVITY CONSIDERED FROM THE EPISTEMOLOGICAL STANDPOINT

## I

### Concepts of Measure and Concepts of Things

THE use, which we can make in philosophy, of mathematics," Kant wrote in the year 1763 in the Preface of his *Attempt to introduce the Concept of Negative Magnitudes into Philosophy,* "consists either in the imitation of its methods or in the real application of its propositions to the objects of philosophy. It is not evident that the first has to date been of much use, however much advantage was originally promised from it. The second use, on the contrary, has been so much the more advantageous for the parts of philosophy concerned, which, by the fact that they applied the doctrines of mathematics for their purposes, have raised themselves to a height to which otherwise they could make no claim. These, however, are only doctrines belonging to the theory of nature. . . . As far as metaphysics is concerned, this science, instead of utilizing a few of the concepts or doctrines of mathematics, has rather often armed itself against them and, where it might perhaps have borrowed a sure foundation for its considerations, we see it concerned with making out of the concepts of the mathematician nothing but fine imaginings, which beyond his field have little truth in them. One can easily

decide where the advantage will fall in the conflict of two sciences, of which the one surpasses all others in certainty and clarity, the other of which, however, is only striving to attain certainty and clarity. Metaphysics seeks, *e. g.*, to discover the nature of space and the supreme ground from which its possibility can be understood. Now nothing can be more helpful for this than if one can borrow from somewhere sufficiently proved data to take as a basis for one's consideration. Geometry offers several data, which concern the most general properties of space, *e. g.*, that space does not consist of simple parts; but these are passed by and one sets his trust merely on the ambiguous consciousness of the concept, which is conceived in a wholly abstract fashion. . . . The mathematical consideration of motion in connection with knowledge of space furnishes many data to guide the metaphysical speculations of the times in the track of truth. The celebrated Euler, among others, has given some opportunity for this, but it seems more comfortable to remain with obscure abstractions, which are hard to test, than to enter into connection with a science which possesses only intelligible and obvious insights."

The essay of Euler, to which Kant here refers the metaphysician, is the former's *Réflexions sur l'espace et le temps,* which appeared in the year 1748 among the productions of the Berlin Academy of Science. This essay sets up in fact not only a program for the construction of mechanics but a general program for the epistemology of the natural sciences. It seeks to define the concept of truth of mathematical physics and contrasts it with the concept of truth of the metaphysician. Materially, however, the considerations of Euler rest entirely on the foundations on which Newton had erected the classical system of mechanics. Newton's concepts of absolute space and absolute time are here to be revealed not only as the neces-

sary fundamental concepts of mathematico-physical knowledge of nature, but as true physical realities. To deny these realities on philosophical, on general epistemological grounds means, as Euler explains, to deprive the fundamental laws of dynamics—above all the law of inertia—of any real physical significance. In such an alternative, however, the outcome cannot be questioned: the philosopher must withdraw his suspicions concerning the "possibility" of an absolute space and an absolute time as soon as the reality of both can be shown to be an immediate consequence of the validity of the fundamental laws of motion. What these laws *demand*, also "is"—and it is, it exists in the highest sense and highest degree of objectivity, which is attainable for our knowledge. For before the reality of nature as it is represented in motion and its empirical laws all logical doubt must be silent; it is the business of thought to accept the existence of motion and its fundamental rules instead of attempting to prescribe to nature itself from abstract considerations concerning what can or cannot be conceived.

This demand, however, illuminating as it appears and fruitful as the methodic stimulus of Euler proved in the development of the Kantian problem,[1] becomes problematical when considered from the standpoint of modern physics and epistemology. Kant believed that he possessed in Newton's fundamental work, in the *Philosophiae Naturalis Principia Mathematica*, a fixed code of physical "truth" and believed that he could definitively ground philosophical knowledge on the *"factum"* of mathematical natural science as he here found it; but the relation between philosophy and exact science has since changed fundamentally. Ever more clearly, ever more compellingly do we realize today that the Archimedean point on which he supported himself and from which he undertook to raise the

[1] For more detail concerning Euler and Kant's relation to him, *cf.*, *Erkenntnisproblem* (7), II, 472ff., 698, 703f.

whole system of knowledge, as if by a lever, no longer offers an unconditionally fixed foothold. The *factum* of geometry has lost its unambiguous definiteness; instead of the one geometry of Euclid, we find ourselves facing a plurality of equally justified geometrical systems, which all claim for themselves the same intellectual necessity, and which, as the example of the general theory of relativity seems to show, can rival the system of classical geometry in their applications, in their fruitfulness for physics. And the system of classical mechanics has undergone an even greater transformation, since in modern physics the "mechanical" view of the world has been more and more superseded and replaced by the electro-dynamic view. The laws, which Newton and Euler regarded as the wholly assured and impregnable possession of physical knowledge, those laws in which they believed to be defined the concept of the corporeal world, of matter and motion, in short, of nature itself, appear to us today to be only abstractions by which, at most, we can master a certain region, a definitely limited part of being, and describe it theoretically in a first approximation. And if we turn to contemporary physics with the old philosophical question as to the "essence" of space and time, we receive from it precisely the opposite answer to that which Euler gave the question a hundred and fifty years ago. Newton's concepts of absolute space and absolute time may still count many adherents among the "philosophers," but they seem definitively removed from the methodic and empirical foundations of physics. The general theory of relativity seems herein to be only the ultimate consequence of an intellectual movement, which receives its decisive motives equally from epistemological and physical considerations.

The working together of the two points of view has always come to light with special distinctness at the decisive turning points in the evolution of theoretical

physics. A glance at the history of physics shows that precisely its most weighty and fundamental achievements stand in closest connection with considerations of a general epistemological nature. Galilei's *Dialogues on the Two Systems of the World* are filled with such considerations and his Aristotelian opponents could urge against Galilei that he had devoted more years to the study of philosophy than months to the study of physics. Kepler lays the foundation for his work on the movement of Mars and for his chief work on the harmony of the world in his *Apology for Tycho,* in which he gives a complete methodological account of hypotheses and their various fundamental forms; an account by which he really created the modern concept of physical *theory* and gave it a definite concrete content. Newton also, in the midst of his considerations on the structure of the world, comes back to the most general norms of physical knowledge, to the *regulae philosophandi.* In more recent times, Helmholtz introduces his work, *Uber der Erhaltung der Kraft* (1847), with a consideration of the causal principle as the universal presupposition of all "comprehensibility of nature," and Heinrich Hertz expressly asserts in the preface of his *Prinzipien der Mechanik* (1894), that what is new in the work and what alone he values is 'the order and arrangement of the whole, thus the logical, or, if one will, the philosophical side of the subject."[2] But all these great historical examples of the real inner connection between epistemological problems and physical problems are almost outdone by the way in which this connection has been verified in the foundation of the theory of relativity. Einstein himself—especially in the transition from the special to the general theory of relativity—appeals primarily to an epistemological motive, to which he grants, along with the purely empirical and physical grounds, a decisive

[2] *Cf.* Helmholtz (29, p. 4); H. Hertz (31, p. XXVII).

significance.[3] And even the special theory of relativity is such that its advantage over other explanations, such as Lorentz's hypothesis of contraction, is based not so much on its empirical material as on its pure logical form, not so much on its physical as on its general *systematic* value.[4] In this connection the comparison holds, which Planck has drawn between the theory of relativity and the Copernican cosmological reform. The Copernican view could point, when it appeared, to no single new "fact" by which it was absolutely demanded to the exclusion of all earlier astronomical explanations, but its value and real cogency lay in the fundamental and systematic clarity, which it spread over the whole of the knowledge of nature. In the same way, the theory of relativity, taking its start in a criticism of the concept of time, extends into the field of epistomological problems not merely in its applications and consequences but even in its first beginnings. That the sciences, in particular, mathematics and the exact natural sciences, furnish the criticism of knowledge with its essential material is scarcely questioned after Kant; but here this material is offered to philosophy in a form, which, even of itself, involves a certain epistemological interpretation and treatment.

Thus, the theory of relativity, as opposed to the classical system of mechanics, offers a new scientific problem by which the critical philosophy must be tested anew. If Kant—as Hermann Cohen's works on Kant urged repeatedly and proved from all angles—intended to be the philosophical systematizer of the Newtonian natural science, is not his doctrine necessarily entangled in the fate of the Newtonian physics, and must not all changes in the latter react directly on the form of the fundamental doctrines of the critical philosophy? Or do the doctrines of the Transcendental Aesthetic offer a foundation, which is broad

[3] *Cf.* Einstein (17, p. 8).
[4] See below, Sect., II.

enough and strong enough to bear, along with the structure of the Newtonian mechanics, also that of modern physics? The future development of the criticism of knowledge will depend on the answer to these questions. If it is shown that the modern physical views of space and time lead in the end as far beyond Kant as they do beyond Newton, then the time would have come when, on the basis of Kant's presuppositions, we would have to advance beyond Kant. For the purpose of the *Critique of Pure Reason* was not to ground philosophical knowledge once for all in a fixed dogmatic system of concepts, but to open up for it the "continuous development of a science" in which there can be only relative, not absolute, stopping-points.

Epistemology, however, closely as its own fate is connected with the progress of exact science, must face the problems which are presented to it by the latter, with complete methodic independence. It stands to physics in precisely the relation, in which, according to the Kantian account, the "understanding" stands to experience and nature: it must approach nature "in order to be taught by it: but not in the character of a pupil, who agrees to everything the master likes, but as an appointed judge, who compels the witnesses to answer the questions which he himself proposes." Each answer, which physics imparts concerning the character and the peculiar nature of its fundamental concepts, assumes inevitably for epistemology the form of a question. When, for example, Einstein gives as the essential result of his theory that by it "the last remainder of physical objectivity" is taken from space and time (17, p. 13), this answer of the physicist contains for the epistemologist the precise formulation of his real problem. What are we to understand by the physical objectivity, which is here denied to the concepts of space and time? To the physicist physical objectivity may appear as

a fixed and sure starting-point and as an entirely definite standard of comparison; epistemology must ask that its meaning, that what is to be expressed by it, be exactly defined. For epistemological reflection leads us everywhere to the insight that what the various sciences call the "object" is nothing given in itself, fixed once for all, but that it is first determined by some standpoint of knowledge. According to the changes of this ideal standpoint, there arise for thought various classes and various systems of objects. It is thus always necesary to recognize, in what the individual sciences offer us as their objects and "things," the specific logical conditions on the ground of which they were established. Each science *has* its object only by the fact that it *selects* it from the uniform mass of the given by certain formal concepts, which are peculiar to it. The object of mathematics is different from that of mechanics, the object of abstract mechanics different from that of physics, *etc.*, because there are contained in all these sciences different questions of knowledge, different ways of referring the manifold to the unity of a concept and ordering and mastering the manifold by it. Thus the content of each particular field of knowledge is determined by the characteristic form of judgment and question from which knowledge proceeds. In the form of judgment and question the particular special axioms, by which the sciences are distinguished from each other, are first defined. If we attempt to gain a definite explanation of the concept of "physical objectivity" from this standpoint, we are first led to a negative feature. Whatever this objectivity may mean, in no case can it coincide with what the naïve view of the world is accustomed to regard as the reality of things, as the reality of objects of sensuous perception. For the objects, of which scientific physics treats and for which it establishes its laws, are distinguished from this reality by their general fundamental

form. That concepts, such as those of mass and force, the atom or the ether, the magnetic or electrical potential, even concepts, like those of pressure or of temperature, are no simple thing-concepts, no copies of particular contents given in perception: this scarcely needs any further explanation, after all that the epistemology of physics itself has established concerning the meaning and origin of these concepts. What we possess in them are obviously not reproductions of simple things or sensations, but theoretical assumptions and constructions, which are intended to transform the merely sensible into something measurable, and thus into an "object of physics," that is, into an object *for* physics. Planck's neat formulation of the physical criterion of objectivity, that everything *that can be measured* exists, may appear completely sufficient from the standpoint of physics; from the standpoint of epistemology, it involves the problem of discovering the fundamental conditions of this measurability and of developing them in systematic completeness. For any, even the simplest, measurement must rest on certain theoretical presuppositions on certain "principles," "hypotheses," or "axioms," which it does not take from the world of sense, but which it brings to this world as postulates of thought. In this sense, the reality of the physicist stands over against the reality of immediate perception a something through and through mediated; as a system, not of existing things or properties, but of abstract intellectual symbols, which serve to express certain relations of magnitude and measure, certain functional coördinations and dependencies of phenomena. If we start from this general insight, which within physics itself has been made very clear, especially by Duhem's analysis of the physical construction of concepts, the problem of the theory of relativity gains its full logical definiteness. That physical objectivity is denied to space and time by this theory must, as is now seen, mean

something else and something deeper than the knowledge that the two are not things in the sense of "naïve realism." For things of this sort, we must have left behind us at the threshold of exact scientific physics, in the formulation of its first judgments and propositions. The property of not being thing-concepts, but pure concepts of measurement, space and time share with all other genuine physical concepts; if, in contrast to these, space and time are also to have a special logical position, it must be shown that they are removed in the same direction as these, a step further from the ordinary thing-concepts, and that they thus represent, to a certain extent, concepts and forms of measurement of an order higher than the first order.

The fact appears even in the first considerations, from which the theory of relativity starts, that the physicist does not have only to hold in mind the measured object itself, but also always the particular conditions of measurement. The theory distinguishes between physical determinations and judgments, which result from measurement from resting and moving systems of reference, and it emphasizes the fact that before determinations, which have been gained from diverse systems of reference, can be compared with each other, a universal methodic principle of transformation and permutation must be given. To each objective measurement, there must be added a certain subjective index, which makes known its particular conditions and only when this has taken place can it be used along with others in the scientific construction of the total picture of reality, in the determination of the laws of nature, and be combined with these others into a unitary result. What is gained by this reflection on the conditions of physical measurement in a pure epistemological regard appears as soon as one remembers the conflicts, which have resulted from the lack of this reflection in the course of the history of philosophy and of exact science.

It seems almost the unavoidable fate of the scientific approach to the world that each new and fruitful concept of measurement, which it gains and establishes, should be transformed at once into a thing-concept. Ever again does it believe the truth and the meaning of the physical concepts of magnitude to be assured only when it permits certain absolute realities to correspond to them. Each creative epoch of physics discovers and formulates new characteristic measures for the totality of being and natural process, but each stands in danger of taking these preliminary and relative measures, these temporarily ultimate intellectual instruments of measurement, as definitive expressions of the ontologically real. The history of the concept of matter, of the atom, of the concepts of the ether and of energy offer the typical proof and examples of this. All materialism—and there is a materialism not only of "matter" but also of force, of energy, of the ether, *etc.*, —goes back from the standpoint of epistemology, to this one motive. The ultimate constants of physical calculation are not only taken as real, but they are ultimately raised to the rank of that which is alone real. The development of idealistic philosophy itself is not able to escape this tendency. Descartes as an idealistic mathematician was at the same time the founder of the "mechanical view of the world." Since only extension offers us exact and distinct concepts and since all clearly comprehended truth is also the truth of the *existing*, it follows, in his view, that mathematics and nature, the system of measurements and the totality of material existence, must be identified. The manner, in which the same step from the logico-mathematical to the ontological concept has been repeated in the development of modern energetics, is known. Here, after energy had been discovered as a fundamental measure, as a measure which is not limited to the phenomena of motion, but spans equally all physical fields, it was made an all-

inclusive substance, which rivalled "matter" and finally took it up into itself completely. But on the whole, we are here concerned only with a metaphysical by-way, which has not seduced science itself from its sure methodic course. For the concept of energy belongs in conception to that general direction of physical thought, which has been called the "physics of principles" in contrast to the physics of pictures and mechanical models. A "principle," however, is never directly related to things and relations of things, but is meant to establish a general rule for complex functional dependencies and their reciprocal connection. This rule proves to be the really permanent and substantial: the epistemological, as well as the physical, value of energetics is not founded on a new pictorial representation to be substituted for the old concepts of "matter" and "force" but on the gaining of *equivalence-numbers*, such as were expressly demanded and discovered by Robert Mayer as the "foundation of exact investigation of nature." (*Cf.* 52, p. 145, 237ff.)

Even in these two examples we can learn that through the whole history of physics there is a certain intellectual movement, which throughout runs parallel to the movement in epistemology that mediates and passes to and fro between the "subject" and the "object" of knowledge. Physical thought is always concerned at first with establishing a characteristic standard of measurement in an objective physical concept, in a certain natural constant. Then it is concerned, in the further development, with understanding more and more clearly the constructive element that is contained in any such original constant, and with becoming conscious of its own conditionality. For, whatever particular properties they may have, no constants are immediately given, but all must be *conceived* and *sought* before they can be found in experience. One of the most pregnant examples of this is found in the his-

tory of the concept of the atom. The atoms were postulated by Democritus as ultimate constants of nature long before thought possessed any means of concretely realizing this postulate. Fundamentally, such a realization, such a strictly quantitative meaning of the concept of the atom, was only reached in the beginnings of modern chemistry in the law of multiple proportion. To the extent, however, that to this particular realization of the concept of the atom in the law of multiple proportion others and still others are added and the concept of the atom finally comes to characterize and to organize intellectually the most diverse fields, its character as a pure principle, which was originally fused with its thing-character, comes to light. The content of the idea of the atom changes and shifts from place to place in the course of the development of physics and chemistry, but the function of the atom as the temporarily ultimate unit of measurement remains. When we pass from the consideration of "ponderable" matter to the consideration of the ether, when we seek a unity, which comprehends not only the mechanical but also the optical and electrical phenomena, the atom of matter becomes the atom of electricity, the electron. In recent physics, there appears further, with Planck's Quanta Theory the thought of an atomistic structure not only of matter but of energy. It would be in vain were one to attempt to combine all these various applications of the concept of the atom in chemistry, in the kinetic theory of gases, and in the doctrine of light and heat radiation, *etc.*, into a unitary picture. But the unity of its meaning requires no such pictorial unity; it is satisfied, indeed verified in a far stricter logical sense, when it is shown that here a common relation, a peculiar "form" of connection, prevails, which as such can be verified and represented in the most diverse contents. The atom shows itself thereby to be, not an absolute minimum of being, but a relative mini-

mum of measure. It was one of the founders of modern philosophy, Nicholas Cusanus, who, with true speculative profundity, anticipated and announced this as the function of the concept of the atom, which was to be actually realized only in the history of natural science. Cusanus' fundamental doctrine of the infinite and of the unity of opposites in the infinite rested entirely on this insight into the relativity in principle of all determinations of magnitude, on the coincidence of the "greatest" and the "smallest." (*Cf.* 7, I, 40ff, 265ff.) Modern criticism of knowledge brings the riddle, with which Cusanus' doctrine of the minimum struggles, to a simple expression. Contradiction only enters when we attempt to unify after the fashion of a thing all the different forms, which the thought of the "smallest" assumes, in the different fields of thought; but it disappears as soon as we reflect that the true unity is never to be sought in things as such, but in intellectual constructions, which we choose according to the peculiarity of the field to be measured, and which are thus in principle possessed of an unlimited variability. It follows from this that, as *what is to be measured* is unlimited in variety, so *what measures* can be represented in infinitely many and infinitely diverse ways. In other words, the unity that we have to seek lies neither in the one nor the other member, but merely in the form of their reciprocal connection, *i. e.*, in the logical conditions of the operation of measurement itself.

This receives new confirmation when we pass from the concept of matter, of energy and of the atom to the real concept of objectivity of modern physics, that of motion. The historical beginnings of the modern theory of motion in Galilei refer directly to the epistemological question, which has received its definitive formulation in the general theory of relativity. What Galilei gained with *his* idea of relativity was the cancelling of the absolute reality of

place, and this first step involved for him the most weighty logical consequences, viz., the new concept of the lawfulness of nature and the new interpretation of the particular laws of dynamics. Galilei's doctrine of motion is rooted in nothing less and nothing more than in the choice of a new standpoint from which to estimate and measure the phenomena of motion in the universe. By this choice, there was given him at once the law of inertia and in it the real foundation of the new view of nature. The ancient view saw in place a certain physical property that produced definite physical effects. The "here" and "there," the "above" and "below," were for it no mere relations; but the particular point of space was taken as an independent real, which consequently was provided with particular forces. In the striving of bodies to their "natural places," in the pressure of air and fire upwards and in the sinking of heavy masses downwards, these forces seemed given as immediate empirical realities. Only when one takes account of these fundamental features, not only of ancient astronomy and cosmology but also of ancient physics, does one understand the whole boldness of the new intellectual orientation, resulting from the Copernican system of the world. One of the most fixed and certain realities on which Grecian thought had constructed its picture of the world now became a mere illusion, a purely "subjective" feature. Even the first adherents of the new doctrine drew the decisive conclusion with reference to the doctrine of place. What Gilbert, *e. g.*, urges against the Aristotelian physics and cosmology is above all this epistemological feature, *i. e.*, that it permits the ideal and the real to flow into each other. Differences belonging merely to our thought, to our subjective reflection, are throughout made into objective oppositions. But in truth no place in itself is opposed to any other, but there are in nature only differences in the mutual positions of bodies and of material masses. "It

is not place which, in the nature of things, works and produces, which determines the rest and motion of bodies. For it is in itself neither a being nor an effective cause; rather bodies determine their mutual place and position by virtue of the forces which are immanent in them. The place is a nothing; it does not exist and exerts no force, but all natural power is contained and grounded in bodies themselves." (7, I, 360f.) It is implied in this that what we call the "true place" is never given to us as an immediate sensuous property, but must be discovered on the basis of calculation and of the "arithmetic of forces" in the universe. All determination of place—as Kepler sharply and clearly expresses this insight which for him resulted equally from astronomical convictions, physiological optics and analysis of the general problem of perception—is a work of the mind: *omnis locatio mentis est opus.* (37, II, 55, *cf.* 7, I, 339.) From this point, the way is open to Galilei's foundation of dynamics: for since place has ceased to be something real, the question as to the *ground* of the place of a body and the ground of its *persistence* in one and the same place disappears. Objective physical reality passes from *place* to *change of place*, to motion and the factors by which it is determined as a magnitude. If such a determination is to be possible in a definite way, the identity and permanence, which were hitherto ascribed to mere places, must go over to motion; motion must possess "being," that is, from the standpoint of the physicist, numerical constancy. This demand for the numerical constancy of motion itself finds its expression and its realization in the law of inertia. We recognize here again how closely, in Galilei, the mathematical motive of his thought was connected with an ontological motive, how his conception of *being* interacted with his conception of *measure*. The new measure, which is found in inertia and in the concept of uniform acceleration, in-

volves also a new determination of reality. In contrast with mere place, which is infinitely ambiguous and differs according to the choice of the system of reference, the inertial movement appears to be a truly intrinsic property of bodies, which belongs to them "in themselves" and without reference to a definite system of comparison and measurement. The velocity of a material system is more than a mere factor for calculation; it not only really belongs to the system but defines its reality since it determines its *vis viva, i. e.,* the measure of its dynamic effectiveness. In its measure of motion, in the differential quotient of the space by the time, Galilei's physics claims to have reached the kernel of all physical being, to have defined the intensive reality of motion. By this reality, the dynamic consideration is distinguished from the merely phoronomic. The concept of the "state of motion," not as a mere comparative magnitude, but as an essential element belonging to the moving system intrinsically, now becomes the real mark and characteristic of physical reality. Leibniz, too, in his foundation of dynamics, stands throughout at this standpoint, which becomes for him a starting-point for a new metaphysics of forces. Motion conceived as a mere change of place in the purely phoronomical sense, he explains, remains always something purely relative; it only becomes an expression of a true physical and metaphysical reality when we add to it an inner dynamic principle, a force conceived as an "originally implanted principle of permanence and change," *principium mutationis et perseverantiae.* (42, VI, 100 *cf.* 5, p. 290ff.) In all these examples, it is evident how sharply, on the one hand the physical thought of modern times has grasped the thought of the relativity of place and of motion, and, on the other hand, how it has shrunk back from following it to its ultimate consequences. If not only place but the velocity of a material system is to

signify a magnitude that entirely depends on the choice of the reference body and is thus infinitely variable and infinitely ambiguous, there seems no possibility of an exact determination of magnitude and thus no possibility of an exact objective determination of the state of physical reality. Pure mathematics may be constructed as the ideal doctrine of the comparison and connection of magnitudes, as a system of mere relations and functions and may come to recognize itself as such ever more clearly, but physics seems necessarily to reach an ultimate limit, a *non plus ultra*, if it is not wholly to lose any basis in reality.

The difficulty, which remains in the structure of classical mechanics in the formulation of the principles of inertia, is expressed in an epistemological circle, from which there seems no escape. To understand the meaning of the law of inertia, we need the concept of "equal times" but a practicable physical measure of equal times can, as is discovered, only be gained by presupposing, in its content and validity, the law of inertia. In fact, since Carl Neumann's work, *Uber die Prinzipien der Galilei-Newton'schen Theorie* (57), which set in motion the modern discussion on the law of inertia, it is customary in mechanics to define equal times as times within which a body left to itself traverses equal distances. Maxwell, too, in his exposition of the Newtonian mechanics, conceived the law of inertia as a pure definition of measure. The first law of Newton, as he explains clearly and pregnantly, tells under what conditions no external force is present. (51, p. 31.) Thus in the progress of mechanics the principle of inertia is *recognized* with increasing distinctness as what it *meant* fundamentally to Galilei. It is no longer taken as a direct empirical description of given processes of nature, but as the "axiom of the field," the fundamental hypothesis by which the new science of dynamics prescribes to itself a certain form of measurement. Inertia appears, not as

an absolute and inherent property of things and of bodies, but as the free establishment of a certain standard and symbol of measurement, by virtue of which we can hope to reach a systematic conception of the laws of motion. In this alone is rooted its reality, *i. e.*, its objective and physical significance. Thus, within the historical development of physics itself *what measures* is separated with increasing distinctness from *what is measured,* with which it at first seems to coincide; the observable data of experience are separated with increasing distinctness from what must be presupposed and used as a *condition* of observation and of measurement.

And what is here seen in a particular example and within a narrow field is repeated, on closer examination, in all the special fields of physics. Everywhere physical thought must determine for itself its own standards of measurement before it proceeds to observation. There must be established a certain standpoint for the comparison and correlation of magnitudes; certain constants must be established at least hypothetically and in preliminary fashion before a concrete measurement can take place. In this sense, each measurement contains a purely ideal element; it is not so much with the sensuous instruments of measurement that we measure natural processes as with our own thoughts. The instruments of measurement are, as it were, only the visible embodiments of these thoughts, for each of them involves its own theory and offers correct and useful results only in so far as this theory is assumed to be valid. (*Cf.* 8, p. 189ff.) It is not clocks and physical measuring-rods but principles and postulates that are the real instruments of measurement. For in the multiplicity and mutability of natural phenomena, thought *possesses* a relatively fixed standpoint only by *taking* it. In the choice of this standpoint, however, it is not absolutely determined by the phenomena, but the choice re-

mains its own deed for which ultimately it alone is responsible. The decision is made with reference to experience, *i. e.*, to the connection of observations according to law, but it is not prescribed in a definite way by the mere sum of observations. For these in themselves can always be expressed by a number of intellectual approaches between which a choice is possible only with reference to logical "simplicity," more exactly, to systematic unity and completeness, of scientific exposition. When thought, in accordance with its claims and demands, changes the form of the "simple" fundamental measuring relations, we stand before a new "picture" of the world with regard to content also. The previously gained relations of experience do not indeed lose their validity, but, since they are expressed in a new conceptual language, they enter into a new system of meanings. The fixed Archimedean point of the former view of the world moves; the previous ποῦ στῶ of thought appears transcended. But it is soon seen that thought, by virtue of its peculiar function, can only transcend an earlier construction by replacing it by a more general and more inclusive one; that it only shifts, among phenomena, the constancy and identity, which it cannot cease to demand, to another and deeper place. That every realization, which the demand of thought for ultimate constants can find within the empirical world is always only conditioned and relative, is guaranteed by the unconditionality and radicalism of precisely this demand. The critical theory of knowledge would not only show this connection *in abstracto*, but for it the concrete movement of thought, the continual oscillation between experience and concept, between facts and hypotheses in the history of physics, forms a perpetually new source of instruction. In the midst of the change of particular theoretical instruments of measurement, the critical theory holds fast to the thought of the unity of measurement,

which indeed signifies for it no realistic dogma but an ideal goal and a never-to-be-concluded task. Each new physical hypothesis erects, as it were, a new logical system of coördinates, to which we refer phenomena, while nevertheless the doctrine is retained, as a regulative idea for investigation, that all these systems converge on a certain definite limiting value. In the confusion and continuous flux of phenomena, the understanding seems at first almost arbitrarily to fix and separate out certain points in order to learn through them a definite law of change, but everything which it regards as determined and valid in this sense proves, in the course of further progress, to be a mere approximation. The first construction must be both limited and more exactly defined logically by the second, this again by the third, *etc.* Thus, ever anew does the temporarily chosen theoretical center of thought shift; but in this process, the sphere of being, the sphere of objective knowledge, is more and more penetrated by thought. As often as it seems that thought is overturned by new facts and observations, which are outside its previously formulated laws, it is seen that, in fact, thought has found in them a new point of leverage, around which moves henceforth the totality of empirically provable "facts." The epistemological exposition and evaluation of each new physical theory must always seek to indicate the ideal center and turning-point around which it causes the totality of phenomena, the real and possible observations, to revolve,—whether this point is clearly marked or whether the theory only refers to it indirectly by the intellectual tendency of all its propositions and deductions.

## II

### The Empirical and Conceptual Foundations of the Theory of Relativity

If there can be no doubt, according to the opening words of the *Critique of Pure Reason,* that all our knowledge begins with experience, then this holds especially when we are concerned with the origin of a physical theory. The question here can never run as to *whether* the theory has issued from experience but merely as to *how* it is based on experience and what is the relation of the diverse elements which characterize and make up the concept of experience as such. There is accordingly needed no special epistemological analysis to make clear the relation of the special and general theories of relativity to experience, to the whole of observation and of physical experiment; such an analysis will only have to decide whether the theory in its origin and development is to be taken as an example and witness of the critical or of the sensualistic concept of experience. Does "experience," as it is used here, mean merely the bare sum of particular observations—*experimentorum multorum coacervatio,* as a sensualistic thinker once described it—or is there involved in it an independent intellectual form? Is the construction of the theory merely a matter of joining "fact" to "fact," perception to perception,—or, in this connection of particulars, have there been effective all along certain universal and critical norms, certain methodic presuppositions? No "empiricism" however extreme can ever seek to deny the rôle of thought in establishing and grounding physical the-

ories, and just as little is there, on the other hand, a logical idealism, which would attempt to free "pure thought" from reference to the world of the "factual" and from being bound to it. The question dividing the two views can only be as to whether thought consists in a simple registration of facts, or whether, even in the *establishment,* in gaining and interpretation of "particular facts," thought reveals its characteristic power and function. Is its work completed in arranging particular data, immediately taken from sense perception, like pearls on a thread—or does it face them with its own original measures, as independent criteria of judgment?

The problem raised here received its first sharp and clear systematic formulation in the Platonic doctrine of ideas. For Platonic idealism, too, the proposition holds that it is not possible to think save on the basis of some perception: οὐ δυνατὸν ἐννοεῖν ἢ ἔκ τινος αἰσθήσεως. But the function of the "logic in us" consists indeed not in finding the sum of the particular perceptions, not in deriving and deducing the "idea of the equal" from the "equal pieces of wood and stone," but the "logic in us" is revealed in discriminating and judging what is given in perception. This discriminating constitutes the real fundamental character of thought as διάνοια, as *discursus.* Not all perceptions and observations stimulate equally the critical and discriminating activity of thought. There are some which do not summon the understanding to reflection since satisfaction is done them by mere sensation, but there are others which in all ways call forth thought as if in their case perception by itself could gain nothing solid. "Not exciting, namely, is that which does not pass into an opposite perception; exciting objects I call those which give opposite perceptions, because here perception gives no more vivid idea of any particular object than of its opposite. Much in perception is a paraclete of thought (παρακλητικὰ

τῆς διανοίας), while other perceptions are not—*such an awakener of thought, namely, is everything, which comes into sense at the same time as its opposite;* but what does not, that also does not arouse thought." (*Republic* 523-524.) In this Platonic characterization of the relation of thought and sensation, of reason and sensibility, we have, as Cohen has urged, "one of the most fundamental thoughts in the evolution of the critique of cognition." (12, p. 16ff.) Just as for Plato thought becomes what it is in assertion and contradiction, in dialectic, so not any arbitrary perception but only one to which this feature corresponds can become its awakener and paraclete. The dialectic of perception summons that of thought to judgment and decision. Where the perceptions, as it were, rest peacefully side by side, where there is no inner tension between them, thought rests also: only where they contradict each other, where they threaten to cancel each other does thought's fundamental postulate, its unconditional demand for unity stand forth and demand a transformation, a reshaping of experience itself.

The evolution of the theory of relativity has furnished a new typical proof of this general relation. It was in fact a fundamental contradiction between physical experiments from which the theory of relativity took its start. On the one side stood the investigation of Fizeau, on the other, that of Michelson, and the two seemed in their results absolutely irreconcilable. Both sought to gain an answer to the question as to how the velocity of light in a moving medium was related to its velocity in a resting medium; and they answered this question in completely opposite ways. The investigation of Fizeau showed that the velocity of light in flowing water was greater than in water at rest; that, however, not the whole velocity of the flowing water, but only a fraction of it was added to the velocity of light in a medium at rest. If we call the

velocity of light in the moving medium $W$ and the velocity of light in a medium at rest $w$ and the velocity of the flowing $v$, it results not simply that $W=w+v$, but rather that $W=w+v\left(1-\frac{1}{n^2}\right)$, in which the magnitude $n=\frac{c}{w}$ signifies the exponent of the refraction of the liquid. This result, as interpreted by the theory of Lorentz, spoke directly for the assumption of a motionless ether not carried along by the body in its movement. But the attempt of Michelson, to discover the consequences of the movement of the earth with reference to this motionless ether, failed. In no way could any influence be shown of the motion of the earth on the velocity of the propagation of light; it was rather shown with increasing evidence that all optical phenomena take place as if there were no translation of the earth against the ether.[5] And behind this conflict of "facts" there stood, as one was forced to recognize more and more, a conflict in general principles, to which the theories of mechanical and of optical and electromagnetic phenomena seemed to lead necessarily. Experiments in the latter could finally be summarized in a single proposition, the principle of the constancy of the velocity of light in a vacuum. The validity of the fundamental equations of electro-dynamics of Maxwell and Hertz involved the assumption that light in an empty space is always propagated with a definite velocity V independently of the state of motion of the body emitting it. From whatever system one made the observation and from whatever source the light issued, there would always be found the same determinate value for its velocity of propagation. But this assumption of the velocity of light as a universal constant the same for all systems, necessarily demanded by the principles of electro-dynamics, now

[5] For more detail concerning the investigations of Fizeau and Michelson as well as concerning the negative outcome of other investigations on the influence of the movement of the earth on optical and electrical phenomena, *cf.* Laue (40), p. 10ff.

comes into opposition with the principle of relativity of the Galilei-Newtonian mechanics. This principle demands that, when any definite Galileian reference body is given —*i. e.*, a body relatively to which a body "left to itself" persists in its state of rest or of uniform motion in a straight line—all the laws, which are valid relatively to this reference body K remain valid when one passes to the system of reference K′, which is, with reference to K, in uniform translatory motion. In the transition from K to K′, the equations of the "Galilei-transformation" hold,

$$x' = x - vt,\quad y' = y,\quad z' = z$$

(where v signifies a constant velocity of K′ with reference to K parallel to the x and x′ axes), to which there is to be added the identical transformation from the time $t' = t$, which is not especially noticed in classical mechanics. If we seek, however, to apply the principle of relativity of mechanics to electro-dynamics, *i. e.*, to recalculate its equations according to the formulae of the Galilei-transformation, it is seen that this cannot be done: the electro-dynamic equations, in contrast to the Newtonian equations of motion, alter their form when we insert the coördinates x′, y′, z′, t′, into them in place of the coördinates x, y, z, t according to the rules of the Galilei-transformation. The effort to unite mechanics and electro-dynamics by carrying over the principle of relativity of the first into the latter thus had to be given up: the Hertzian theory, which represented such an attempt, came into irreconcilable conflict with assured experimental results. Physical investigation stood before the dilemma of giving up a principle which had been verified without exception in all the phenomena of motion and which formed a corner-stone in the structure of classical mechanics—or of retaining it within its field but denying its applicability to optical and electromagnetic phenomena. In both cases, the unity of the explanation of nature, the unity of the very concept of nature,

seemed destroyed. Here in fact the condition set up by Plato, of the intellectual fruitfulness of experience was fulfilled: here experience stood at a point at which assured observation seemed to pass directly into its opposite. The conflict between the principle of the constancy of the propagation of light and the principle of relativity of mechanics became the "paraclete of thought"—the real awakener of the theory of relativity.

But how did physical thought go about overcoming this conflict, since it was bound to the outcome of observation as such, since it could neither set aside the facts expressed in the principle of the constant velocity of light in a vacuum, nor those expressed in the principle of relativity of mechanics? If we look back on the historical development of the theory of relativity, we recognize that the latter has followed here a counsel which was once given by Goethe. "The greatest art in theoretical and practical life," wrote Goethe to Zelter, "consists in changing the *problem* into a *postulate;* that way one succeeds." In fact, this was the course which Einstein followed in his fundamental essay, *'Zur Elektrodynamik bewegter Systeme* of the year 1905. The principle of the constancy of the velocity of light was given first place as a postulate, but,—supported by the negative result of all attempts to establish an "absolute" motion with reference to a chosen system of reference, *i. e.,* the "motionless ether,"—the *supposition* was made that there correspond to the concept of absolute rest no properties of phenomena in either mechanics or electro-dynamics, but rather that the same electro-dynamic and optical laws hold for all systems of coördinates of which the mechanical equation hold. And this *"supposition"* does not continue such, but is expressly "made a presupposition," *i. e.,* a shaping of theory is demanded which will simultaneously satisfy the conditions of the principle of relativity and those of the principle of

the constant propagation of light. (*Cf.* 16, p. 26.) The two assumptions are indeed not compatible according to the means and habits of thought at the disposal of the kinematics generally accepted before the establishment of the theory of relativity, but they—*ought* no longer to be incompatible. The demand made of physical theory was that it remove this incompatibility by subjecting precisely these means and habits of thought themselves to a critical examination. By an analysis of the physical concepts of space and time, it now appears that in fact the incompatibility of the principle of relativity with the law of the propagation of light is not to be found; that rather there is only needed a transformation of these concepts in order to reach a logically unobjectionable theory. The decisive step is taken when it is seen that the measurements, to be gained within a system by definite physical methods of measurement, by the application of fixed measuring-rods and clocks, have no "absolute" meaning fixed once for all, but that they are dependent on the state of motion of the system and must necessarily result differently according to the latter. There now arises the purely mathematical problem of discovering the laws of permutation, according to which the space-time values of an event are changed in going from one reference body to another, which is in uniform translatory motion with regard to the first. This problem is solved, as is known, by the fundamental equations of the "Lorentz-transformation":

$$x' = \frac{x - v t}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$y' = y \qquad z' = z$$

$$t' = \frac{t - \frac{v}{c^2} x}{\sqrt{1 - \frac{v^2}{c^2}}}$$

On the basis of these equations, we see that the law of the propagation of light in a vacuum is equally fulfilled for

all justified systems K and K'; on the other hand, it is seen that Maxwell's fundamental equations of electrodynamics do not change their form when the formulae of the Lorentz-transformation rather than those of the Galilei-transformation are applied to them. There is thus a universal principle of relativity, which comprehends the totality of physical phenomena; the laws, according to which the states of physical systems change, are independent of whether they are referred to one or the other of two systems of coördinates in uniform translatory motion relative to each other. (*Cf.* 16, p. 29). The principle of relativity of classical mechanics is so little contradicted by this general principle that it is rather contained in it as a special case; the equations of the Galilei-transformation directly issue from those of the Lorentz-transformation when one considers only such velocities v as are very small in comparison with the velocity of light so that the values $\frac{v}{c^2}$ $\frac{v^2}{c^2}$ can practically be left out of account. It follows from this that the principle of relativity of electrodynamics, carried over to mechanics, can come into conflict with no empirical result, while the converse carrying-over of the principle of relativity of mechanics to electrodynamics proves to be impossible, as the collapse of Hertz's theory showed. More closely considered, however, in the special theory of relativity, the electrodynamic processes are not used as a key to the mechanical, but a truly universal principle, a heuristic maxim of investigation in general, is established, which claims to contain a criterion of the validity and permissibility of all particular physical fields and theories. Thus it is seen that the initial contradiction, appearing between the principles of mechanics and those of electrodynamics, has shown the way to a far more perfect and deeper unity between them than previously existed. And this result was not reached entirely by heaping up experiments, by newly instituted investigations, but it

rests on a critical transformation of the system of fundamental physical concepts.

On the purely epistemological side, there thus appears with special distinctness in this intellectual process in which the theory of relativity originates, that peculiar "Copernican revolution," that variation in the conceptual foundations of the theory of nature, which we have previously traced in the example of classical mechanics and the older physics. An essential part of its achievement seems based on the fact, that it has shifted the previous logical constants of physical knowledge, that it has set them at another place than before. For classical mechanics, the fixed and immovable point was the assumption of the identity of the spatial and temporal values gained by measurement in the various systems. This identity was taken to be the unquestionable and sure foundation of the concept of objectivity in general: as that which first really constituted the object of "nature" as a geometrical and mechanical object and distinguished it from the changeable and relative data of sensation. τὸ μὲν σχῆμα καθ' αὑτό ἐστι, τὸ δὲ γλυκὺ καί δλως τὸ αἰσθητὸν πρὸς ἄλλο καὶ ἐν ἄλλονς—thus runs the proposition, which Democritus brought into the foundations of atomism, and which in modern times was taken up by Galilei to support the fundamental distinction between "primary" and "secondary" qualities, and thus the whole "mechanical" view of the world. Although the principle here established proved to be very fruitful and has been frequently confirmed in mathematical physics, the modern evolution of physics shows, with increasing evidence, that it was conceived too narrowly in a philosophical and methodological sense. The true goal of science is not mechanism but unity—as Henri Poincaré once formulated the guiding maxim of modern physics. But concerning this unity the physicist does not need to ask *whether* it is, but merely *how* it is; *i. e.*, what is the minimum of presup-

positions that are necessary and sufficient to provide an exact exposition of the totality of experience and its systematic connection. (72, p. 172ff.) In order to maintain this unity, which seemed endangered by the conflict of the principle of the constancy of the velocity of light and the principle of relativity of mechanics, and to ground it more deeply and securely, the theory of relativity renounces the unity of the values of spatial and temporal magnitudes in different systems. It surrenders the assumption that the temporal interval between events is a magnitude fixed once for all independently of the state of motion of the reference body and that in the same way the spatial distance between two points of a rigid body is independent of the state of motion of the reference body. By going back to the method of measuring time and to the fundamental rôle that the velocity of light plays in all our physical time measurements, it discovers the *relativity of the simultaneity* of two processes and further leads to the insight that the magnitude of the length of a body, of its volume, its form, its energy and temperature, *etc.*, are, as results from the formulae of the Lorentz-transformation, to be assumed as different according to the choice of the system of reference in which measurement takes place. But these "relativizations" are not in contradiction with the doctrine of the constancy and unity of nature; they are rather demanded and worked out in the name of this very unity. The variation of the measurements of space and time constitutes the necessary condition through which the new invariants of the theory are discovered and grounded. Such invariants are found in the equal magnitude of the velocity of light for all systems and further in a series of other magnitudes, such as the entropy of a body, its electrical charge or the mechanical equivalent of heat, which are unchanged by the Lorentz-transformation and which thus possess the same value in all justified systems of refer-

ence. But above all it is the general form of natural law which we have to recognize as the real invariant and thus as the real logical framework of nature in general. While the special theory of relativity limits itself to regarding all reference bodies K′ which are moving uniformly in a straight line relatively to a definite justified reference system K, as equivalent for the formulation of natural laws, the general theory extends this proposition to the assertion that all reference bodies KK′, whatever their state of motion may be, are to be taken as equivalent for the description of nature. (17, p. 9; 18, p. 42.) But the path by which alone this true universality of the concept of nature and of natural law, *i. e.*, a definite and objectively valid description of phenomena independent of the choice of the system of reference, is to be reached, leads, as the theory shows, necessarily through the "relativization" of the spatial and temporal magnitudes, that hold within the individual system; to take these as changeable, as transformable, means to press through to the true invariance of the genuine universal constants of nature and universal laws of nature. The postulate of the constancy of the velocity of light and the postulate of relativity show themselves thus as the two fixed points of the theory, as the resting intellectual poles around which phenomena revolve; and in this it is seen that the previous logical constants of the theory of nature, *i. e.*, the whole system of conceptual and numerical values, hitherto taken as absolutely determinate and fixed, must be set in flux in order to satisfy the new and more strict demand for unity made by physical thought.

Thus reference to experience, regard for phenomena and their unified exposition, proves to be everywhere the fundamental feature, but at the same time it is seen that,

in the words of Goethe, experience is always only half experience; for it is not the mere observational material as such, but the ideal form and the intellectual interpretation, which it is given, that is the basis of the real value of the theory of relativity and of its advantage over other types of explanation. As is known, the investigation of Michelson and Morley, which gave the impetus and starting-point for the development of the theory of relativity, was explained as early as the year 1904 by Lorentz in a manner which fulfilled all purely physical demands. The Lorentzian hypothesis, that each body moving with reference to the motionless ether with a velocity v undergoes a certain shortening in the dimension parallel to the motion, and indeed in the ratio of $1 : \sqrt{1 - \frac{v^2}{c^2}}$, was sufficient to give a complete explanation of all known observations. An experimental decision between Lorentz's and Einstein's theories was thus not possible; it was seen that between them there could fundamentally be no *experimentum crucis*.[6] The advocates of the new doctrine accordingly had to appeal—a strange spectacle in the history of physics—to general philosophical grounds,—to the advantages over the assumption of Lorentz which the new doctrine possessed in a systematic and epistemological respect. "A really experimental decision between the theory of Lorentz and the theory of relativity," Laue, *e. g.*, explains in his exposition of the principle of relativity in the year 1911, "is indeed not to be gained, and that the former, in spite of this, has receded into the background, is chiefly due to the fact, that, close as it comes to the theory of relativity, it still lacks the great simple universal principle, the possession of which lends the theory of relativity . . . an

[6] For more detail *cf. e. g.* Ehrenfest (15a), p. 16ff.

imposing appearance."[7] Lorentz's assumption appeared above all to be epistemologically unsatisfactory because it ascribes to a physical object, the ether, definite effects, while at the same time it results from these effects that ether can never be an object of possible observation. Minkowski too explains in his lecture on space and time that the Lorentzian hypothesis sounds extremely fantastical; for the contraction is not to be conceived as a physical consequence of the resistence of the ether but rather purely as "a gift from above," as an accompaniment of the state of motion. (47, p. 60f.) What thus, in the last analysis, decided against this assumption was not an empirical but a methodological defect. It conflicted most sharply with a general principle, to which Leibniz had appealed in his struggle against the Newtonian concepts of absolute space and time, and which he formulated as the "principle of observability" (*principe de l'observabilité.*) When Clarke, as the representative of Newton, referred to the possibility that the universe in its motion relatively to absolute space might undergo retardation or acceleration which would not be discoverable for our means of measurement, Leibniz answered that nothing fundamentally outside the sphere of observation possessed "being" in the physical sense: *quand il n'y a point de changement observable, il n'y a point de changement du tout* (5, p. 247ff). It is precisely this principle of "observability," which Einstein applied at an important and decisive place in his theory, at the transition from the special to the general theory of relativity, and which he has attempted to give a necessary

[7] 40, p. 19f.; *cf.* 41, p. 106. *Cf.* also the characteristic remark of Lorentz himself in his Haarlem lecture: "The estimation (of the fundamental concepts of Einstein's theory of relativity) *belongs to a very large extent* (*grösstenteils*) *to the theory of knowledge*, and one can leave the judgment to the latter in confidence that it will consider the questions mentioned with the necessary thoroughness. But it is certain that it will depend for a great part on the type of thought to which one is accustomed, whether one feels drawn more to the one or the other conception. As far as concerns the lecturer himself, he finds a certain satisfaction in the older conceptions, that ether possesses at least some substantiality, that space and time can be sharply separated, that one can talk of simultaneity without further specification." (46a, p. 23.)

connection with the general principle of causality. Any physical explanation of a phenomenon, he urges, is *epistemologically* satisfactory only when there enter into it no non-observable elements; for the law of causality is an assertion concerning the world of experience only when *observable facts* occur as causes and effects. (17, § 2). Here we stand before one of the fundamental intellectual motives of the theory of relativity—a motive which not only gives it the advantage over the empirically equivalent hypothesis of Lorentz, but which also produces the advance from the more limited interpretation of the postulate of relativity in the special theory to the completely universal formulation.

The way in which this advance has taken place is especially suited to make clear the conceptual and empirical presuppositions of the theory and their reciprocal connection. The special theory of relativity rests, as has been shown, on two different assumptions, which stand, equally justified, side by side: on the postulate of the uniformity of the propagation of light in a vacuum and on the presupposition that all reference systems in rectilinear, uniform and non-rotary motion relatively to a definite justified system K are equally permissible for the formulation of the laws of nature. If one considers these presuppositions, which stand in inseparable connection in the empirical structure of the special theory of relativity, from a purely methodological standpoint, it is seen that in *this* respect they belong to different strata. On the one side, stands the assertion of a general fact, a constant of nature, which results from the experimental findings of optics and electrodynamics; on the other side stands a demand, which we make of the *form* of natural laws. In the first case, it is empirically established that there is a peculiar velocity with a definite finite value, which retains this value in any system independently of the state of motion of the latter. In

the second, a general *maxim* is established for the investigation of nature, which is to serve as a "heuristic aid in the *search* for the general laws of nature." In the formal limitation, which is placed on natural laws by this maxim, lies—as Einstein himself has urged—the characteristic "penetration" (*Spürkraft*) of the principle of relativity. (18, pp. 28, 67.) But the two principles, the "material" and the "formal" are not distinguished from each other in the shaping of the special theory of relativity. The fact that this distinction is made and that the general and "formal" principle is placed above the particular and "material" principle constitutes, from the purely epistemological standpoint, the essential step taken by the general theory of relativity. And this step seems to lead to a strange and paradoxical consequence; for the particular result is not taken up into the general, but rather is cancelled by it. From the standpoint of the general theory of relativity, the law of the constancy of the velocity of light in a vacuum no longer possesses unlimited validity. According to the general theory of relativity the velocity of light is dependent on the gravitation potential and must thus in general vary with places. The velocity of light must always depend on the coördinates when a field of gravitation is present; it is only to be regarded as constant when we have in mind regions with a constant gravitation potential. This consequence of the general theory of relativity has often been regarded as a refutation of the presupposition from which the special theory of relativity took its start and on which it based all its deductions. But with justice Einstein rejects any such conclusion. The special theory of relativity, he explains, is not rendered valueless by the fact that one comes to see that its propositions refer to a definitely limited field, namely, to the phenomena in an approximately constant field of gravitation. "Before the establishment of electrodynamics, the laws of electro-

statics were regarded as the laws of electricity in general. Today we know that electrostatics can only describe electrical fields correctly in the case, that is never exactly realized, in which the electric masses are exactly at rest relatively to each other and to the system of coördinates. Is electrostatics overthrown by Maxwell's electrodynamical equations? Not in the least! Electrostatics is contained as a limiting case in electrodynamics; the laws of the latter lead directly to those of the first for the case that the fields are temporarily unchangeable. The most beautiful fate of a physical theory is to point the way to the establishment of a more inclusive theory, in which it lives on as a limiting case." (18, p. 52.) In fact, in the advance from the special to the general theory of relativity, we have only a verification of the same principle of the construction of concepts of natural science that is found in the advance from classical mechanics to the special theory of relativity. The constants of measurement and of the theory of nature in general are shifted and magnitudes, which were regarded as absolute from the earlier standpoint, are again, with the gaining of a new theoretical unit of measurement, made into merely relative determinations valid only under definite conditions. While classical mechanics, like the special theory of relativity, distinguished between certain reference bodies relatively to which the laws of nature were valid and certain relatively to which they were not, this distinction is now cancelled. The expression of the universal physical laws is freed from any connection with a particular system of coördinates or with a certain group of such systems. To be expressed the laws of nature always require some definite system of reference; but their meaning and value is independent of the individuality of this system and remains self-identical whatever change the latter may undergo.

Only with this result do we reach the real center of the general theory of relativity. Now we know where lie its truly ultimate constants, its cardinal points, around which it causes phenomena to revolve. These constants are not to be sought in particular given things, which are selected as chosen systems of reference from all others, such systems as the sun was to Copernicus and as the fixed stars were for Galilei and Newton. No sort of things are truly invariant, but always only certain fundamental relations and functional dependencies retained in the symbolic language of our mathematics and physics, in certain equations. This result of the general theory of relativity, however, is so little a paradox from the standpoint of the criticism of knowledge, that it can rather be regarded as the natural logical conclusion of an intellectual tendency characteristic of all the philosophical and scientific thought of the modern age.[8] To the popular view and its habits of thought the radical resolution of "things" into mere relations remains as ever suspicious and alienating, for this view believes that it would lose with the thing-concept the one sure foundation of all objectivity, of all scientific truth. And thus, from this side not so much the positive as the negative aspect of the theory of relativity has been emphasized; what it destroys, not what it constructs has been comprehended. But it is remarkable to find this interpretation not only in the popular expositions of the theory of relativity but in the investigations of its general "philosophical" significance; and to meet in the latter also the view that it brings an element of subjective arbitrariness into the formulation of the laws of nature and that, along with the unity of space and time, the unity of the concept of nature is destroyed. In truth, as closer consideration shows, the theory of relativity is characterized through-

[8] Here, indeed, I can only make this assertion in a general way; for its proof I must refer to the more specific explanation in my work *Substanzbegriff und Funktionsbegriff*. (8, pp. 148-310).

out by the opposite tendency. It teaches that to attain an objective and exact expression of natural process, we cannot take without further consideration the space and time values, gained by measurement within a definite system of reference as the only and universal values, but that we must, in scientifically judging these measurements, take account of the state of motion of the system from which the measurement is made. Only when this is done can we compare measurements which have been made from different systems. Only those relations and particular magnitudes can be called truly objective which endure this critical testing, that is, which maintain themselves not only for one system but for all systems. That not only are there such relations and values, but that there must be such, in so far as a science of nature is to be possible, is precisely the doctrine the theory of relativity sets up as a postulate. If we start, as practically we must do at first, from a definite system of measurement, we must bear in mind that the empirical values, which we gain here, do not signify the final natural values but that, to become such, they must undergo an intellectual correction. What we call the system of nature only arises when we combine the measurements, which are first made from the standpoint of a particular reference body, with those made from other reference bodies, and in principle with those made from all "possible" reference bodies, and bring them ideally into a single result. How there can be found in this assertion any limitation of the "objectivity" of physical knowledge is not evident; obviously it is meant to be nothing but a definition of this very objectivity. "But it is clear," says Kant, "that we have only to do with the manifold of our presentations and that X, which corresponds to them (the object), since it is to be something distinct from all our presentations, is for us nothing; the unity, which makes the object necessary, can be nothing else than the formal

unity of consciousness in the synthesis of the manifold of presentations. Thus we say: we know the object when in the manifold of intuition we have produced synthetic unity." The object is thus not gained and known by our going from empirical determinations to what is no longer empirical, to the absolute and transcendent, but by our unifying the totality of observations and measurements given in experience into a single complete whole. The theory of relativity shows the whole complexity of this task; but it retains the postulate of the possibility of such a system all the more strenuously and points out a new way to satisfy it. Classical mechanics believed itself at the goal too soon. It clung to certain reference bodies and believed that it possessed, in connection with them, measures in some way definitive and universal, and thus absolutely "objective." For the new theory, on the contrary, true objectivity never lies in empirical determinations, but only in the manner and way, in the function, of determination itself. The space and time measurements in each particular system are relative; but the truth and universality, which can be gained nevertheless by physical knowledge, consist in the fact that all these measurements correspond mutually and are coördinated with each other according to definite rules. More than this indeed knowledge cannot achieve, but it cannot ask for more, if it understands itself. To wish to know the laws of natural processes independently of all relation to any system of reference, is an impossible and self-contradictory desire; all that can be demanded is that the content of these laws not be dependent on the individuality of the system of reference. It is precisely this independence of the accidental standpoint of the observer that we mean when we speak of the "natural" object and the "laws of nature" as determinate in themselves. Measurements in *one* system, or even in an unlimited number of "justified" systems would

in the end give only particularities, but not the true "synthetic unity" of the object. The theory of relativity teaches, first in the equations of the Lorentz-transformation and then in the more far reaching substitution formulae of the general theory, how we may go from each of these particularities to a definite whole, to a totality of invariant determinations. The anthropomorphism of the natural sensuous picture of the world, the overcoming of which is the real task of physical knowledge,[9] is here again forced a step further back. The mechanical view of the world thought to have conquered it, when it resolved all being and natural process into motion and thus put every where pure magnitudes in place of qualitative elements of sensation. But now it is seen that precisely the determination of these values, the measurements, which it applies to motions, are still bound to certain limiting presuppositions. Reflection on the manner in which we make empirical measurements of space and time shows how anthropomorphism reaches into this field that was thought withdrawn from it in principle. It is, as it were, this earthly remainder still belonging to classical mechanics with its assumption of finite fixed reference bodies and motionless inertial systems, from which the theory of relativity seeks to free itself. The conceived unit of connection determined by a system of mathematical equations here takes the place of any sensuously given, and also sensuously conditioned, unit of measurement. As is seen, there is involved here not a cancellation but a critical correction of the empirical concept of objectivity, by which a correction of our empirical spatial and temporal measures and their transformation into the *one* system of natural laws are gained.

We are brought to the same outcome by consideration of the historical problems out of which the theory of rela-

[9] *Cf.* Planck (66) p. 6ff. and (67) p. 74.

tivity has grown. To give the propositions of abstract mechanics, especially the principle of inertia a definite physical meaning had been attempted repeatedly by trying to point out some empirical systems for which they would possess strict validity. But these attempts were all thwarted, in particular, by the discovery of the motion of the solar system and of the fixed stars; to find a fixed and clear empirical meaning for the equations of the Galilei-Newtonian mechanics, nothing remained save to postulate, with Carl Neumann, an absolutely motionless body $\alpha$ at some unknown place in space. But such a postulate of the existence of a particular physical object, a body which can never be discovered by observation, remains the strangest anomaly, from the epistemological standpoint. (8, p. 238ff.) The absolutely motionless ether too, which seemed for a time to offer the lacking physical reference system of the Galilei-Newtonian mechanics, showed itself unsuited to this purpose; since the negative outcome of Michelson's investigation the question seemed to be decided here also. At this point, as has been seen, the theory of relativity begins. It makes a virtue out of the difficulty into which philosophical thought had fallen in its attempt to find a particular privileged system of coördinates. Experience had shown that there is no such system, and the theory, in its most general interpretation, makes it a postulate that there *cannot* and *must* not be such. That, for the physical description of the processes of nature, no particular reference body is to be privileged above any other is now made a principle. "In classical mechanics, as well as the special theory of relativity," says Einstein," a distinction is drawn between reference bodies K relatively to which the laws of nature are valid and reference bodies K′ relatively to which they are not valid. With this state of affairs no consistently thinking man can be satisfied. He asks: how is it possible that certain reference bodies (and their states

of motion) are privileged over other reference bodies (and their states of motion)? In vain, I seek in classical mechanics for something real to which I might trace the difference in the behavior of the body with reference to the systems of reference K and K'." (18, p. 49.) In this argument from the principle of *insufficient reason*, the physicist seems to move on slippery ground. One is inevitably reminded of the argument of Euler, who thought that he *proved* the principle of inertia of classical mechanics by explaining that, if a body changed its state of motion without the influence of external forces, there would be no reason why it should choose any particular change of magnitude and direction of its velocity. (23.) The circle involved here, namely, that "the state of motion" of a body is assumed to be a determinate magnitude, while it is only defined as such by the law of inertia itself, is easily seen. In Einstein's appeal to the "principle of reason," there is doubtless involved a more general and deeper epistemological motive. If we assume that the final objective determinations, which our physical knowledge can reach, *i. e.*, the laws of nature, are provable and valid only for certain chosen systems of reference, but not for others, then, since experience offers no certain criterion that we have before us such a privileged reference system, we can never reach a truly universal and determinate description of natural processes. This is only possible if some determinations can be pointed out, which are indifferent to every change in the system of reference taken as a basis. Only these relations can we call laws of nature, *i. e.*, ascribe to them objective universality, whose form is independent of the particularity of our empirical measurements of the special choice of the four variables $x_1$, $x_2$, $x_3$, $x_4$, which express the space and time parameters. In this sense, one could conceive the principle of the universal theory of relativity, that the universal laws of nature are

not changed in form by arbitrary changes of the space-time variables, as an analytic assertion; as an explanation of what is meant by a "universal" law of nature. But the demand, that there must in general be such ultimate invariants, is synthetic.

In fact, it can be shown that the general doctrine of the invariability and determinateness of certain values, which is given first place by the theory of relativity, must recur in some form in any theory of nature, because it belongs to the logical and epistemological nature of such a theory. To start from the picture of the world of general energetics—Leibniz, in establishing the law of the "conservation of *vis viva*" as a universal law of nature, referred to this logical element in it. He first defines the *vis viva* of a physical system as a quantity of work; he determines that forces are to be called equal, when they are able to perform equal mechanical work, no matter what their properties may be in detail; thus if they produce an equal degree of tension in an equal number of elastic springs, raise an equal weight to the same height, communicate to an equal number of bodies the same amount of velocity, *etc.* In this definition it is assumed that measurement of the *vis viva* by different systems of measurement will give results equivalent to each other, and thus that forces, which, when measured by a certain effect, prove to be equal or in a definite relation of greater or smaller, will retain this same relation if we measure them by any other effect. If this were not the case, Leibniz adds, and did there result a different relation of forces according to the different effect which one uses as a measure, nature would be without laws; the whole science of dynamics would be superfluous; and it would not be possible to measure forces, for forces would have become something indeterminate and contradictory, *quiddam vagum et absonum.* (42, III, 208ff.; VI, 209f.; *cf.* 5, p.

305ff.) The same process of thought has been repeated on broader physical lines in the discovery and grounding of the modern principle of energy. Here, too, the energy of a material system in a certain state was defined—*e. g.*, by W. Thompson—first as the amount of all the effects, expressed in mechanical units of work, called forth outside the system when the system passes in any way from its state into a definite but arbitrarily defined state of nullity. This explanation at first leaves it entirely undecided as to whether there exists a determinate value of what is here called "energy," *i. e.*, whether the results of the measurement of the amount of work of a system turn out the same or differently according to the method of bringing the system from the given state into a definite state of nullity. But that this determinateness in fact exists, that there always results the same amount of energy no matter what effect we use as the measure of work and what type of transition we choose, is precisely what the principle of the conservation of energy affirms. This affirms nothing else and has no other physically comprehensible meaning than that the amount of all the effects, measured in units of mechanical work, which a material system calls forth in its external environment, when it passes from a definite state in any arbitrary manner to an arbitrarily defined state of nullity, has a determinate value, and is thus independent of the type of transition. If this independence did not exist—and that it exists only experience can teach us—it would follow that what we called "energy" is not an exact physical determination; energy would not be a universal constant of measurement. We would have to seek for other empirical values to satisfy the fundamental postulate of determinateness. But it holds, conversely, that if energy is once established as a constant of measurement, it thus becomes a constant of nature also, a "concept of a definite object." Now from a physical

standpoint a "substantial" conception of energy can be carried through without arousing suspicion; energy can be regarded as a sort of "reserve supply" of the physical system, the quantity of which is completely[10] determined by the totality of the magnitudes of the states, which belong to the system involved. From the epistemological standpoint, it must be remembered that such an interpretation is nothing more than a convenient expression of the relations of measurement, that alone are known, an expression which adds to them nothing essential. The unity and determinateness of measurement can be immediately understood and expressed as the unity and determinateness of the object, precisely because the empirical object means nothing but a totality of relations according to law. It follows from this analogy from a new angle that the advance in "relativization" which takes place in the theory of relativity, represents no contrast to the general task of objectification, but rather signifies one step in it, since, by the nature of physical thought, all its knowledge of objects *can* consist in nothing save knowledge of objective relations. "Whatever we may know of matter," here, too, we can cite the *Critique of Pure Reason,* "is nothing but relations, some of which are independent and permanent and by which a certain object is given us." (34, p. 341; *cf.* Müller's Trans. p. 232.) The general theory of relativity has shifted these "independent and permanent relations" to another place by breaking up both the concept of matter of classical mechanics and the concept of the ether of electrodynamics; but it has not contested them as such, but has rather most explicitly affirmed them in its own invariants, which are independent of every change in the system of reference. The criticism made by the theory of relativity of the physical concepts of objects springs thus from the same method of scientific thought,

[10] In more detail in Planck (63) p. 92ff.

which led to the establishment of these concepts, and only carries this method a step further by freeing it still more from the presuppositions of the naïvely sensuous and "substantialistic" view of the world. To grasp this state of affairs in its full import we must go back to the general epistemological questions offered to us by the theory of relativity; we must go back to the transformation of the *physical concept of truth* involved in it by which it comes into direct contact with the fundamental problem of logic.

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* This article is a selection from a translation of Professor Ernst Cassirer's *Zur Einstein'schen Relativitätstheorie.* A translation of Professor Cassirer's *Substanzbegriff und Funktionsbegriff,* with the whole of the former essay as a supplement, will be published in the course of the year by THE OPEN COURT PUBLISHING COMPANY as one volume under the title *Substance and Function.* The translators are W. Curtis Swabey, Ph.D., and Marie Collins Swabey, Ph.D.

# CRITICISMS AND DISCUSSIONS

## SCIENCE AND SELF-SACRIFICE

THIS generation has seen illustrated in full a paradox more startling than those of Kant. We have seen thesis and antithesis spread upon the front pages of our newspapers, and the whole world is working out the solution. The thesis was put in what seemed unassailable terms in the political campaign cries of a dozen years ago. "The full dinner pail" and "dollar diplomacy," the free silver agitation, free trade and protection, each rested its case on a foundation which only a few visionaries questioned, and to them no one paid heed. This foundation was the belief that self-interest was the dominant factor in life. That the individual's will to live, to enjoy life and play a large part in it, to conquer foes and live on the spoils of the victory, was the fundamental motive in all action. The Christian teaching of the value and necessity of self-sacrifice played no part in practical politics or in business life. The world believed in morality, but it very distinctly disclaimed any belief in Christian morality. Justice meant something, a man had a right to his own, but if he had nothing, it was no one's duty to sacrifice *themselves* to give him of their plenty. In broad outline this was the active and real creed of the world up to 1914. Where exceptions are to be noted, especially in the foreign policy of the United States toward Cuba and toward China, they were looked upon as the magnanimous throwing to weaker folk of the crumbs spilled from our overflowing table. A man or a nation had first to look out for itself, then and only then would other things be added to *it*.

No one needs to be reminded of the change today. Four years have worked a revolution. There are men still living in the past, aging politicians who cannot read the signs of the times. But the tide of popular demands rolls on, and a socialist government in once imperial Germany is called moderate, and our prayers go up for its continuance, lest something more radical threaten us. To wage war, politicians and statesmen had to appeal to something other than self-interest. To give up money, ease, comforts, and life itself was but a glorious opportunity to fulfil the highest demands of life. The completeness of this change we have not yet realized. Self-sacrifice is taken as a matter of course, and if my brother has

less than I, it is my duty to feed him, even though I take from the scanty store of my own table. Self-sacrifice and service are the foundation of the appeals to which the world is responding.

The self-sacrifice given by millions has won the great struggle, yet the war is not formally ended nor decently interred before self-interest, and conflicting claims as to mutual rights and interests are again heard. The swing of the pendulum was so rapid that there is question whether it may not swing back as rapidly. Thesis and antithesis are before us in plain sight. The problems of the solution have come upon us before we were ready.

In the eagerness to down the foe, men forgot for a moment their doubts, but now those doubts return. Can men be depended upon to give themselves to service and to self-sacrifice when the war passion has passed? Was it not perhaps the Berserker fury of war, after all, and not the gentle pleading of Jesus of Nazareth that brought about this outpouring of self? If man continued to act on the basis of the Sermon on the Mount, would it pay? Is there any real place in the world for the Christian doctrine of self-sacrifice? Does a man benefit when he does right? Are those forces in the world which make for righteousness powerful enough to overcome the operation of the natural laws of self-interest and the struggle for the survival of the fittest? Does a moral Being who rewards righteousness and punishes iniquity rule the material universe? Or does blind force determine the destinies of men? Is the end of all the struggle and sacrifice to be that some time the last man, though he be benefited by all that the generations have done for him, must perish weakly in some natural upheaval? Is all that we have gained in the years of war at the mercy of some wandering star that may any moment end all life upon the earth? These old questions return again to men's minds.

These doubts return, but one even more fundamental comes with them. There has been noted in this war the same tendency to fatalism on the part of the soldiers that has appeared in wars of the past. Not what a man does, but what fate has in store for him, determines his safety or death. A man should strive to the utmost, but that striving will not assure safety. This is perhaps inevitable under circumstances where doing one's duty so frequently leads to greater and greater danger. The will to serve faithfully is the will that frequently leads to death. Were the soldier not so faithful he might perhaps escape. He has no thought of this, but leaves to fate the result. For the soldier faithful service assures no material

reward. Under the impulse of the combat, with the war spirit strong, this brings no questioning, but when the ardor of conflict dies, the question recurs with redoubled force. Of what use is faithfullness since it gives no material reward? The soldier does what he must. Of what avail for him is the struggle to be faithful overmuch?

This is but the restatement of the problem of the place in the world of the will to do good. A statement by Professor H. C. Warren of Princeton (*Journal of Philosophy*, August 15, 1918, p. 464), puts this problem distinctly before us. "All human activity, including deliberation and selective volition, is completely mechanistic." If all life is fundamentally mechanistic, functioning as a supermachine, then it is not a righteous Providence watching to reward faithful service that rules the universe, but a blind medley of forces, which may at any moment be completely altered in character by the coming of a stronger force. The arrow sent by chance killed King Ahab, not any retribution at the hand of a righteous God. The connection between Ahab and the arrow is mechanical, not moral. So, fundamentally, is every connection of life if this theory be true. We need then to face squarely the issue. If all life is mechanistic, explainable and governed by unmoral forces, then self-sacrifice, the devotion of the soldier as well as of the saint, is quixotic and useless.

There is a plain opposition between mechanical force as a theory of existence and a moral deity. Yet the solution is not as simple, nor the differences as easy to state as *it* seems at first. Facts are facts, and form the material with which either theory of life must build. Moreover, either theory must use all the blocks in the puzzle. In the final building all the facts must have their appointed place. Mechanism must account for morality as a fact in the world, and theism must account for the death of King Ahab by a chance shot. We turn first to the facts of morality.

Evolution did not cease when man reached his present physical form. More change has *occurred* in his methods of obtaining a living, more changes on the surface of the earth due to modern civilization than in any previous period in the world's history of many times the duration of our modern world. The evolution is now an evolution of ideas and collective ability. An united family can do more than a single individual. More game falls to the hunter hunting with his brothers than when alone. The clan and tribe *is* more efficient than the single family in providing for human needs. A nation brings greater security and material prosperity to its citizens

than independent and warring tribes. It is at least a present dream that an united world will be more efficient and prosperous than separate hostile nations.

The one thing that has made these larger social groups possible is morality. As men's ideas of duty toward their fellow men *has* grown and broadened, so the social unit has enlarged. As men have learned to sacrifice and serve each his brother, evolution has moved forward. Social morality has made possible life in the family, in tribes, in great cities, in nations and in empires. Without the teaching of duty to one's neighbor man would be even more solitary than packs of dogs in their hunting. The dogs find it possible to work together without sacrificing each his own private interests. This man cannot do. Men must either fight or serve. For savage tribes a stranger is an enemy. Only when bound by ties which impose mutual obligation can another man be trusted.

This social obligation has been inforced by the tribal and national religions. One's duty to one's brother is strengthened by one's duty to one's tribe. Because he, too, is a member of the tribe I must aid him. Because he, too, worships the tribal god I must give to fill his need. As tribal religions make possible the tribe, so the Christian religion makes possible a world brotherhood.

Morality, and the religion that reinforces morality, *has* played a large part in the progress of the human race. They are factors that must be included in any study of existence. If the world is mechanistic, the mechanistic theory must explain morality and religion, and account for their work as evolutionary factors.

As the fact of morality must be accounted for, so must its dependence on the material world. It must be brought into relationship with that world. Just how is the chance death related to the intentional infliction of the death penalty? Just how far does the field of morality extend? Morality has had to do not only with intra- and inter-tribal life, it has also almost as often, in its primitive stages, concerned itself with the relation of the savage to the animals and plants around him. He has at times sought to force the rain to fall, and to this end has enforced rigorous prohibitions upon the members of the tribe, and especially upon the priests or leaders. If some modern views of morality have divorced it from the material world and ignored that world, the savage never fell into that error. He built up a tribal morality, enforced certain ceremonies and required certain forms of self-denial in an effort to gain power over animal and inanimate nature. As he sought to

force other tribes of men to his will, so he sought to rule the wild animals of the forests, and even the rain and the stars. He never sought, as we do, to build up a morality unconcerned with the material needs of life. It is to a later stage that the statement belongs that the rain falls on the just and on the unjust like. For the savage, only to those who live the proper life and perform the proper ceremonies will the spirits of the rain be gracious. That the rain does not come in response to the charms of the medicine man we know, but that knowledge has not stopped man's effort to rule nature.

If we were content to rest in modern ethical theories, it would be enough to prove the existence of morality as a necessary factor in *human* life. The problem would then be for the mechanist to build his theory in accordance with this fact. But we cannot rest content with this. If only while one lives has morality any place, then the fields of Europe mark the downfall of all ethical theories. If all that man dies for is that his fellowman may live and in his turn die, we cannot escape the question, of what avail is it? If there is nothing in the best human character that is not always at the mercy of a chance arrow, if the whole life of a city which men have stricken to make pure, for which they have toiled and suffered, may in an hour be utterly destroyed by a volcanic outburst, and only the criminal in jail escape, our ethical and religious theories fall. Better for Red Guards to take and enjoy while they may, than to suffer to no avail. Religion must face this problem squarely, if religion is to continue in the world.

The case is not as dark for religion and morality as our statement might indicate, nor need man take refuge just yet in mysticism and declare physical life an illusion and not to be taken into account. The savage took it into account, and the modern man takes it into account. Modern morality is not confined to making men live together as brothers. We combine not merely in order to defend and protect ourselves against our fellow man, but also to gain power over the material world. A race that spans sea and land with railroad and telegraph, that sends boats beneath the waves, and men to fly thousands of feet in the air, that predicts the weather, even though it cannot control it, that provides as a matter of course against the cold of winter and the heat of summer, has gone far in the conquest of nature. Complete conquest is not achieved, but every year sees some step forward in this mastery of the universe. All this would be impossible without modern morality. Only by those combinations which the sense of moral obligation makes pos-

sible has man increased or held his advantage over inanimate life. Religion, in supporting and strengthening morality, is the driving force in this conquest of the world. Theories of religion must take account of this fact. The savage failed in his attempt to rule all existence only because he did not have the proper weapons. The same morality which he used, only differently applied, the same sense of mutual obligation, is the foundation of our successes in the conquest of the world.

The fact that men from before the dawn of history up to the present have set themselves to this conquest of the world, and to that have subordinated everything, ease, wealth, even the individual life, forces us to change our question. It was the passion of war that made men forget self and made them willing to die for an idea. It was the passion of the fighter that has led to the voluntary suffering of every reformer. It is in the heat of the conflict that through all the ages the hero has shed his blood. When the conflict has come, when love of ease and self has been opposed by love of *tribe, and the* will to war against an enemy of that tribe, whether that enemy be man or matter, only the coward and craven has sought to save self. In war, self-sacrifice is more fundamental than self-seeking. On this rock *every form* of utilitarianism falls in pieces. The call to stand, even in death, side by side with one's brother awakes a stronger response than the desire for personal safety. It is more primitive. It comes more directly from the womb of nature. Morality apart from this cannot exist. And religion and morality are bound indissolubly together. It is self-seeking and not self-sacrifice that needs to be explained. As in this war just over, so in all conflicts of the past, the burden of proof rested upon the man who declined to sacrifice self or possessions. The man who gave of self or of money gave as a matter of course, and without question. Our problem must then be restated. We do not need to account for self-sacrifice; but we have still to deal with the modern question as to its wisdom.

The fact that self-sacrifice is primitive and instinctive goes a good way toward proving it of value. Just as the instincts of the lower animals are developed in order better to equip the animal in the struggle for existence and the better to fit him to his environment, so human fundamental traits play the same part. We have at times looked too high for our justification of morality. We have valued the human race so little that only those things are called of value which aid in the everlasting preservation of the race. To

maintain the race for a day, if tomorrow the race is destroyed, we hold of no account. Nature does not so count value. Whatever even merely tends to preserve a race, though it may not assure its safety, is of value. Man is in the world. If he is to continue in the world a certain amount of co-operation between man and man is necessary. Morality assures this co-operation. Therefore, morality is of value in the material world.

The continued existence of the race demands self-sacrifice of the individual. This he renders, in a sense from necessity, in a sense also, of his own free will. No man acts solely after an exhaustive study of the facts and of the probable results of a contemplated action. No man lives to himself alone. The old idea of individualism, as well as of the value of the individual soul, is not Christian, and is the result of non-Christian influences. The insistence of Jesus that he who wills to save his soul shall lose it, and that he who loses his soul shall save it, should have warned us that our current theology was in error. Nowhere does Jesus teach that a man apart from his fellows is of infinite value. "The whole creation groaneth and travailleth together until now." It is neither instinctive nor Christian for a man to demand personal advantage to himself from every action he takes. Thus the Christian does not demand an answer to the question, whether sacrifice of self is of benefit to the individual. It is enough for him that the race, his brothers, benefit by his toil and suffering.

The man who is not a Christian, who asserts the right of the individual to demand personal benefit from every personal endeavor, still insists upon his question, of what advantage to me is self-sacrifice. When the call for service comes, a man finds himself bound to his fellowmen by many ties. The sinking of the Lusitania, the sufferings of the Belgians and Armenians, pull at his emotions. He can refuse to heed the appeal, but in so doing he breaks those ties which were a fundamental part of his nature. He does violence to his own truest self. This is not religious cant, but a statement of biologic and natural law. The man who refuses to sacrifice self for the benefit of his brothers is no longer the man he was or might be. He has cut off a part of himself. Material safety he may find, but at personal loss. His future acts, assisting only in the well being of one only among the millions of creatures on the earth, cannot have the value that self-sacrifice would have had. It is the refusal to serve that is without value in this material world of ours.

A man must serve, must sacrifice self, or be no true man. Yet, for all this his labor, for all the benefit that may come to his brothers from his toil, there may be in the end no result. If the race is of no value to the universe, if man makes but a brief and impotent appearance upon the stage of life, then man's fate is tragic. Serve and sacrifice self he must, or be untrue to his fundamental instincts, yet when he is the nearest to his ideal, a few ages may sweep away all that he achieved. If the end of the world be a cold lifeless planet, then morality has no final place in time. Then those forces in the world which make for morality are not fundamental in the universe. Man for his own peace of mind may live gloriously as a hero while he lives, but like the Scandinavian gods, for all that the heroes may do, their Asgard is bound to go down to destruction. Whether man's lot is thus tragic is the real problem.

To answer such a question is beyond our present purpose. Yet we need not therefore sit down beside the rivers of Babylon and weep. What the future may hold in store for the world we do not and cannot know. It is not, however, a matter to be decided by the natural sciences alone. It is a fact in the material world, that the human race exists. The material forces have given a place to man. In a sense morality and religion are natural phenomena, concerned with the existence of a part of the material world. Whatever the nature of the fundamental forces of the universe, those forces cannot be described in terms of geology and astronomy alone. Those forces are the forces which include spiritual forces. On the other hand spiritual forces concern and imply material forces. The Logos Doctrine has at least this analogy to modern scientific theories, that it tends to portray God as being forced by his own nature to express Himself, and that expression includes expression in material form. This is the assumption of all theories of the Incarnation. Neither for Christian theology nor for science can we divorce morality and religion from the material realm. As part of that realm morality has value. Even if the world become lifeless, that lifeless husk will bear traces of man. It will not be the same word it would have been without man. What the value of a lifeless trace of an extinct race may be, is not our present question. Man has changed the world which he found. Morality has made possible those changes. Therefore, morality has a value for the material universe. Men through the influence of morality and religion have played an *irradicable* part in existence.

More than this. As science must explain how from lifeless matter came living organisms, so for the future it must explain how those spiritual forces can be gathered back again into a lifeless void. In neither case can we rest content with the present scientific theories. As no mechanistic theory can be suffered to call itself truly scientific if it ignores the place of morality and of religion as we have sketched it, so no astronomical theory can be allowed to assert unchallenged that the spiritual forces are at the mercy of the material forces. No man can look far into the future. The only sure foundation for prophecy is the look forward we obtain from a tower built on the foundation and erected from all the facts of life. When the astronomer takes into account the spiritual forces, then, and not until then, need religion concern itself with his prophecies. So long as he omits religion and morality and organic life from his calculations, he cannot be sure that his conclusions are true. The neglected factors may upset all his carefully calculated results.

The moral forces are forces active in the natural world. Yet they are not chemical or mechanical. We do not need to delve into the structure of being to maintain this contention. Nor are we just at this moment arguing against the mechanistic constitution of matter. Moral and religious influences can neither be described, correlated, nor explained, in the terms of a chemical or mechanical science. History and ethics do not use the terms nor the methods of chemistry and mechanics. Whatever may be their constitution, whether the result of chemical reactions or not, the moral forces as we see them active in the world are of a different type from the inorganic reactions. We can explain a man sitting in the sun on a cool day as we explain the tropisms of the unicellular organisms, much as we explain the affinity of certain substances for one another, but we have no chemical terms that will account for the elaborate heating systems of modern days. This is not due merely to the ignorance of the chemist. Scientific theory to be efficient must account for details. The chemical history of the plans and purposes involved in the building of a house would be so complex and involved that it would be useless as a working theory. The simple statement of the purpose of the builder is effective as the chemical account would not be. Chemical and mechanical theories of life, therefore, do not apply to morality and religion, and the forces that bring about a moral life—that is, the spiritual forces—have a right to treat the world from this moral point of view. These moral forces are what the religious man calls the power of God. We have, there-

fore, a right to include God among the natural forces. In any complete survey of existence He must be taken into account. In this sense, but in this sense only, we have proved His existence.

Just so far as man has dominion, so far has man's god dominion. Whatever triumphs lie before man, his god will lead him to them and share in them. So long as men live in communities on the earth and their combined endeavors rule inanimate nature, so long will the god who rules them exercise his power over the universe. All this, however, does not answer the question raised by the prophecy of a dead world where man does not exist. Nor does it answer the questions raised by the escape of the criminal and the death of all the righteous in the volcanic eruption. This latter point, however, is easily set aside. It is easily conceivable that in the end man may succeed in harnessing the volcanoes, and other natural forces. In this present struggle to master them, no one claims that he has won the final victory. Each defeat but spurs him to greater exertion and some day we may expect to see the result. For this it is worth while to maintain and increase the efficiency of the moral forces which make it possible for men to work together. A momentary defeat is no disproof of the value of morality.

The first objection still stands. If the fate of the earth is to become a dead world, cold and still, or to end in some explosion of superheated gasses, man cannot hope finally to rule all existence. Even today he cannot influence the stars. There is the possibility which some extreme visionaries dream of, the ability of man to pass through interstellar space before the end of life on this planet and to transfer to another world, Venus or some more distant star, the life that has begun here. The bare possibility of this the astronomer cannot deny, but this can bring little solace to the moralist. To pass interstellar space would not be possible to even a completely moral world unless there *shall* by that time be discovered some physical means to pass beyond our atmosphere and still live. The continued existence of man would then depend not on morality but on physical science. Not a moral God, but an all-knowing God would or could bring this about. In discussing such a dream we are no longer in the sphere of morality. We may therefore acknowledge our limitations and leave the question. If a man waits to be moral until he is assured that the results of his moral action will assure human life into endless ages, he will never act at all.

There is another side even to science. Natural forces are to be understood when most fully revealed. The forces which brought

this earth into being are most clearly revealed in earth's highest product, the human race. To the human race we must turn in order to understand what the natural forces really are. There is a scientific theory that every physical event leaves some trace, has some effect on the universe. As we have already noted, man and morality must therefore have an everlasting influence, even though man himself cease to exist. To this corresponds the religious conception of the eternal value of the soul.

Religion has two aspects. It is both social and mystic. Men have sought to divorce the two, but always in every religion that has endured, the two are to be found inextricably intermixed. We have been speaking of morality as simply *a* natural *phenomena*. It is more than this even for science. Morality is a matter of man's consciousness. It is therefore a proper study of the science of psychology. It is still more. In acting from a sense of duty man is self-conscious. Personality comes into play. It is no longer possible to exhaust the problems of the moral will by a purely objective study. We, therefore, pass over to the world of introspection.

When a man is moral, when he resists some suggestion on the ground that it is not "right" for him so to do, he is conscious of his own will. The same is true on the positive side. When a man enlists in the army because he thinks that he ought to, he places side by side the idea of country and his duty to his country. It is himself that he studies and is conscious of, his own will and that focus of activities we call the self. This is true of any deliberated action. For religion it is also true. In fact we have the claim of the higher religions of the right to exercise dominion over the whole will of man. Each and every action is either right or wrong. The questions which we raised in the beginning have no place here. A man has no right to require proof that the act will benefit him, before he acts. The categorical imperative is an element in all religions. Man must obey first and reason afterward. Social pressure is to the same end. Conform to custom first and attack it afterward, but conform. Safety for the human soul consists in this conformity. By conforming, the soul attains immortality, whatever immortality may be. Right action, action that expresses the will to do right, is of value altogether apart from results. I am to love my brother, to give him alms, present day Moslem and *Medeaeval* monk agree, not so much to help him as to store up virtue for myself. I am to be moral not to save others but myself. Not to save the race of men, but to do God's will is the aim of religion. Whether the race

continues to exist or not is of little value. In fact religion has at times taught that very little of the race will be saved. The world is due for destruction. Only those who are to be saved out of the world are to be considered. The scientific problems of the future of this earth are thus ignored.

We have to face this attitude clearly. Though often crudely put, yet in its essence it is scientific. As a matter of fact man does get satisfaction in expressing himself in action regardless of the result. "After me the deluge" expresses much of human conduct. So long as my will has expression, so long as I do what I intend, I may not care whether the effects last long or not. Only when my purpose concerns all men yet to be born do I need to raise the question as to the future of the human race. The statesman who builds for all time must face the question whether it is worth while to labor for a passing and temporary result. The man of religion who is content to save one soul from the burning may well pass lightly over the problem. We have to ask whether this religious attitude has any possible scientific justification. Has morality any place in reality apart from its results on the material universe?

A will or purpose is a conscious state which seeks to perpetuate itself. If the purpose relates to the outer world, then the perpetuation sought is in the material universe. If the purpose concerns the individual consciousness alone, then the end sought is the continuance of that conscious state. When the end sought is found, the man is far from content. Other purposes may come, but that purpose or desire is expressed in the result attained. The picture the artist paints, so far as he attains to his ideal, is the expression of the beauty he desires to portray. The moral will share this characteristic. The moral act, as the expression of the will to do right, is the fulfilment of that purpose. The moral act expresses the intention to be moral. Since morality is the expression of the effort of the human race to continue to exist and to expand, the moral act is the expression of traits fundamental in humanity. It could not be the human race and not labor to build up co-operative effort. It could not form great kingdoms, empires, and republics except by the expression of itself in moral action. By moral action we mean to include self-sacrifice. Self-sacrifice, fundamental and primitive, expresses one of the primitive and essential instincts in man. Whatever the results of moral action, it has at least this value always, that it gives expression to this fundamental human instinct. It expresses the power of the race over the individual, and gives con-

sciousness to the race soul. In this is a fact which must form the basis of any scientific study of man.

It is here that morality is tied up indissolubly with religion. Morality has this value in itself only as the expression of human instincts and forces which seek expression. As morality concerns itself with the actions resulting, so religion deals directly with those forces. All that religion does come back in one way or the other to the effort to build up a moral dynamic. Religion thus has an inescapable value to the human race. A desire becomes a purpose and hardens into will only as it attains a certain measure of expression. The willing of an action means the accomplishing of that action so far as the individual can bring it about. Religion strengthens those desires and brings them more readily to expression. Selfsacrifice is instinctive, but the constant insistance by the Christian teachers on its necessity undoubtedly brings mankind more readily to this act. The moral strength of the allied powers finally overmatched the limits of self-sacrifice of the Germans largely because the religious element was more clearly present with the allies. Defeat did not daunt them as it did the Germans, because they were more prepared for self-denial. Self-sacrifice as a religious duty thus has its value apart from its results. Because it gives expression to man's true self, it is of value in itself.

The moral will, and especially the will to deny one's self, not merely expresses a fundamental instinct, it is conscious of that self-fulfillment. This is the mystical element in religion. Even the mystic has often misunderstood it, and tried to divorce his will to deny himself from the race for which that denial is to be made. The Buddha tried this, but even he stayed in the world to teach his brothers the truth. Compassion for them affected even the Enlightened One. The saint who denies himself for his brother's sake is fulfilling the highest function of a member of the human race. He is doing that for which he came into the world. He is expressing his manhood.

Man is an expression of the meaning of existence. The universe may have other more complete expressions of the real significance of its life, but human kind is at least a partial expression. No other race or existence known to us reveals as much of what life means as does man. He is for us the highest point attained by the cosmic powers. Those instincts which express most truly his nature therefore reveal best the meaning of the universe. Self-denial is fundamental to man, and expresses his truest nature. Self-denial

is therefore a true expression of the meaning of life. Those forces which reveal themselves in morality and religion are revealing the true nature of the universe. Thus, again, religion and morality have a value apart from any temporary or incidental results. We do not ignore the problem as to the relation of man to the material universe. Religion and science must face these problems, but no solution of those problems can take from the value we have found for self-denial. The Christian doctrine shows itself to be scientific and true to the facts. One question we can answer finally. Since morality and religion have and represent a truer conception of the facts than is possible to material science, no conclusions of mechanics or chemistry can overthrow our results. They may explain life for their own purposes in mechanical terms but the true explanation of the facts of human life rests with the moralist and the saint.

Not merely is the true nature of man revealed in the will to serve, but in addition man is conscious of his will. He knows what this true self is, even more surely than he knows it to be his true self. We have been arguing all this time as to whether morality is fundamental in human nature, but no argument is needed to tell man what morality is. He knows by direct experience what his will and spirit *is*. So far as the true being of the universe is revealed in man, in knowing himself he also knows that true being. In man's will to serve he therefore comes into direct contact with this true being. This true being is what he calls God. In willing to sacrifice himself for his fellows, and in being conscious of that will to serve, in recognizing that it is his true self, man also recognizes that he touches directly the divine spirit. Such language seems to affront both science and religion. Such affront appears only because even today science and religion are too far sundered. If God is real, he must influence and be the true being of the material universe. As such he concerns both science and religion. He who truly knows God knows the truth of all existence, and he who knows the real being and nature of the world knows God.

In this way, mysticism, the conscious presence of the divine spirit in man, attains a valid place in scientific study. We need to notice that it is a mysticism that reveals the world, and not a mysticism that withdraws from the world. Only to the man who wills to serve is it possible to become conscious of God's presence. Ancient and classic mysticism failed because in the effort to gain self-consciousness there was left out everything of which one could be conscious. Yet the search was partially successful. Men found by

searching within something which they recognized as their true being. If we, in the fullness of our understanding of the necessity of service, come to self-consciousness, we will not fail as they failed in making that communion with God really conscious and distinct.

We have not been directly discussing the question whether God exists. If we define God as the true being, or as the truth of existence, then by definition He exists. He may not be personal, or infinite, or endowed with power, but whatever life is, that is God. Our problem in such a case is to find out what this truth of existence is. In conscious devotion and self-sacrifice man finds contact with this true being. So, in the religious phrase, he finds God. He finds Him to be a conscious power. Meaning is given to man's life by this consciousness of service. As this will to serve reveals man's true nature, so it reveals God, who is the true being of the universe. As man's will is a conscious power, so God is known as a conscious power. What more He may be, whether consciousness is the highest form of life, needs no decision here. As man is the highest form of life known to us, so the revelation of *God* in man is the highest revelation possible to us. God is at least all that man knows him to be.

The paradox that we stated at the beginning is thus brought to a certain kind of solution. The purpose to provide for one's self and conquer the world is primitive and fundamental in man, but so also is the instinct to conquer by combining with one's fellows. For this self-sacrifice is essential. The solution is that the instinct for self-preservation is really the effort to express the true nature of our existence. This, however, is better expressed by the spirit of co-operation, for it is co-operation which raises man above the brute. Self-expression rather than self-preservation is the real fundamental instinct. Sometimes it shows itself in the effort of the individual to gain for himself such power that none may oppose his will. Sometimes it shows itself in the self-sacrifice of the individual as he thereby expresses his purpose of joining with his fellows in a common warfare. This is the synthesis by which the opposing ideals of the last generation and of ours are gathered to one common end.

This synthesis or solution is purely in scientific terms. We have asked neither the metaphysical nor the religious value and meaning of our problem. We have put aside the questions as to the final outcome of man's effort to subdue nature. We have been dealing with facts and not values. As a matter of fact we found that self-sacrifice, along with self-assertion, expresses man's true being,

and in so far as man is part of the universe, expresses partially the true being of our universe. In this sense we have found out something about the nature and being of God. For all this we do not need ourselves to be religious. Only if we are not religious we must take our facts at second hand. To the teacher and leader in religion, such a study as this must be made and the results taken into account if he is to make the right connection between religious and secular life. The pendulum will of necessity swing back again perhaps to its old extreme of each man for himself unless it is stopped or slowed up in its swing.

For theology, the study of the meaning of God's life and of man's relation to Him, such a conclusion as ours must be taken into account. A metaphysics can come to conclusions concerning the meaning and relation of what is already known to us. It cannot bring us new facts. Theology can relate the facts known to us, as a philosophy of the Christian religion, but so long as it remains a philosophy, an interpreter merely, it can give us no new facts. To prove God's existence we need facts. If already in fact we have known God, then philosophy or theology can explain these facts. Prove it, however, it cannot. Meaning is not fact. If God is a being who is a fact in existence, then He will be known as a fact and not as an inference. What He is will be known as a fact, and that knowledge be subjected to scientific study. As "fact" we have found God and learned something of His nature.

Yet, once more, we have been studying facts, not explaining them. How it comes to pass and why, that man is called to self-sacrifice which may destroy him in spite of his instinct to live, how and why it is that man is the highest point so far attained in the evolution of the world; what the end and the value of human life may be; upon all these questions we gain no light from our present study. Whether that spirit which we have met as the true spirit of man is destined to rule all existence, whether our God is omnipotent and eternal, we know no more than before. All this remains for future study. We rest content at this point with having found man's religious nature to be fundamental to his existence.

GEORGE A. BARROW.

CHELSEA, MASS.

## DEATH

By D. T. Praigg

Oh, Death! thou long maligned and dreaded foe
Of that inherent spark which comes unsought,
But unto which we cling as though it were
Of priceless worth, I come to thee with meed
Of praise too long delayed, for thou hast been,
Of all the friends of man, the truest, best,
And steadfast most in loyalty and love;
And hast evinced from him a sympathy
Which thy detractors, lost in blind conceit,
And dreading change from fickle light of Day
To cloudless Night, can ne'er appreciate,
Nor plaudit give for duty well performed.

Thou dost come to man when others from him
Turn away, and he becomes an outcast
On the paths of earth, shunned, reviled, abused
By all his fellows, and afflicted sore
By heavy hand which Time upon him lays;
And thou dost give to him the gentle boon
Of rare forgetfulness of worldly griefs,
Op'ning wide to him the regal chamber
Of honored guest, where blest Oblivion
Close draws the curtains of her silence oe'r
The din of conflict in a world of strife,
And gives to tired life her sweet repose.

Thou art the loyal friend, oft tried, of strength,
The enemy of weakness, self-approved,
Yet thou dost come unto the old, infirm,
And long despairing with reward of rest,
And lead them far away from earthly paths
On which they tread with falt'ring step and are
A burden to themselves and earth and time;
And for this sad estate dost give to them
Thy blest eternity of voiceless Calm.
And taking thus the debris from the paths,
Which Time doth litter with the wrecks of men,
Thou givest Youth an unobstructed course
On missions that unveil the New and make
The roads of earthly progress bud and bloom
With fragrance and with beauty unexcelled.

Thine is the heritage of blissful Calm,
In which the Present reigns supreme, undimmed
By clouds that lower o'er the buried Past
And from misgivings of the Future free,
In an abode where Hope's illusions cease
To beckon on to dull Despair, and Time,
Of pow'r despoiled, incites no more to aim
Whose inspiration is its earthly tomb.
Thus, then, to pay thee tribute and declare
The regal worth of thy decree, I come,
And on thy paths I strew the bloom of earth
And crown thee Mercy's noblest gift to man.

## THE ECONOMICS OF MORALITY

THE national welfare may be called the national economy, since in the national welfare a plurality of interests becomes a community, or economy, of interests. The national welfare is the one interest that virtually represents all of the interests of the nation; therefore, it is the most important interest of the nation. And every other interest of the nation is half dependent on this leading interest, just as the final, or real, effectiveness of every individual workman is half dependent on the officials, the leaders, of the business in which he is engaged.

Now, as an interest, the national welfare is dealt with in two very different, though complementary, ways. The first way is by means of codes of political laws. It is extremely difficult for many, if any, men to grasp and retain in mind the end, or ideal, of national welfare in its relation to present conditions; and therefore, these codes of political laws are substituted in men's minds for the real end of national welfare. It would be out of the question to decree that all men act in whatever way gives best promise of helping to attain the end of national welfare, for since most men's minds are very confused in regard to the proper method of attaining this ideal, there would result confused and conflicting actions and modes of life. It would be impossible to establish the guilt of offenders. But codes of political laws are easy to understand, and therefore to obey. They deal with particular modes of action, rather than action in general. They make clearer to men what their duties are with respect to political, or civil life, and in so doing they enable courts to punish offenders more readily.

Codes of laws are based solidly on the experience of the race in trying to attain the end of national welfare. Whatever modes of action have proven useful in attaining the end of national welfare are made legal; whatever modes of action have proved injurious are made illegal. Now it sometimes happens that the experience on which a law is based is forgotten by the nation, and that the nation fails to enforce the law, or changes it, until experience again proves

its wisdom. But as a general rule laws are enforced, even if their *raison d'etre* be temporarily forgotten. Authorities realize that it is very hard for most men to realize the importance of many laws, or to foresee the full consequences of disobeying any law, and therefore authorities enforce laws with a strictness corresponding to the importance of the laws. It is demanded of men that they obey laws faithfully and somewhat blindly.

It is very necessary that men establish codes of laws to assist in attaining the end of national welfare; but codes of laws are not all that men need. They are intended chiefly for men's *wills*, since they must be obeyed rather blindly, for men's intellects cannot have anything like free play in respect of laws. Yet the intellect is just as important as is the will, and provision should be made for the intellect. The will should be obedient, and the intellect clear.

To satisfy this need men have developed the science of political economy. Principles of political economy are not enforced, as are political laws; for things that are left to the intellect are to be decided by the intellect. Principles of political economy are based solidly on the experience of the race, as are political laws; but political laws are *commands*, whereas principles of political economy are *assertions*, or *suggestions*. Political laws compel the individual, principles of political economy assist the individual. People *must* have political laws, since there can be no government without laws; and people *should* have principles of political economy, since governments cannot be efficient where political economy is unknown. Between the right-of-ways of laws there is much individual freedom of action; and in order to make the most of this freedom of action, individuals are taught political economy, as far as is practicable. It is not necessary for all persons to understand political economy—only a small part of the population of any nation understands it—yet the welfare of all nations is at least double what it would be if political economy were unknown.

Now *morality* is also an economy; it is the *universal* economy of interests. Morality is the one interest that represents all other interests; it unites all interests. In morality all other interests have their being. All parties to life are irrevocably committed to morality; it is the chief end of life. Whenever another interest becomes exaggerated, conflicts with morality, it must sooner or later give in. Its very life depends on morality, as a branch depends on its vine. In attaining the moral end it is important to use two different, though complementary methods analogous to those used in attaining the

end of national welfare. In the experience of the race certain modes of action have proved beneficial; these have become morally legal. Other modes have proved injurious; these have become morally illegal. It is hard for most men to foresee the consequences of disobeying these moral laws, and therefore they are expected to be obeyed almost blindly as well as faithfully, like political laws.

With the universal welfare, morality, as with the national welfare, it is important that people develop and teach principles of economy—in this case principles of *moral* economy. Men should be obedient to moral laws, and should understand principles of moral economy. Between the right-of-ways of moral laws there is always room for a very great amount of individual freedom of action. Therefore, the individual will always be able to do more good if he understands principles of moral economy. And just as the national welfare *is now* twice what it *would be* if political economy were not taught, so it is that morality, the universal welfare, *would be* twice what it *now is* if moral economy were taught.

It cannot be denied that morality is more than a question of economy; for morality cannot be adequately defined in terms of any interest, even as virtue, happiness, wealth, honor, or economy, since these are all elements in all interests. Morality is like a cord of many threads. All of the threads lead to the end of the cord, to the moral end; and therefore any thread will serve as a guide. Because, to be truly moral an act must satisfy all of the threads of cord; it must satisfy at once the demands of virtue, happiness, honor, wealth, economy. If it fails to satisfy all of these it will be found to fail to satisfy any. And if an act fully satisfies any one of these it will be found to satisfy all of these. If one had exhaustive knowledge of the nature of any one side of morality it would suffice to guide one to right action in all things, without knowledge of the nature of any other side of morality. But it is impossible to have such knowledge. Therefore, one should not attempt to live up to the requirements of only one side of morality, to regard morality as a question of only one interest; for just as an object is seen better in two lights than in one, so morality is understood better in two lights than in one. Morality is without doubt just as much a question of economy as it is a question of virtue, happiness, wealth, or honor. And in connection with this it may be said that the national welfare is a question, not only of economy, but also of virtue, happiness, wealth and honor, since all of these are inseparably connected with the national welfare. The difference between the

national welfare and morality, the universal welfare, is therefore mainly a difference in degree; the national welfare is a limited interest, whereas morality is an unlimited interest.

Just as morality is the representative of all other interests, and therefore the equal in importance to all other interests combined, so moral economy is the representative of the economies of all other interests combined. Moral economy is the economy of the whole body of interests; and the economy of *any whole* is exactly equal in importance to the combined economies of *all parts*.

It may be thought by some that teaching principles of moral economy means the interference on the part of some men in the internal affairs of others, but such is not the case. For teaching general principles is part of the work in all sciences, yet no one regards teaching other sciences as an interference on the part of some men in the internal affairs of others. For the individual is almost wholly dependent on general principles in dealing with his particular problems. The individual will always have his individual problems, and no science attempts to solve them for him. Principles of moral economy, like principles of political economy, seem clearer as they are studied in relation to one another.

The following are fundamental principles of moral economy, and give an idea of the general nature of the subject:

1. The more leading one's position is in any field, the greater is his chance of seeing the needs of that field in their true relation.

The officials of any business see the needs of that business more clearly than do the employees of that business; the leaders of an army see the needs of that army, the materials and movements necessary, better than do men in the ranks. It is the same with all governments, whether city, state, or national; it is the same with all interests *in* the world, and with the general interest *of* the world. There is much unrest at present, and therefore people do not readily see the truth of this principle. Now it is true that there are some cases in which leaders do not see the needs in their field so clearly as do some of their followers, but the cases are relatively very few. The exceptions cannot disprove the rule; with this, as with a mosaic, the parts do not give a perfect idea of the whole. It is easier to see the truth of this principle in dealing with classes than it is in dealing with individuals; for there can be no doubt but that the leading classes in all fields see the needs of their fields more clearly than do other classes, and as we come down the scale of classes we find that

the lower the class in any field, the less clearly it sees the needs of its field.

II. The more urgent any need in any field becomes, the greater becomes its chance of receiving the attention of leaders in its field.

This is moral economy's law of supply and demand. It is based on Principle I. All true needs are *moral demands;* and whenever any need becomes more urgent, whenever there is a greater moral demand for anything, the need receives greater attention from leaders in its field. All needs, all interests are related, and if any need becomes sufficiently urgent it will not only receive attention from the leaders in its immediate field, but it will also receive attention from the leaders in related, often more important, fields. There are, of course, exceptions to this principle. However, there are fewer real exceptions to it than there are apparent ones. For instance, most people have exaggerated ideas of the importance of some needs; and the leaders in the fields of these needs, who see the real importance of these needs, and who give them the proper share of their attention, seem to be neglecting these needs. This seeming neglect causes people to think that leaders do not see needs in their proper relations to one another, or that, if they do, they do not attend to needs according to their urgency. In this, people are assuming as a basis of argument the very thing that they undertake to prove. Of course, leaders, like their followers, have a limited amount of attention to divide among needs; they must be frugal with their attention. Though leaders work to the limit of their capacity, they cannot give any need all of the attention it deserves. They, therefore, endeavor to find out which needs deserve their attention most, and which needs may best be left to their subordinates. This principle does not mean that leaders should be left *alone* to attend to needs; for no leaders can accomplish much more than other men, unless their followers do their share of the work. Followers must co-operate with leaders, everyone must make as great an effort as possible; but since followers often interfere with leaders by trying to advise them, or by acting with too little confidence in them, it is important that followers keep in mind the nature of leadership. This principle should be remembered in choosing work, whether it be work of a few hours, or life work. For it is seldom that people do not overvalue some lines of work. It is quite the usual thing for people to think that certain lines of work are more important, and more neglected than they are, unless this law of supply and demand is considered. There are everywhere misfits, who

know they are misfits, yet who continue their present work, thinking it to be important, and neglected. In a few instances they are right, but in most instances they are wrong. If they all understood this principle, most of them would change to other occupations which are more useful, and at which they have more ability.

Now a philanthropist may understand scientific philanthropy, and yet not know how to do most actual good with his money. For scientific philanthropy is not so comprehensive as is moral economy; a gift may seem of greatest value from the standpoint of scientific philanthropy, and yet, from the more comprehensive standpoint of moral economy the gift may seem of value, but not of greatest value. For moral economy is concerned in all needs, no matter what their nature; it overlooks none.

III. The greater the difference in position in any field between two men in that field, the less the chance of its paying society for one of them to attempt to advise or consult the other about matters concerning that field.

There is little that ordinary men in any field know that men of importance, leaders, in that field do not also know; and therefore men should be careful in advising men of more importance than themselves in any field about matters concerning that field. Also, men should be certain that their need for advice is sufficiently urgent before they ask men of more importance than themselves for advice; for advising requires time, and the more important the man, the more important his time. Men of more importance should also remember this—they should be certain that they can get good advice from the man of lower position before they ask him for advice, and they should be certain that the man of lower position is sufficiently in need of advice before they take time from their important work to advise him. Of course, large bodies of men, through their spokesmen, are more likely able to give useful advice than single men, or small bodies of men; and large bodies of men are more likely to be worth advising than single men, or small bodies of men. Three things, therefore, should be considered in advising, or consulting; the *urgency* of the need for advice, or the *certainty* that one's advice will prove helpful; the *size* of the body of men that needs advice, or wishes to advise; and the *importance* of the man or group, of men that needs advice, or wishes to advise.

IV. In choosing one's work, whether it be work of a few hours,

or life work, one should give equal consideration to two things: the usefulness of the work; and one's ability to do the work.

Many people choose their work according to their ability to do it; many others choose their work according to the usefulness of the work. Everyone should give equal consideration to both factors, and choose the work that, from both factors, promises to be the most useful in which he can engage. This work will in most instances be neither the work for which he has the very greatest ability, nor that which is most useful, or important. Because, one usually has *much greater ability* for some *much less useful* lines of work, and also *much less ability* for *much more useful* lines of work. One may have very little ability as a statesman, and very great ability as a janitor, and decide to be neither, to be an engineer, or journalist. In estimating the usefulness of any line of work one should not forget Principle II, for it is impossible to make anything like a correct estimate, unless moral economy's law of supply and demand is considered.

On these fundamental principles of moral economy it is possible to build a system quite as vast as that of political economy, and in all probability much more so, since moral economy is the representative of all other economies, since moral economy is fully comprehensive.

Now morality seldom receives the credit or attention it deserves; there is a vague feeling among some men that, even were there no moral order, things would somehow, in some order, function, and perhaps, in some material respects, grow or develop. Few see that morality is the sole interest that, like gravitation, at once holds things together, and properly separates them. As was shown above, the efficiency of all bodies of men or interests is dependent half on the leaders of the bodies. The vast majority of men have a very indefinite idea of the proportion of welfare, or efficiency, that is due their leaders. Almost all admit that the leaders are the most important men of any body of men with respect to the efficiency of that body, but few men admit that half of the efficiency of most bodies of men is due to the leaders. As a general thing, leaders individually receive *in amount more*, and *in proportion to the value of their services less*, than do followers. The individual usually overrates his own importance, and underrates that of his leaders; he usually over-rates the interest, or economy, in which he is interested, and underrates the leading interest of all—morality—and the leading economy of all—moral economy. This is simply part of the universal tendency

to bring things to a general level in respect of rewards, or credit for, services rendered. Few men will agree that moral economy deserves a place in education, and in thought, ahead of political economy; but to one who has considered all sides of moral organization this conclusion is unavoidable.

In the field of every economy there are possible lines of action ranging all the way from those whose economic value is clear to those whose economic value is obscure. All in all, common sense is less than half so adequate as it is if combined with knowledge of principles of economy. Principles of moral economy should not overrule moral laws, but should be applied where they do not conflict with moral laws. Moral laws should receive all of the attention that they now receive, and probably more; but principles of moral economy should receive, in relation to moral laws, just as much attention as do principles of political economy, in relation to political laws.

There is already education in morals to quite an extent; but there is no education in moral economy whatsoever. The science of moral economy is very important, very urgent; the development of this science will yield greatest returns for the effort spent on it. Scientific philanthropy, social efficiency, political engineering, and kindred subjects, are somewhat like moral economy; but all of these are relatively narrow in scope, they might be called *branches* of moral economy. Principles of moral economy are universal principles; they are useful to all men who are trying to do good, useful in all walks of life.

B. H. Somerville.

Norwood, Virginia.

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# THE MONIST

## EMILE BOUTROUX

THE death of Emile Boutroux, the well-known philosopher and one of the most profound thinkers of modern times, will be regretted by all readers of THE MONIST. The one work by which he is best known is his celebrated thesis, *De la Contingence des Lois de la Nature,* which he wrote when quite a young man, about the same time inaugurating his teaching of high philosophy at the Ecole Normale. Appointed Professor of History and Modern Philosophy at the Sorbonne in 1888, he always belonged to the University of Paris, of which he was honorary professor at the time of his death—an event accelerated by the loss, a short time previously, of his wife, the faithful and devoted collaborator in all his activities.

He wrote a number of books which gained a high repute, among them *Socrate, Fondateur de la Science Morale, De l'Idée de Loi Naturelle dans la Science et la Philosophie Contemporaines, Science et Religion,* and *Etudes d'Histoire de la Philosophie.* A Director of the Fondation Thiers, he was brought in contact with, and exercised a strong educative influence on quite an élite of young men who lived there during the final stage of their career as students of philosophy. He was elected a member of the Institute in 1898 and of the French Academy in 1912. In addition to various other distinctions he was Grand Officer of the Legion of Honour and his name had recently been put forward in connection with the Nobel Prize.

His was a rare and singularly attractive nature, one quite unspoiled by the sordidness of so much we see around us in modern civilized life. Combining a delightful disingenuousness with profound erudition and insight, he exercised the most beneficial influence on the world of letters in France, while his engagements in England and America considerably increased his circle of friends and well-wishers. So magnetic and sympathetic was he that it was impossible to feel anything else than at home in his presence. He would give the best of himself and elicit quite naturally and unconsciously the best from his interlocutor. In his correspondence, too, is manifested an unusual power of self-effacement and interest in another's concerns. This precious quality effectually endeared him to the hearts of many; one had in his company a strange feeling of upliftment as though one were breathing a purer and more refined atmosphere.

That which, in addition to the sense of a perfectly balanced judgment, struck one most in his personality, was that the intellect had not been cultivated at the expense of the heart. Those who had read his illuminating sketches of Pascal and Jacob Boehme must be vividly conscious of the author's subtle mysticism and power to grasp the inner and hidden realities of life. Deep, too, was his insight into German mysticism and philosophy, only equalled by a Gallic clarity and power of exposition that sheds a flood of light on the most recondite of thoughts.

In his home and university life he was deeply revered by his pupils. During the whole course of the war, the Fondation Thiers was a hive of activity, for like practically every other public building in France it had been converted into a hospital for war service. While engaged as an orderly at the Hôtel Majestic in the Avenue Kléber, it was frequently my duty—and a welcome change—to conduct some of the convalescent wounded to a select concert given at his

residence, or to go there alone, when I had a little spare time, to enjoy the intimacy of his study and occasionally listen to him as he sang, from a collection of old French ballads, some of his favorite songs, such as *En passant par la Lorraine,* or the *Chanson des Métamorphoses.*" or again *Derrière chez mon père* (*Les Trois Princesses*). This he would do without any thought of musical accompaniment, just giving way to the impulse of the moment in a spirit of delightful spontaneity.

Altogether his was a life of singular usefulness and beauty, nor will it be easy to fill the place of a master from whose intuition and learning such intellects as those of Henri Bergson and Pierre Lassere received their inspiration. Unostentatious and unassuming, neither courting the public gaze nor appealing to the masses, he yet exercised a great and beneficent power both in his spoken and in his written words, and, if he did not actually create a new school of philosophy, he powerfully moulded the though of his age and did perhaps more than any of his European contemporaries to humanize philosophy and ethics. He had considerable intellectual affinity with William James; a warm personal friendship existed between the two great thinkers of the Old and the New World.

His loss must have been keenly felt; and *emeritus* is indeed a term that may most fittingly be applied to so noble and devoted a character.

*Sit illi terra levis!*

FRED ROTHWELL.

LONDON, ENGLAND.

# THE PHILOSOPHY OF EMILE BOUTROUX

BY THE DEATH of Emile Boutroux (which occurred in November of 1921), contemporary French philosophy has been deprived of one of its champions and international thought laments the loss of a valued thinker. Boutroux preserved a wonderful vitality, and an alertness of mind which was astonishing for one who was seventy-six. Shortly before his death the present writer had the pleasure of visiting him in Paris and noted his keen interest in social events and in approaching developments in political thought. It was characteristic of Boutroux to look to the future for inspiration rather than to the past. His mind seemed a particularly good manifestation of that *élan vital* of which his pupil Henri Bergson has written, being intensely active and pushing as it were continually *"en avant."*

Boutroux's mental pilgrimage throws an interesting light upon modern French philosophy. He was a notable thinker of the group whose ideas came to dominate French thought in the last quarter of the last century, the New Spiritualists or Idealists. These later thinkers rejected, not only the doctrines of materialism, naturalism and positivism, against which they took the field in determined opposition, but also the older idealism, and vague teachings of Cousin and his followers. The Eclecticism of Cousin influenced a whole generation of his countrymen. He upheld spiritual ideals, but his philosophy was very largely

an importation from a foreign country. He had spent some years in Germany and incorporated the doctrines of Schelling and Hegel along with other ideas from the ancients to form a romantic idealism. With this he combined certain doctrines which came from the Scottish school of common sense. By this wide interest, we must admit, Cousin did much to establish and encourage the study of the history of philosophy. At times, however, it seems that he was prone to confuse the history of philosophy with philosophy itself. There is perhaps no branch of science or art so intimately bound up with its own history as is philosophy, but we must certainly beware of substituting an historical survey of problems for an actual handling of those problems themselves. Cousin's own aim was to found a metaphysics spiritual in character, based upon psychology. The chief defect in his own philosophy was simple but disastrous. The older idealism had no place within it for positive science. Philosophizing was to be dependent upon introspection. Now it was precisely because of this vagueness in Cousin's teaching that such a welcome was accorded by many minds to the positivism of Comte.

In Comte, modern science made a claim to consideration by philosophers. Much of Comte's science has been surpassed, his neglect of psychology (and ethics) was a serious defect. His dogmatism called forth the denunciations of the great thinker Charles Renouvier. The several currents of development, however, in France did not follow out Renouvier's *néo criticisme* and *personnalisme*. He and Comte share between them the highest honors of the century in France as far as philosophy is concerned, but neither were professional, academic teachers in the University and for this reason their doctrines came but slowly before the French public.

An important event occurred in the very year of Cousin's death, an event which heralded the development of

the best that was to be in the intellectual life of the century. This event was the foundation of the New Spiritualism, by Ravaisson's celebrated manifesto to idealists, for such was his *Rapport sur la philosophie dans le dixneuvième Siècle* issued in 1867 for the "Exposition Universelle" at the request of the French Ministry of Education. Ravaisson had at an earlier date opposed Cousin by his praise of Maine de Biran. Cousin was so annoyed by Ravaisson's criticism that he excluded him from the Institute.

Ravaisson's Report laid the foundations of a new Idealism and dealt a blow to both the eclectic school of Cousin and to the followers of Auguste Comte. Ravaisson himself wrote little but his influence was powerful and ultimately made itself felt upon the minds of the younger men in the University of Paris, notably, Lachelier, Boutroux and Bergson. A noble tribute to his memory was given by this last thinker when he took Ravaisson's place at the *Académie des Sciences Morales et Politiques* in 1904.

It was Ravaisson's chief merit that he was able to show that the utter inadequacy of Cousin's vague idealism lay in its premature assumptions, its scorn of science and the lack of the discipline which comes from a study of the positive sciences. Ravaisson saw that a valid idealism must not scorn science, but work along with it; even if it finds science inadequate it will not judge it false.

With this inspiration from Ravaisson, Lachelier continued the expression of the New Spiritual Philosophy in his brilliant little thesis on "Induction," and the important article "Psychology and Metaphysics" which attempted what Cousin had been unable to effect. Lachelier finds the pure mechanism of efficient causes inadequate to explain reality. Some principle of final causes operates, and an understanding of this is, he shows, necessary for philosophy. Only by realizing the need for an outlook and interpretations beyond the purely scientific can philosophy pro-

ceed. Such a procedure involves a certain critique of science, a discussion of spiritual values, and of the possibility of freedom.

It was at this point that Boutroux took up the subject and made his influence felt. He appeared upon the philosophical field and entered the arena of discussion at a critical and interesting time. Science, philosophy and religion were each endeavoring to justify their existence. The rigorous positivism of Taine differed from that of Comte. Its narrowness and dogmatism appeared crushingly untrue to some souls. A real crisis had arrived in French thought, a conflict between the dogmatism of finalism of science on the one hand and the claim of man's spirit and the assertion of his beliefs on the other. It was a conflict of naturalism, *la science* v. *la conscience*. Into this intellectual *milieu* came Boutroux.

Born in the department of the Seine in 1845, he had been through the best schools of Paris, the Lycée Henri-Quatre, the Ecole Normale Superièure and the Sorbonne. After taking his "agrégation" in philosophy he, like Cousin, passed for a time under the influence of German thought and culture. He went, just prior to the Franco-Prussian War, to Heidelberg, where he studied under Zeller, the great authority on Greek philosophy, part of whose work he later translated for his own countrymen. Already young Boutroux observed a change in Germany from the days of Cousin. He felt a foreboding as he saw that the Germany of Goethe and other inspirers of the human race had given place to a less refined spirit, born of materialism and imperialism, and dragging along a third power, militarism, to complete her trio of disgraces.

Returning to France, Boutroux presented his Thesis *Sur la Contingence des Lois de la Nature* and obtained his *Docteur ès Lettres* degree in 1874. This thesis, which was published in 1879, was dedicated to Ravaisson. This is a

significant indication of Boutroux's position. He was to throw his influence on the side of the New Spiritualist movement begun by Ravaisson and carried forward by Lachelier. To make clear how he did this is the purpose of this paper. Before passing from his career, however, we may observe that after teaching philosophy at Caen, Montpellier, Nancy, then at the Ecole Normale, he became Professor of Philosophy and of the History of Philosophy at the Sorbonne in 1888. Then in 1902 he became Director of the institution known as the "Fondation Thiers" or "Institut Thiers," where about a score of picked men from the University carry on research work in various pursuits, both literary and scientific, living a communal life for three years.

By this time Boutroux had an international reputation and was consequently appointed Gifford Lecturer for 1904-5. He delivered courses of lectures in Scotland on "La Nature" and on "L'Esprit," but these lectures have not been published. At the International Congress of Philosophy held in 1908 at Heidelberg, Boutroux gave a paper on "French Philosophy since 1867." He selected this date because it enabled him to carry on his survey from the point where Ravaisson had concluded his "Report." He succeeded in showing that French thought in the closing years of the century merited the highest attention of serious students of human thought. In the same year he issued his *Science and Religion in Contemporary Philosophy*, translated into English the following year, and in this book many of his views on Nature and Spirit find their expression. He then became President of the *Académie des Sciences Morales et Politiques* and in 1914 was elected to the *"Académie francaise."* He delivered the Hertz Lecture to the British Academy in the same year, taking as his subject "Certitude and Truth." This, along with other papers dealing with German political and intellectual de-

velopment, was published during the war (1916) in a collected volume under the title *Philosophy and War*. The close of the war brought out a false report of his death. The French press contained long obituary notices. In 1919 his wife died. It is worth noticing that she was a sister of Henri Poincaré, the eminent French mathematician. Pierre Boutroux, the illustrious son, now holds the Chair of Mathematical Science at the Sorbonne. Emile Boutroux, a beloved and respected figure in French life and thought, passed away in November of 1921.

The particular work he performed, his place in philosophical thought, can only be appreciated when one has grasped the precise nature of the intellectual environment in which he found himself as a young man. This having been shown by tracing the development from Cousin, onwards through Comte and Renouvier to Ravaisson and Lachelier, we can now see how Boutroux's thought opened up the way for that of Bergson, Le Roy and Blondel.

The new spiritualist philosophers had set the notion of freedom and of the spontaneity of the spirit in the forefront of their philosophy as watchwords in the intellectual fight. Under the work and influence of Boutroux, these ideas were further emphasized and worked out more definitely to a position which assumes a critical attitude to the dogmatism of modern science and establishes a contingency in all things. Boutroux's chief fame and importance in the development of the spiritualist philosophy rests upon his thesis on *"The Contingency of the Laws of Nature."* In 1894, he published a course of lectures given at the Sorbonne in the years 1892-3, On the Idea of Natural Law, a book which in some respects supplements the thesis. There was a demand for the republication of the thesis in the following year. This was mainly due to the fact that the work of Ravaisson and Lachelier was attaining a recognition formerly denied to it. Also the book of Henri

Bergson, one of Boutroux's pupils, on *Les Données immediates de la Conscience* (or to give it its more descriptive English title *Time and Free-Will*) had appeared in 1889. The masterpiece of Blondel, *L'Action,* came in 1893. It was seen how important were Boutroux's ideas in relation to the development of this current of thought. He had combined the attitude of Ravaisson with that adopted by Lachelier. The totality of the laws of the universe manifests, according to Boutroux, a contingency. No explanation of those laws is possible apart from a free spiritual activity. The stress laid upon the contingency of the laws of nature thus leads up to the question of freedom and to the philosophy of a spiritual activity indicated in the later thought of Bergson, Le Roy and Blondel. In addition the critique of science which marks Boutroux's work profoundly influenced thinkers like Hannequin, Payot and Milhaud,[1] and in the twentieth century appears in the work of Duhem and of Boutroux's brother-in-law, Henri Poincaré, whose books on science and the philosophy behind the sciences are well known.

Boutroux has certain affinities in his attitude to science with two thinkers whom we have already mentioned, Renouvier and Comte. This is because of his insistence upon the discontinuity of the sciences, upon the element of "newness" found in each which prevents the higher from being deduced from the lower, or the superior explained entirely by reference to the inferior. Boutroux opposes Spencer's doctrines and is a keen antagonist of Taine and his claim to deduce all from one formula. Such a notion as that of Taine is quite absurd according to Boutroux, for there is no necessary bond between one and another science.

[1] Hannequin's notable work is the *"Essai Critique sur l'Hypothèse des Atomes,"* 1896.

Payot's chief book is *"La Croyance,"* 1896, while Milhaud's critique of science is contained in his *"Essai sur le Conditon et les Limites de la Certitude Logique,"* 1894, and in the volume, *"Le Rationel,"* 1898.

This is Boutroux's main point in *La Contingence des Lois de la Nature.*

By a survey of laws of various types, logical, mathematical, physical, chemical, biological, psychological and sociological, Boutroux endeavors to show that they are constructions built up from facts. Just as nature offers to the scientist facts for data, so the sciences themselves offer these natural laws as data to the philosopher for his constructed explanation of things which is metaphysics.

"In the actual condition of our knowledge," he remarks, "science is not one, but multiple; science conceived as embracing all the sciences is a mere abstraction," a remark which recalls Renouvier's witty saying, "I should very much like to meet this person I hear so much about called 'Science.'" We have only sciences, each working after its own manner upon a small portion of reality. Man has a thirst for knowledge, and he sees, says Boutroux, in the world an *"ensemble"* of facts of infinite variety. These facts man endeavors to observe, analyze, and describe with increasing exactness. Science, he points out, is just this description.

It is futile to attempt a resolution of all things into the principle of identity. "The world is full of a number of things," and therefore, argues Boutroux, the formula A=B can never be strictly and absolutely true. "Nature never offers to us identities but only resemblances." This has important bearing upon the law of causality of which the sciences make so much. For there is such a degree of heterogeneity in the things to which the most elementary and general laws of physics and chemistry are applied that it is impossible to say that the consequent is proportional to the antecedent, that is to say, it is impossible to work out absolutely the statement that an effect is the unique result of a certain invariable cause. The fundamental link escapes us, and so for us there is a certain contingency in

experience. There is, further, a creativeness, a newness which is unforeseeable. The passage from the inorganic to the organic stresses this, for the observation of the former would never lead us to the other, for it is a creation, a veritable "new" thing. Boutroux is here dealing hard blows at Taine's conception. He continues it by showing that in the conscious living being we are introduced to a new element which is again absolutely irreducible to physical factors. Life, and consciousness too, are both creators. The life of the mind is absolutely "sui generis," it cannot be explained by physiology, by reflex action, or looked upon as merely an epiphenomenon. Already Boutroux finds himself facing the central problem of Freedom. He recognizes that as psychological phenomena appear to contain qualities not given in their immediate antecedent, the law of proportion of cause to effect does not apply to the actions of the human mind.

The principle of causality and the principle of the conservation of energy are in themselves scientific "shibboleths," and neither of them, asserts Boutroux, can be worked out so absolutely as to justify themselves as ultimate descriptions of the universe. They are valuable as practicable maxims for the scientists, whose object is to follow the threads of action in this varied world of ours. They are incomplete and have merely a relative value. Philosophy cannot permit their application to the totality of this living, pulsing universe. For cause, we must remember, does not in its strictly scientific meaning imply creative power. The cause of a phenomenon is itself a phenomenon. "The positive sciences in vain pretend to seize the divine essence or reason behind things."[2] They arrive at descriptive formulae and there they leave us. But, as Boutroux well reminds us in conclusion to his thesis, formulae never explain anything, because they cannot even ex-

[2] *Contingence des Lois de la Nature,"* p. 154.

plain themselves. They are simply constructions made by observation and abstraction and which themselves require explanation.

The laws of nature are not restrictions which have been, as it were, imposed upon her. They are themselves products of freedom, they are in her what habits are to the individual. Their constancy is like the stability of a river-bed which the freely running stream at some early time hollowed out.

Boutroux, summing up his thesis, indicates clearly in his concluding chapter, his belief in contingency, freedom and creativeness. The old adage, "Nothing is lost, nothing is created," to which science seems inclined to attach itself, has not an absolute value, for in the hierarchy of creatures contingency, freedom, newness appear in the higher ranks. There is at work, no doubt, a principle of conservation, but this must not lead us to deny the existence and action of another principle, that of creation. The world rises from inorganic to organic forms, from matter to spirit and in man himself from mere sensibility up to intelligence with its capacity for critizing and observing, and to will capable of acting upon things, modifying them by freedom.

Boutroux inclines to a doctrine of finalism somewhat after the manner of Ravaisson. The world for him is attracted to an end, the beautiful and the good are ideals seeking to be realized, but this belief in finality does not, he expressly maintains, exclude contingency. To illustrate this, Boutroux uses a metaphor from seamanship: the sailors in a ship have a port to make for, yet their adaptations to the weather and sea *"en route"* permit of contingency along with the finality involved in their making for port. "So it is with beings in nature. They have not merely the one end, to exist amid the obstacles and difficulties around them, they have an ideal to realize, and this

ideal consists in approaching to God, to his likeness each after his kind. The ideal varies with the creatures, because each has his special nature and can only imitate God in and by his own nature."[3]

Boutroux's doctrine of freedom and contingency is not opposed to a teleological conception of the universe, and in this respect he stands in contrast to Bergson, who, in the rigorous application of his theory of freedom, rules out all question of teleology. With Renouvier and with Bergson, however, Boutroux agrees in maintaining that this freedom which is the basis of contingency in things is not and cannot be a datum of experience, directly or indirectly, because experience only seizes things which are actually realized, whereas this freedom is a creative power, anterior to the act. Heredity, instinct, character and habit are words by which we must not be misled or overawed into a disbelief in freedom. They are not absolutely fatal and fully determined. The same will, insists Boutroux, which has created a habit *can* conquer it. Will must not be paralyzed by bowing to the assumed supremacy of instincts or habits. Habit itself is not a contradiction of spontaneity, it is itself a result of spontaneity, a state of spontaneity itself, and does not exclude contingency or freedom.

Metaphysics can therefore, according to Boutroux, construct a philosophy of freedom based on the doctrine of contingency. The supreme principles according to this philosophy will be laws, not those of the positive sciences, but the laws of beauty and goodness, expressing in some measure the divine life and supposing free agents. In fact, the triumph of the good and the beautiful will result in the replacement of laws of nature, strictly so called, by the free efforts of wills tending to perfection, that is to God.

Further studies upon the problem of freedom are to be found in Boutroux's lectures given at the Sorbonne in

[3] *La Contingence des Lois de la Nature*, p. 158.

1892-3 in the course entitled *De l'Idée de la Loi naturelle dans la Science et la Philosophie contemporaines.*" He there recognizes in freedom the crucial question at issue between the scientists and the philosophers, for he states the object of this course of lectures as being a critical examination of the notion we have of the laws of nature, with a view to determining the situation of human personality, particularly in regard to free action.[4] Boutroux recognizes that when the domain of science was less extensive and less rigorous than it is now it was much easier to believe in freedom. The belief in Destiny possessed by the ancients has faded, but we may well ask ourselves, says Boutroux, whether modern science has not replaced it by a yet more rigorous fatalism. He argues that modern determinism rests upon two assumptions, namely, that mathematics is a perfectly intelligible science and is the expression of absolute determinism, also that mathematics can be applied with exactness to reality. These assumptions the lecturer shows to be unjustifiable. Mathematics and experience can never be fitted exactly into each other, for there are elements in our experience, in our own nature, which cannot be mathematically expressed. This Boutroux well emphasizes in his lecture (XIII) upon sociological laws, where he asserts that history cannot be regarded as the unrolling of a single law, nor can the principle of causality strictly speaking be applied to it. An antecedent certainly may be an influence, but not a cause as properly understood. He here agrees with Renouvier's position and attitude to history.

Instead of the ideal of science, a mathematical unity, experience shows us, Boutroux affirms, a hierarchy of beings, manifesting variety and spontaneity, in short, freedom. So far, therefore, from modern science being an advocate of universal determinism it is really, when rightly

[4] *De l'Idée de la Loi naturelle*, Lecture IV, p. 29.

regarded, a demonstration, not of necessity but of freedom. Boutroux's treatment of the problem of freedom thus demonstrates very clearly its connection with that of science, and also with that of progress. His docrine of contingency is directly opposed to any rigid pre-ordained plan of reality or progress, but it does not prevent the spirit from a creative teleology, the formation of a plan as it advances. This is precisely, is it not, that creative determinism, the combination of free action and of teleology which we find in our own lives? Boutroux is thus able to side with Ravaisson in his claim to see tendencies to beauty and truth and goodness, the fruits of the spirit, which it creates and to which it draws us, while at the same time he maintains freedom in a manner quite as emphatic as Lachelier, and he carefully reminds us that "not all developments are towards perfection."

The world is an assembly of beings and its vitality and nature cannot be expressed in a formula. It comprises a hierarchy of creatures, rising from inorganic to organic forms, from matter to spirit, and in man it displays an observing intelligence, rising above mere sensibility and expressly modifying things by free will. In this conception Boutroux follows Ravaisson and he is also influenced by that thinker's belief in a spiritual power of goodness and beauty. He thus leads us to the sphere of religion and philosophy, both of which endeavor in their manner to complete the inadequacy of the purely scientific standpoint. He thus stands linked up in the total development with Cournot and Renouvier, and in his own group with Lachelier also, in regard to this question of science.

We have said that much of Boutroux's work was critical of science and that the critique of science was carried on by several other thinkers. These, however, were not always in line with the spiritualist development of thought. They represent rather a sub-current running out and sep-

arated from the main stream in which Boutroux's thought flowed. This is shown prominently in the fact that while Boutroux's critique of science is in the interests of a valid idealism and the maintenance of some spiritual values, much of the subsequent criticism of science is a mere empiricism, which, being divorced from the general principles of the new spiritualist philosophy, tends merely to accentuate a vein of uncertainty, indeed, scepticism of knowledge. Such is the general standpoint taken by Milhaud, Payot and Duhem.

Boutroux's aim was not of this kind. His critique of science was a serious task not undertaken in any light spirit, but it was only a means to an end. The end for him was the indication of the principles of a truly spiritual philosophy, not one which, like that of Cousin, suffered from vagueness and had no place for science with which it found itself in conflict, but a valid idealism which could boast of having passed so to speak through the fire, the discipline of strict scientific principles, and attained triumphantly a position beyond them, but not in opposition to them. This Boutroux rightly realized to be the task of philosophy in his own and other lands.

Boutroux's devotion to *La Nature* did not obscure his study of *L'Esprit*. He looked upon life steadily and endeavored to see it whole. He was fully conscious of the importance of those disciplines of the human mind which make for the study of spiritual ideals and values other than those which are contained in the narrow rationalism of the positive sciences. He wrote on ethics, on education and on religion. From his pen came the preface to the French edition of William James' work on *The Varieties of Religious Experience*.[5] Boutroux delighted in the study of Jacob Boehme, the old German mystic. In the interesting conclusion to his book on *Science and Religion in Contem-*

[5] He also wrote a monograph on James.

*porary Philosophy,* Boutroux, after affirming that the essential piety of religion is found in all searchings of man's spirit for truth, for goodness and beauty, sums up in the words of the old mystic his attitude to the diversity of religious opinions. "Consider the birds in our forests, they praise God each in his own way, in diverse tones and fashions. Think you God is vexed by this diversity and desires to silence discordant voices? All the forms of being are dear to the Infinite Being Himself." Boutroux was too clear and well balanced to adopt towards religion the hostile attitude of the French thinkers of the eighteenth century. They thought that the human mind would very quickly come to reject all that could not be proved true on strictly rational grounds They sadly miscalculated the bases upon which religion reposes; they stressed with consequent disaster and reaction one aspect of the human consciousness to the exclusion of others the emotions and the will. Boutroux recognized this, and, while the mind could have been more intellectually clear and honest, he realized the limitations of a severe rationalism which should ignore the other elements in man's nature. Consequently, his mind went back, not to the doctrines of Voltaire, but to the thought of Pascal, which clung devotedly to science as well as religion; he went back also to that doctrine of "Nous" which was a feature of Greek thought at its best. To the Greek mind this conception did not imply merely a cold exercise of intellect or rationalizing power. The "Nous" or supreme quality of mind lay in a harmony, a balance of the whole mind with its powers of knowing and of feeling and willing. This was indeed the Supreme Beauty, seeking to ally itself at once with both Truth and Goodness.

Boutroux indeed went back for these fuundamental principles but only in order to project into the future what he thought was truest and noblest among human thoughts and aspirations. In the increasing striving towards the furtherance of these ideals he saw the course of that true spiritual development, at once strictly positive and idealistic, which he himself, by his work and his personality, had done so much to promote.

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# THE TREND OF MODERN THOUGHT[1]

AT A FIRST GLANCE, there would appear to be something contradictory in the attitude of men nowadays on the subject of philosophy. It is the custom to disparage it as being productive of empty abstractions, and yet, at the same time, we find in quite a flourishing state a philosophic literature that stirs to enthusiasm a serious and numerous reading public. It is even remarked that statesmen and journalists, novelists and critics, eagerly take up subjects of philosophic import and are given credit for profound knowledge, though they deal with their material in a very cursory fashion.

This apparent contradiction is less evident if we consider that the mode of philosophizing in vogue at the present time is very different from that of the past. From a philosophy unrelated to life and science and claiming to find in pure reason all the elements and objects of its existence and development we turn away, regarding it, for the most part, as empty formalism, an artificial structure, a survival of scholasticism. We see in it only something that satisfies the mind, something that is worthless to those who, through contact with positive science and living reality, have acquired a sense of certainty. On the other hand, however, we extend an eager welcome to such philosophic thought as would seem to be the legitimate product of a collaboration between the mind and the things of the sen-

[1] Authorized translation by Fred Rothwell.

sible world, presenting itself as a sincere interpretation of science and life, not as a more or less ingenious and novel exegesis and combination of concepts worked out by philosophers of the past.

Our age is tired of a philosophy which claims to be self-sufficing, to nourish itself exclusively on its own substance. Rather does it demand one based on experience, on reality universally recognized as such, and on positive science. It demands an answer to such questions as the following: What is the world? What are we? How are we to act if we would play our part as human beings to the best of our ability?

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Not by chance has philosophy, for some time past, seemed jealously bent on creating a sphere of its own and sufficing unto itself, apart from the science of the world of sense.

Ancient philosophers, Plato and Aristotle, for instance, regarded nature as susceptible of divinity, as herself more or less divine. Thus spirit, for the living of its own life, could rely on nature to summon her to itself. Its whole ambition, moreover, consisted in contemplating nature and finding in her the dominion of the eternal laws of reason —a dominion wherein it had a direct participation. With the advent of Christianity, however, nature underwent a change. Now, she is no more than an inert thing, wholly external to spirit, which created her *ex nihilo*. On this point, modern science is strangely at one with the Judaeo-Christian religion, regarding nature as a crude machine wherein the eternally identical play of given and immutable material forces of itself produces all the phenomena of life, without there ever being room for a guiding thought.

How then can spirit, in its dealing with nature, find for itself an element of life and growth? To unite with

nature would be to abandon and betray—to do away with —itself. On the other hand, by setting itself over against nature, spirit becomes distinctly conscious of what is proper to itself; it ensures for itself full liberty of action and of expression. The reduction of nature to principles that are anti-spiritual thus becomes for spirit, in so far as it escapes from them, the occasion of a new and powerful affirmation of its own distinctive life and originality.

Now, however, positive science, originally well pleased to explain by mechanical principles what are called external phenomena and respectful of the mystery which has seemed to envelop life and thought, has come to consider itself possessed of methods which enable it to subject to its own laws all forms of being without exception. Even the prodigiously increased dominion of mankind over things now makes the position of spirit with reference to nature appear in a new light. If man can modify the course of phenomena to this extent, then he is himself a phenomenon analogous to the rest. The sage of old, who could do no more than contemplate the eternal laws of being, was unable to feel himself one therewith as does the modern scientist who utilizes these laws. If wind and stream combine their action, then they are homogeneous forces. To control nature is to form part thereof.

Besides, where could spirit, apart from nature, find the fixed center, the principle of determination necessary for it to act, i. e., to be? In the past there was God. Modern criticism considers that in the concept of this God there existed many elements taken from nature herself, and that if an attempt is made to reduce it to its strictly suprasensible content, the concept is found to disappear. Amongst the main streams of contemporary thought, one of the strongest is that which turns us aside from the transcendental heaven of Epicurus—where it is not known if there are beings that toil and labor on earth—and leads us in

the direction of the world of matter and temporal life, the object of science, the unapparent though sure basis of all our actions, desires and thoughts.

Such, then, at a time when we are enquiring if idealism is still a possible attitude to take up, are the terms of the problem: Given that spirit, so far as we know anything about it, is inseparable from matter—a matter whose laws seem self-sufficing—can there yet be any free original life of spirit? Thus stated, it must be confessed that the problem appears singularly embarrassing.

In the first place, a grave concession is inevitable. It is by no means contradictory, it is even far simpler, both logically and practically, to acknowledge that naturalism is right. It is possible to live a purely natural life, all that is needed is to give oneself up to the stream of events, not to resist the law of inertia, which, of itself, is realized in all that is.

Naturalism is one possible solution of the problem of human life; does it follow that it is the necessary solution? Suppose I refuse to be content with it, has any one the right to bring against me the reproach that my attitude is the expression of anything more than individual fancy?

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The general idea that results from any investigation into contemporary thought may be formulated as follows: man is either less or more than, as a rule, he thinks himself to be.

If positive science alone is the standard of truth and possibility, man is less than he thinks he is. Individuality and personality, dignity and moral worth, the special rôle and higher destiny he persists in attributing to himself, are in contradiction not only with the actual conclusions, but also—a more serious matter—with the principles and methods, nay, with the very spirit of positive science. If science is the whole of true knowledge, we must see in the

ideas on which human life is based something more than vain traditions which arise out of our predecessors' mistakes and ignorance.

But if science, of itself alone, is not the standard of the true, we must cease to set it up in opposition—as a judge from whom there is no appeal,—to spirit, which is bent on being and acting. Indeed, spirit aims at proceeding hand in hand with science, but what exactly is it that science imposes on it?

The difficulty of the question is found in the idea we form of the relation between science and spirit. Is science the expression of truth, an absolute *per se,* something stereotyped, which spirit can only consider passively from without and endeavor to describe? In that case, its postulates are, to us, the ultimate expression of truth; i. e., it is our duty to regard mechanical determinism and crude fact as fundamental principles of Being. Any notion, then, that contradicts this mechanism, must be regarded as an illusion; nor is it difficult to show that this is the case with every principle that gives human life its form.

But perhaps science, as also language, art, civil laws, and religions, may be regarded not as something external to spirit but as an activity of spirit itself, so that even its deepest principles can be understood in their true meaning only as they refer to the thought on which they are built up and moulded. In that case there would exist nothing rigid or stereotyped, either from or for eternity, whether in science or in the objects around us. Spirit is life and creation. If scientific determinism is its product, it shows forth as a mould, whose consistence and rôle are not necessarily immutable. Scholasticism was simply the transformation of the living word into a dead and finished system. The substitution indeed for active thought, of scholasticism or of thought stereotyped by instruction, is in nature, for it is only an application of the natural and

general law of habit. But it is not therefore necessary. Man may retain his activity and spiritual power by reacting against the sloth which renders him disposed to abdicate in favor of his habits.

Up to quite recently the human mind was fascinated by the clarity, the utility of science, and was dominated by it; at present it has a tendency to see that science is eminently life and activity, an urge in the direction of something better, and to restore science to that interior life whence it really proceeds.

This is equivalent to saying that human mind is determined to break through pure naturalism, and, while relying on nature, to seek after ends that transcend nature. What is it that determines these ends?

There is one system still favored by many philosophers, which is regarded as calculated to satisfy the mind; seeing that, whilst transcending naturalism, it endeavors to avoid the danger inevitable in individual predilections. That system is intellectualism.

Certainly this system frees us from the tyranny of the immediately given, granting a life other than that of the senses. But the principles it seeks behind sensible facts are still themselves facts in reality, rough impenetrable data which, when we try to discover their raison d'être, like the inert symbols that writing substitutes for living thought, maintain a solemn silence. The motto of intellectualism is: ἀνάγκη στῆναι, i. e., motion implies rest; the divisible implies the indivisible; the contingent implies the necessary and time implies eternity. An artificial motto, impossible to imagine as capable of realization; for neither can the analysis of change lead to the immutable, nor can there be any intuition enabling us absolutely to grasp primary elements. Intellectualism represents the despondency of spirit retreating before an endless task and demanding rest as the price of its effort. This, however,

reality refuses: it does not weary of creating, however man may weary of conceiving. Instead of its life being only the mechanical gesticulation of a dead body, it really does live. Nobody has ever succeeded, a priori, in dictating laws to reality. We think erroneously, first observing what nature has done and then endeavoring to classify her productions with the object of discovering, if possible, some of her habits. Our knowledge, ever relative to the extent of our observation and our mental adaptation, remains under subjection to things, and we have no right to be shocked if, because things cannot be reduced to our abstractions, they demonstrate to us that they really are.

Passing then beyond both naturalism and intellectualism, we have to find a point of view which will maintain the reality and value of nature without plunging spirit therein, and will ensure the supremacy and activity of spirit whilst also recognizing its union with nature.

Eucken regarded Fichte's philosophy as indicating the path to be followed in solving the problem. According to Fichte, spirit, being essentially active, dominates everything, but its activity is exercised by means of nature and the intellect. So it is in the Fichtean sense that Eucken would build up the concrete idealism which he imagined contemporary thought to be seeking.

On the one hand he established the reality that is distinctive of spirit as life and potency of creation by basing it on the reality and originality of the all. Spirit wills to be in self and for self; now, according to its essence this existence cannot be something superior both to objectivity pure and simple—or existence for others—and to objective immutable thought, which so far is no more than an abstraction. Spirit *is* only if it acts. It is not a thing, something susceptible of acting; it is action and life itself. Everything within it unfolds, is opposed to inertia, generates, creates and engages in self-creation.

On the other hand spirit does not function in a vacuum; its activity consists in reducing things to itself, in permeating and spiritualizing them. It is not an addition to nature like Kant's freedom-noumenon; it is immanent therein, guiding nature's action, of which itself is essentially the prime author.

The new idealism then instead of being set up apart from science, art, religions, given realities, according to the dualistic conception, finds, in the given, the very matter by whose aid it endeavors to realize spirit.

Before the natural tendency of the creature to remain fixed in its mode of being and detach itself from creative spirit, its task is to react against this inertia and constantly to awaken life in the human soul by resolving it into its principle.

Des Menschen Tätigkeit kann allzuleicht erschlaffen,
Er liebt sich bald die unbedingte Ruh.

Let us not leave to Mephistopheles the task of shaking man out of his natural sloth. The spirit of affirmation and creation is also movement and effort. It also is true action, for to deny and destroy is but to yield to the blind force of dissolution, which tends to drive things on in the direction of sheer annihilation or nonentity.

Eternal life is not a contradiction in terms if it is no more than the organization, by spirit, of matter that is infinite.

Nourished by science and the experience of practical life, the philosophic spirit, which is within us the most immediate expression of the universal spirit, is no simple efflorescence of given reality. It is reason, and at the same time faith and risk: ein Suchen und Versuchen, ein Wet-

ten und Wagen. Knowledge, thought, the spoken word, are all necessary, as also is toil and effort after that which is uncertain. If the value of the intention remains whole and entire, whatever happens, the perfection of the product and its capacity for maintaining life can never be known until afterwards. The greatest creations are always those that call forth the greatest number of new creations.

EMILE BOUTROUX.

# MENTAL INSTABILITY AS A FACTOR IN PROGRESS

OF the recent developments whch have contributed in a marked degree to revise and enlarge our ideas of human progress, that of psycho-analysis probably stands pre-eminent. Concerned in its beginnings mainly with the study and treatment of mental disease it is no longer confined to pathology but has extended its boundaries into the fields of education, sociology, philosophy and art. The movement, moreover, is as yet but in its infancy and we may with confidence predict that its growth will furnish us with still further vistas of human possibilities. At the moment, however, psycho-analysis would appear to be suffering the fate of all new developments: namely, the effects of an easy popularization, with the inevitable concomitants of dogmatism, fanaticism, shallow thought, sentimentality and dilettantism. This tendency is apparent not only in the narrower field concerned with pathology but also in the wider field concerned with education in its broader sense. In this paper I propose to discuss and criticize certain current ideas regarding what psychoanalysts term sublimation, by which is meant the directing of primitive, and largely anti-social tendencies into channels of a worthier kind.

The term "sublimation" is rapidly becoming clothed with sacred and religious meanings and we may soon expect it to be proclaimed as a new evangel replacing the

older schemes of salvation. Under the plea of avoiding or resolving conflicts, of harmonizing the various passions and tendencies in human nature and of adjusting these to the social customs and environment, psycho-analysts and their followers may become as solicitous about the welfare of the individual soul as are the salvationists. A healthy attitude of scepticism is justified in dealing with these schemes of so-called harmonization for it is quite possible that what may pass under the name of sublimation may become a mode of repression. Sublimation in its widest sense is no mere affair of the drawing-room or the academy: it is growth and life itself. What we are concerned with is not a superficial culture but rather the deeper forces moving in human nature. "Shall a man lose himself in countless masses of adjustments?" asks Whitman, "and be so shaped with reference to this, that, and the other, that the simply good and healthy and brave parts of him are reduced and clipped away like the bordering of box in a garden? You can cultivate corn and roses and orchards —but who shall cultivate the mountain peaks, the ocean and the tumbling gorgeousness of the clouds?" Professor McDougall in his *Social Psychology* envisages an ideal individual who—having attained character in the fullest sense and a completely generalized will, is raised above moral conflict and exhibits to the world that finest flower of moral growth—serenity. "His struggles are no longer moral conflicts but are intellectual efforts to discover what is most worth while, what is most right for him to do."

The criticism to be urged against this ideal is two-fold: first, it does not take sufficiently into account the recent psychology of the unconscious which shows forces at work of a strange and often disruptive order: secondly, the ideal appears to stand for a static, rather than for a dynamic, view of human personality, in that it envisages a more or less mechanical application of a completed character rather

than the expansion of the personality itself. Contrast the above statement of McDougall's with the affirmation of Nietzsche—"Yea, a thing unbearable is within me, a thing that blasteth rocks. . . . It is called my Will. . . . And this secret did Life itself utter unto me. 'Behold,' it said, 'I am whatsoever *must* surpass itself.'"

A somewhat similar line of criticism may be urged against the Freudians. In this case, we find the tendency to take the average type of human being as the normal: which type becomes a standard of reference, all variations from it being regarded as abnormal as soon as they show signs of neurosis. The main cause of the neurosis having been traced to the activity of some submerged "complex," the treatment consists in bringing this to the surface and thus restoring the patient to a "normal" condition, when the work of the psycho-analyst is supposed to be finished. The deeper and all-important problem is left untouched; namely, as to whether the so-called normal is desirable: in short, as to whether it is not the undesirable environment conditioning the neurosis that needs the surgeon's knife. This is not to suggest that primitive passions and tendencies should be allowed free-play: for that would involve the dissolution of society. Controlling forces embodied in social organization are indispensable for progress and the problem confronting us is concerned with the nature of these controlling forces and the manner in which they are to be applied. Childhood, for example is a period of continual conflict between primitive tendencies and the customs and ideals of society, and according to the direction given to the energy engendered depends the future character and life of the individual.

In the light of the foregoing consideration it is refreshing and stimulating to find a point of view being put forward which is at once a challenge to popular ideas on the subject and an indication of a line of advance full of im-

mense possibilities. I refer to a recent work by Dr. W. H. R. Rivers: *Instinct and the Unconscious.* The main purpose of this book is to suggest a biological view of psychoneurosis by linking together the two conceptions indicated in its title. For the purpose of this paper, however, I am concerned only with a single chapter in the book, that on Sublimation in which Dr. Rivers raises the whole question of the signifiance of conflict in relation to human progress. It is quite possible, the author points out, that by the processes of suppression and sublimation a person may become completely adapted to his environment and thereby attain a highly peaceful and stable existence. But is such an existence conducive to *exceptional* accomplishment, i. e., to sustained creative effort in the fields of science, art, literature or of action? Dr. Rivers is inclined to think not, and suggests that all great achievement necessitates a certain degree of *instability* in the unconscious and subconscious strata of the mind which form the scene of the conflict between instinctive tendencies and the forces by which they are controlled.

A study of the lives of many geniuses certainly tends to substantiate the above view: and the close connection between genius and insanity opens up the wider question of the relation between pathology and art. Are pathological conditions the cause of artistic activity or vice versa, or are both but the concomitants of still deeper causes? Nietzsche, by virtue of his own bitter experience, was fully aware of the close connection between disease and art. "It is the exceptional conditions," he writes in *The Will to Power,* "that make one an artist; all those states which are related to, and profoundly interwoven with, pathological phenomena; so that it appears impossible to be an artist without being diseased." In a series of illuminating articles recently contributed to the *New Age,* Mr. Janko Lavrin asserts that Nietzsche owes his penetrating analysis,

his biting satire, his ecstatic Hymns to Life, largely to the grave disease from which he suffered and which led him to progressive paralysis and madness. He suggests that physical disease by stirring up some of those mental currents which normally remain unconscious, may produce the intellectual sensitiveness characteristic of genius. Nevertheless, I hold that there need be no final or inevitable connection between disease and art; and that Nietzsche's assumption that his art depended on his disease may be regarded as a rationalization on the part of the sufferer. Otherwise the problem of producing exceptional men would resolve itself into one of producing the most suitable forms of disease. This question of arousing and directing the unconscious powers in man is no new one and it is more than probable that many, if not all, of the ancient systems of Yoga current in the East, were means to this end. But the whole problem has received a totally new orientation as a result of Western social development, which opens up new possibilities for the individual through a modification of the total environment in his favor. It is certain that up to the present we have barely touched the fringe of these possibilities, but there are indications already of what might be accomplished.

The stupendous advances in science and in industry involved in, and following, the Industrial Revolution have resulted in a dominance over material factors, and made possible the linking up and organization of the whole world. The rise and growth of the modern democratic states have provided new opportunities for self-government among the peoples concerned; and the facilities of education, literature, science and art have been opened up to the individual on an altogether unprecedented scale. The whole of this modern movement might fitly be called the Democratic movement, using the term in its widest sense to include not government merely, but in Whitman's phrase,

"the ploughing up in earnest of the interminable average fallows of humanity." On the other hand, the growing recognition of all these possibilities is giving rise to increasing criticism, directed against the shortcomings of the existing social and economic system. It is urged that the enormously enhanced powers of production are being used not to lighten the burden of man's labor but to enhance the power of the few: that industry has become centralized, with the result that all power is concentrated at the apex of a pyramid, the base of which holds masses of workers as mere bound supporters of the whole system. In the sphere of politics self-government is seen often to be an illusion: control over policy residing ultimately with small financial or governmental groups. With regard to education it is pointed out that the large classes in working-class schools fail to provide children with the individual attention they require, while the higher facilities are still only open to those with sufficient money. Again, the forcing of boys and girls into industry at a critical age and irrespective of their natural aptitudes, results in warped natures, misfits, discontent, with all the consequent loss of energy and talent. Finally in our domestic and social life the free growth and expression of the individual is hampered and distorted by the existence of a network of customs, traditions and taboos out of which escape is well-nigh impossible.

The existence of these grounds for complaint results in the production of a wide-spread dissatisfaction with things as they are, which—where it escapes from sinking into apathy—finds expression on the one hand in movements of reform, or on the other along channels of violence or vice. Thus in dealing with social affairs we are confronted with the same problem that we met with in the case of the individual, namely, the relation between conflict and development; and we are led to enquire as to how far the existing conflicts are justified and desirable. Leaving

on one side those extremists who would encourage and intensify conflicts of all kinds, and also those at the other extreme who seek to abolish all conflicts, we may say that existing conditions set up conflicts between society and the individual, and between different societies, of an altogether undesirable kind. If, as seems probable, the direction taken by the energy engendered in conflict is partly determined by the nature of the conflict, then many of the causes at present operating to produce conflict stand condemned by their results. This is obvious in the case of international strife and warfare; but it is equally evident in our ordinary life. The causes that compel casual laborers to struggle like wolves at the docks each morning for the chance of a job, or that condemn millions of workers to a grim fight with poverty and sickness amid hideous and nauseating surroundings, cannot be justified in view of the immense resources of modern production. Of what use to talk of sublimation to a miner living with his family in a miserable hovel yet with artistic yearnings and possibilities in his soul? It may be noted in passing, however, that much of the social and religious activity of the present day, whatever other function it may perform, affords a mechanism for that easy harmonization against which I have protested above. This point is admirably illustrated by the following deliciously frank statement in a recent number of the *British Journal of Industrial Welfare,* "In others it is the religious attitude towards life that makes the situation tolerable. The lack of satisfaction seems unimportant and the discouragements of this life seem trivial in the light of the fuller life that is promised by religion which reconciles the worker to social inequalities and enables him, under hard economic and social conditions, to continue cheerfully productive."

It is at this point that it becomes possible to formulate our exact charge against existing social conditions. This

charge is not that the latter generate conflicts but rather that they do so in a senseless, brutal and unnecessary fashion. The case against the existing order is, briefly, that by imposing limitations of an arbitrary and chance character upon the individual it prevents the latter from coming up against his own *natural* limitations or those of a healthy and rational social environment. And the tragedy is that so large a number are ignorant of the fact that they are limited in this way; the average man, in short, does not realize the tremendous potentialities dormant within him; his energies are either taken up in the getting of bread and butter or with the trivialities of life. When men do awaken to the situation the chances are that, after a struggle against conventions, they end either by accommodating themselves to the existing order, or by becoming neurotic. Regarding all healthy creative effort as resulting from a balance between unconscious forces and the forces by which they are controlled, there is a loss of energy in either case, since one of the factors becomes dominant *at the expense of the other;* whereas *what is required is that the two factors should meet in such a way that they issue forth in some form of creative activity;* just as the upper and nether millstones meet in grinding corn or as the two carbons of an arc-lamp first meet and then spring equally apart to produce the light.

It is in connection with this aspect of the question that I want to refer to another fascinating suggestion put forward by Dr. Rivers. This suggestion is concerned with the origin of the energy which is engendered in the conflicts "made necessary by the highly complex character of the past history of our race." "There are two chief possibilities," continues Dr. Rivers, "one, that it is derived from the instinctive tendencies which through the action of the controlling forces fail to find their normal outlet; the other, that the *energy so arising is increased in amount*

through the conflict between controlled and controlling forces." It is this second alternative that Dr. Rivers is inclined to accept, and many facts and analogies might be cited in support of it. The effect of climate on body and mind is a case in point, and it has been proved that the production of bodily heat in winter increases by as much as 150 per cent. Similarly in human affairs opposition, by arousing a greater force of antagonism to overcome it, carries many a movement much farther than would otherwise be the case. Again Neitzsche could not have felt the blasting force of his "Will" apart from the limitations imposed by his own nature and by his social environment. But I would suggest that before an *increase* of energy can come into being, certain important conditions must be fulfilled, chief among which is that the energy engendered by conflict shall find some form of expression. Just as a flame is produced by the fusion of two gases which in the act of combining develop so much heat that their products become incandescent, so the flame of genius only arises as a result of the welding of opposing elements into some adequate form of expression. In each case we have the same cycle of contact, conflict, and fusion, with the consequent increment of energy issuing in a synthetic function. According to this view the increase of energy noted by Dr. Rivers comes in the shape of an "unearned increment" arising from the association of the factors concerned. The great achievements in life therefore are those in which conflict has issued in conquest, just as suffering is only raised to the level of tragedy by becoming aware of its own poignant depth and meaning.

To summarize the conclusion of this essay, psychoanalysis has revealed the immense part played by the unconscious factors in man's nature and we have reason to believe that these forces are stimulated by conflict with other opposing forces; and further, that the outcome of

the conflict, whether for good or evil, depends partly upon the nature of the conflict itself and partly on the direction given to it by the individual or by his environment—using the latter term in its widest sense. We have seen that the conflicts forced upon the majority under existing social arrangements result in a collossal waste of energy and talent, and that these arrangements must be modified in favor of the individual. But if conditions are made more favorable for the development of exceptional persons, these latter can only arise by virtue of transferring the struggle with society to a struggle with their own limitations and those of nature. "He who cannot command himself shall obey," said Nietzsche, or in the words of Franz Hartmann, "He who cannot evolve a world within his own soul needs the external world to evolve his soul." Moral codes and sanctions may be necessary for the large majority, but the higher virtues can only be attained by self-discipline going hand-in-hand with self-knowledge. Finally, we have arrived at the conclusion that mental instability may, or may not be, a factor making for human progress, according to the nature of its outcome. In itself it is an *indication of power,* but whether that promise shall be realized depends on whether it is able to find some outlet in creative activity. In the case of the genius the elements are fused or *integrated* and the energy directed into some positive and specific function In the case of the confirmed victim of psycho-neurosis there is *disintegration,* resulting in loss of potential power and causing any available energy to be driven into subterranean channels or into distorted forms of expression. Between these two extreme types there are of course infinite gradations and also the ever-present possibility of a rapid alternation from one to the other. Hence, there is no room for instability as a cult since it can wreck as well as build. In fact, we may liken it to the katabolic forces at work within the body, whereby in the

breaking down of the tissues heat and motion are generated. These processes, however, must be counter-balanced by others of the anabolic order, whereby worn-out tissues are renewed. Both processes are necessary—representing flux and constancy, instability and stability—and are included in the metabolism by which the life and health of the body are maintained.

This view corresponds with that expressed in a passage in George Santayana's *Life of Reason* which I cannot resist quoting at length. "A barbarous mind," he writes, "cannot conceive life like health, as a harmony continually preserved or restored and containing those natural and ideal activities which disease merely interrupts. . . . Its deification of unreason, instability and strife comes partly from piety and partly from inexperience. There is piety in saluting nature in her perpetual flux, and in thinking that since no equilibrium is maintained for ever, none, perhaps deserves to be. There is inexperience in not considering that wherever interests and judgments exist, the natural flux *has fallen, so to speak, into a vortex,* and created a natural good, a cumulative life and an ideal purpose. . . . To adjust all demands to one ideal and adjust that ideal to its natural conditions—in other words, to live the Life of Reason—is something perfectly possible; for those demands, being akin to one another in spite of themselves, can be better fulfilled by *co-operation than by blind conflict,* while the ideal, far from demanding any profound revolution in nature merely expresses her actual tendency and forecasts what her perfect functioning would be."

MILTON HARRISON.

NELSON, ENGLAND.

# THE RELATION OF SPACE AND GEOMETRY TO EXPERIENCE

## IV

### The Extension of Space Beyond the Bounds of Experience*

In our last lecture, we arrived at a definition of a general triad of parallel or concurrent lines, instead of what we were looking for—a definition of a general triad of concurrent, not of parallel lines. We wish now to ascertain to what extent the definition we obtained may take the place of that for which we searched in the definition of a point as a class of concurrent lines. Now, it is one of the commonplaces of elementary geometry that parallel lines share many of the most important properties of intersecting lines. Two coplanar lines, for example, must either be parallel or intersect; three planes not all possessing a line in common may intersect either in three concurrent or in three parallel lines; and so on indefinitely. We have just given a definition of a generalized point in terms of the relation of concurrence among three lines: we said that a generalized point was a class $a$ of lines, in the sense in which lines have been already defined, such that there are

* Professor G. A. Pfeiffer of Columbia has called to my attention the fact that the definition of a point as a set of convex solids each intersecting each and excluding no solid intersecting all, is too inclusive. It includes the set of all convex solids intersecting all the sides of a given triangle. The definition should be amended to read: A point is a set of convex solids each intersecting each, and such that if every solid of the set intersects some convex solid of a set $\sigma$, then some solid of $\sigma$ belongs to the set.

two intersecting lines, $l$ and $m$, which are so related to $a$ that $a$ is the class made up (1) of all the lines $x$ which are of such a nature that $l$, $m$, and $x$ are concurrent, and (2) of the lines $l$ and $m$ themselves. The analogy between intersection or concurrence, on the one hand, and parallelism, on the other, suggests to us an interesting logical experiment: let us substitute the word "coplanar" (i. e., parallel or intersecting) for "intersecting" in the above definition, and the phrase "concurrent or parallel" for the word "concurrent," and let us see what we shall obtain as the result. The amended definition will now read as follows: A generalized point is a class $a$ of lines, in the sense in which lines have been already defined—that is, to all intents and purposes, a class of all the linear segments which span that region of space wherein convex solids are experienced to intersect—such that two such lines, $l$ and $m$, which lie in the same plane can be assigned which determine $a$ in such a way that it is made up (1) of all lines $x$ such that $l$, $m$, and $x$ are either concurrent or parallel and (2) of the two lines $l$ and $m$ themselves. Let us consider this definition thoroughly, and see what it means. Two alternatives and only two are open: either the $l$ and $m$ by which $a$ was determined intersect one another, or they do not intersect one another, although they are by hypothesis coplanar—that is, they are parallel. First, suppose that $l$ and $m$ intersect. Then there can be no lines parallel to both, for no line can be parallel to both of two intersecting lines in a space in which the lines we have defined have the properties we have attributed to them—that is, in a space in which our initial experience of the intersection of convex solids has the properties that we should naturally attribute to it. Consequently, all the lines which are either parallel or concurrent with both $l$ and $m$ must be concurrent with both $l$ and $m$. For this reason, the generalized point $a$ is made up of all the lines which, when taken together with $l$ and $m$

form concurrent triads—or in other words, which pass through the intersection of $l$ and $m$ but are distinct from either—together with the lines $l$ and $m$ themselves. $a$, that is, is the class of all the lines, in the sense in which we have already defined lines, which pass through what we should ordinarily call a certain point—namely, the intersection of $l$ and $m$. That is, all the things that we wished to call generalized points remain generalized points when we replace our original definition of a generalized point by the one just given. There are, however, things that are generalized points in accordance with our new definition which would not be called generalized points under our old definition. If the $l$ and the $m$ by which our generalized point $a$ is defined do not intersect, then since they are coplanar, they must be parallel. It is manifestly impossible for three lines, two of which are parallel, to be concurrent, so that all the lines $x$ which are parallel or concurrent with $l$ and $m$ both must be parallel with $l$ and $m$, and consequently the generalized point $a$ reduces to $l$ and $m$ and all the lines parallel to them: that is, to all of those lines that we have defined which point in a single direction. These generalized points which consist in all the lines in the region accessible to our experience pointing in some direction and are generalized points by our second definition and not by our first tentative one we shall call *irregular* generalized points, for example, a direction may be regarded as an irregular generalized point. If our experience of the intersection of convex solids behaves as it should, every generalized point either corresponds to a point of ordinary geometrical space or is a direction.

It will be noticed that the definition of generalized points on which we finally agreed satisfies that criterion which we have insisted that every definition in this paper should fill—that it introduces no notions other than that of our experience of the intersection of convex solids or

notions which have already been defined in terms of this. This is the case because it involves only the notions of a plane and of concurrence or parallelism, which have already been defined in the requisite manner. Here is perhaps as good a place as any to repeat at some little length the reason why we are always so insistent that our definition shall involve no other notion than that of our experience of the intersection of convex solids and notions already defined in terms thereof, except, of course, such purely abstract and logical notions as those of class, member, disjunction, etc. It will be remembered that our object in this course of lectures is to show that the geometrical properties of space are due, not to any peculiar formal properties of our original spatial experience, but to the method in which the points, lines, planes, distances, etc., of geometry are defined as constructions from or tabulations of our original spatial experience. Therefore, while it is perfectly in order for us to make use of the conventional geometrical notions of lines, points, planes, etc., and of the theorems of geometry concerning them that we may show that the things within our system of definitions have such not purely formal properties as we should naturally expect entities of their respective names to have, we must not introduce into our definitions any notions which we do not either explicitly start from or reach by some chain of definitions starting from these explicit primitive notions alone. Otherwise, we have not given an adequate characterization of the method by which we attain to our final geometrical concepts, and are hence not able to give a satisfactory proof that the fact that these notions obey the laws of geometry results simply and solely from the method by which they are reached and defined in terms of the notions which we have explicitly taken as primitive. It is for this reason that we do not make use, for example, of the first definition of generalized points which we gave; we had in

our possession no criterion of the intersection of two lines or of the concurrence of three lines which was independent of our every-day geometrical notions and depended solely on our experience of the intersection of convex solids, which we have taken as primitive, so that our first definition of a generalized point, which was essentially dependent on the notions of intersection and of concurrence, failed to satisfy our demand that the purely geometrical properties of the entities defined by it should result wholly from the method in accordance with which they were derived from our fundamental experience.

Our final definition of generalized points is, then, an adequate definition, and covers all the entities which one would naturally call points, but it still suffers from a defect which, it is true, is essentially different from the one that constituted an objection to our very first definition of points, which this new definition of generalized points was designed to replace. Our first definition of points was defective in that it did not give us enough points—instead of yielding us all points in space, it only yielded us all those points in a certain region of space—while our definition of generalized points has the disadvantage of covering, not only such entities as our good ordinary every-day points, but also such entities as directions, which would not be termed in points in ordinary Euclidean geometry. The problems then arise, how can we distinguish our good, ordinary, every-day points from the things which we have called directions or irregular generalized points? and, can this be done by finding some property, possessed by all directions, not possessed by any other generalized points, and definable in terrms of our experience of the inetrsection of convex solids alone? We shall discuss both these questions in the next lecture and give reason why we should answer the second of them in the affirmative. We shall thereby have given a definition

of a point which, to say the least, apply to a set of entities correlated in a one-one manner with the points of ordinary space (providing that the experience of the intersection of convex solids has those properties which we should naturally expect it to have), which might possibly be said to *be* the set of all the points of ordinary space, seeing that we do not yet know what the points of ordinary space are. This set is the set of all generalized points, with the irregular generalized points removed from it. The definition of these points will not, however, be that which we shall accept as final in this course of lectures, for this definition of "point" will not secure to space its ordinary geometrical properties, whatever the formal attributes of our experience of the intersection of convex solids may be, whereas we desire to arrive ultimately at definitions of points, lines, distances, etc., which will of themselves secure to space its geometrical properties, and will entail as consequences the usual theorems of geometry.

Another thing to which I desire to call attention again in this place is the essential difference between the things which we called points in accordance with our first definition of that term and generalized points. The class of all our generalized points is not made up simply of all the points, in the first sense in which we defined points, together with certain additional entities; all our generalized points are classes of the lines that we have already defined and all such lines are classes of points, in the first sense we gave to that term. Consequently, none of these latter points can any more be generalized points than a collection of collections of tables can be a table, or a group of families of human beings can be a human being. Nevertheless, to a point in our first sense, there may happen to correspond a certain generalized point. If our experience of the intersection of convex solids has the properties that it would naturally be expected to have, the class of all the

lines, in the sense in which we have already defined lines, which contain as a member a certain point, in our first sense, constitutes a generalized point which, we may say, corresponds to or represents the point, in our first sense. It is not by any means true, however, that, whatever formal properties our initial experience of the intersection of convex solids might have, it would follow *merely from the definitions of points, lines, and generalized points in terms of our initial experience* that the class of all the lines passing through some point, in our first sense, is a generalized point. Now, we desire to give a definition of the generalized points corresponding to given points, in our first sense, which, while it will yield the same result as the definition just suggested in the case where our fundamental experience behaves itself, and will make the generalized point consisting of all the lines that we have defined which contain a given point, in our first sense, correspond to this point, will not be dependent for its importance, to all intents and purposes, on the hypothesis that our initial experience of intersection has not been misbranded and possesses all those properties which we should naturally associate with its name. This result we secure by saying that if a generalized point contains two distinct lines as members, while these lines themselves both contain a certain point of our first sort as members, the generalized point shall correspond to the latter point. If points in our first sense are analogues of those of the points of an ordinary geometrical space which lie within a certain closed convex region, as they are when our initial experience behaves itself, and generalized points are analogues of pencils of all the linear segments intercepted by the surface of this region which are concurrent or parallel with a given coplanar pair of such segments, including the linear segments belonging to this pair, then, since if any such pencil contains two segments both containing a given point it contains all of our

segments that pass through this point, any generalized point which contains two distinct lines passing through a given point, in our first sense, will consist of all of those of our lines that pass through this point. If our initial experience does not behave itself, however, and has other formal properties than those which we have attributed to it in this and previous lectures, it will be a much more common thing for a generalized point to contain two lines passing through a given point than for it to contain all the lines passing through that point, so that our new definition of the correspondence between a point, in our first sense, and a generalized point will be much more frequently applicable than the one which we have rejected. It should be noticed that our new definition of this relation of correspondence offers us no security that to each of the points, in our first sense, there shall correspond one point and one only, nor that the converse correspondence is unequivocal. To one generalized point there may correspond many points in our first sense, and to one point in our first sense there may correspond many generalized points. We can define unequivocally, however, the *class* of generalized points corresponding to a given class of points in our first sense. This class is made up of all the generalized points which correspond to any member of the class of points in our first sense, whether this correspondence is one-one or not. If we possess a definition of the class of all the points, in the first sense, in some region, we may define in this manner the class of all the generalized points in the same region.

We have completed, then, our discussion of generalized points. To be able to make any use of generalized points, however, we must be in a position to assert geometrical statements concerning them. The first and most important of all geometrical statements are those which concern the collinearity of points or the concurrence of lines. To

be able to make such statements we must know what a line is and when three points are collinear. Since our generalized points are classes of lines, in a sense of the term which we have already defined, such that all the lines belonging to a given generalized point are either parallel or concurrent, and since, when these lines are concurrent, the ordinary geometrical point to which our generalized point corresponds is the apex of the pencil of lines constituting the generalized point, it might be thought that a necessary and sufficient condition for the collinearity of three generalized points would be that they should contain in common a member—i. e., a line in our previously defined sense. One should bear in mind, however, that the things which we called lines in accordance with our former definition did not correspond to the complete lines of ordinary geometry, but rather to the linear segments intercepted by the surface of a certain closed convex region of space to which we shall refer briefly as R, whereas there are generalized points representing all the points in space. Since R is closed, there are lines in space which do not intersect R. Let *l* be one of these, and let A, B, and C be three distinct points on *l*. Since, to put it crudely, a generalized point is made up of all the linear segments intercepted by the surface of R which are aimed at some point or in some given direction, if we should adopt the definition of collinearity which would make three distinct generalized points collinear when and only when they all contain some member (i. e., some line in the sense already defined) in common, A, B, and C cannot be collinear, for there is no segment of a line intercepted by the surface of R that points at all three of them. We must therefore search about for another definition of collinearity.

The definition of collinearity that we have just rejected is, however, a sufficient condition of collinearity and an adequate preliminary definition of it, provided that the

three generalized points A, B, and C, whose collinearity we assert, are on a line which passes through the region R. It is fairly obvious, too, that if A, B, and C be any three generalized points, if $l$ is a member of A, $m$ is a member of B, and $n$ is a member of C, and if $l$, $m$, and $n$ are coplanar, in the sense in which we defined the coplanarity of our lines in our last lecture, we should naturally say that A, B, and C are all in the plane of $l$, $m$, and $n$. Furthermore, if R is any solid region in space and $l$ is any set of points in space, it is obvious that if at least two planes, $p$ and $q$, can be contructed in such a manner that each contains $l$ and each possesses a planar area in common with R, $l$ is made up solely of the points in some line. These two planes have only the points of the single line $l$ in common. A, B, and C, then, are collinear, if they both belong to two distinct planes passing through R in planar regions. That is, by virtue of the fact which we stated just a second or so ago, if $l$ and $l'$ are members of A, if $m$ and $m'$ are members of B, and if $n$ and $n'$ are members of C, while $l$, $m$, and $n$, on the one hand, and $l'$, $m'$, and $n'$, on the other, form triads of coplanar lines, A, B, and C are collinear, provided that the plane of $l$, $m$, and $n$ is different from that of $l'$, $m'$, and $n'$. This latter condition may be formulated as follows: there shall not be any two distinct intersecting lines of our first sort meeting at a point $a$ of our first sort which are each cut in a point other than $a$ by every one of the six lines, $l$, $m$, $n$, $l'$, $m'$, and $n'$. On account of this fact and on account of the fact that the coplanarity of the lines of our first sort can be defined solely in terms of our experience of the intersection of convex solids, the necessary condition which we have given for the collinearity of three generalized points is formulable purely in terms of notions that can themselves be defined ultimately in terms of our experience of the intersection of convex solids alone. To show that we can regard this condition as a definition of

the collinearity of three generalized points, it simply remains for us to prove that this criterion is also a sufficient criterion of the collinearity of three of those generalized points that correspond to ordinary geometrical points, and to investigate what is the natural interpretation in the language of ordinary geometry of the relation of collinearity thus defined when it relates directions to one another or to ordinary generalized points. Then we may be sure that the relation which we have defined as collinearity will have the properties one would naturally associate with its name when it connects ordinary generalized points, and we shall know the translation into ordinary geometrical language of the relation we call collinearity, when it holds among directions or between them and other generalized points. We shall finally close this chapter with a definition of "generalized lines"—i. e., lines made up of generalized points—in terms of the relation of collinearity.

Our first remaining task is, as we have said, to show that the condition we just gave for the collinearity of three points is a sufficient one. We wish to prove, that is, that if A, B, and C are three generalized points which we should naturally call collinear, then six lines in our first sense of the term—namely, $l$, $l'$, $m$, $m'$, $n$, and $n'$—can be assigned in such a manner that $l$ and $l'$ shall belong to A, $m$ and $m'$ to B, and $n$ and $n'$ to C, while $l$, $m$, and $n$, on the one hand, and $l'$, $m'$, and $n'$, on the other, form coplanar triads of lines in distinct planes. Returning to the geometrical interpretation of the lines in our first sense, we see that this condition will be fulfilled if it is always possible, given a convex solid region in space, and a set of three collinear points anywhere in space, to draw six lines, two passing through each of the points in question, in such a manner that all pass through the solid region, and that there are two distinct planes, each containing one line through each point. This is practically equivalent to the obvious propo-

sition that two distinct planes can be drawn, passing through any line you please, and cutting a given solid region of space in planar areas. Consequently, if our experience of the intersection of convex solids lives up to its name, the relation which we have defined as the collinearity of generalized points, in so far as it applies to generalized points other than directions, is at least the precise analogue of the relation known by the same name in ordinary geometry. All that remains now is to consider the properties of the relation we have defined as collinearity when it relates irregular generalized points to one another or to ordinary generalized points.

There seem, at first sight, to be three possible ways in which directions may enter into relations of collinearity. It seems *a priori* possible that either (1) three directions should be collinear, or that (2) two directions and one ordinary generalized points should be collinear, or that (3) one irregular and two ordinary generalized points should be collinear. However, we shall see that, provided our initial relation of apparent intersection behaves itself, the second alternative just suggested cannot occur. For, if it does occur, let us suppose that the two directions in question are A and B, and that the ordinary generalized point is C. Then, by the definition of collinearity, there are two distinct planes, each containing a member of A, a member of B, and a member of C. Since A and B are each sets of parallel lines, and represent different directions in space, the two planes in question are parallel, for it is an elementary geometrical theorem that two distinct planes such that one of them contains two given lines in different directions and the other of them contains two lines parallel, respectively, to these lines are parallel. Since, however, each of the two planes contains a member of C, and since C, being an ordinary generalized point, is made up of segments of intersecting lines, the two planes in question intersect. We

thus obtain a flat contradiction in the case where two of three collinear generalized points are directions and the third is not, and where our initial experience has the formal properties that are attributed to it in ordinary geometry.

Let us, then, consider the first of the two remaining alternatives. What are the conditions under which three directions will be collinear in accordance with our definition? Let these irregular generalized points be A, B, and C. By our definition, they will be collinear if there are two distinct planes such that each of our three points contains a line in each of the planes as a member. By what we said in the last paragraph, these two planes must be parallel. Three directions, that is, are collinear when and only when each of them possesses as a member a line, in the sense already defined, in each of two given parallel planes. Since, however, a direction is made up of all the lines, in our first sense, parallel to a given line of that sort, and since if two planes are parallel each of them contains some line parallel to any line you please of the other, the condition that we have just given for the collinearity of three directions is equivalent, provided that our points, lines, etc., have the ordinary geometrical properties, to the condition that some member of A, some member of B, and some member of C be coplanar, or in other words, since we should naturally say that a point lies in the same plane as a line through it—that is, as one of its members—it is equivalent to the condition that A, B, and C all belong to some ordinary plane. To sum up, in a space that obeys the laws of geometry, the collinearity of directions is equivalent to coplanarity, in some ordinary geometrical plane.

The only remaining alternative is that the three points A, B, and C should consist of two ordinary generalized points and one direction. Let A and B be the ordinary generalized points, and let C be the direction. If A, B, and C are to be collinear, there must be two distinct planes, each

of which contains a member of A, a member of B, and a member of C. We should naturally say that A and B are on the intersection of these two planes, since each is situated on (i. e., contains as a member) a line in each plane. Since the two members of C lying, respectively, in each of these planes are parallel, as C is made up of all those of our lines that point in some single direction, both of these lines must, to put it crudely, be parallel with the intersection of the two planes, the line AB, for it is a theorem of elementary solid geometry that if $p$ and $q$ be any two intersecting planes, if $l$ is a line in $p$, and if $m$ is a line in $q$, the only condition under which $l$ and $m$ can be parallel is that both should be parallel with the line of intersection of $p$ and $q$, unless one of them coincides with this line, and the other is, of course, parallel with it. For suppose the contrary to be the case. Then the line of intersection of $p$ and $q$, since, by the definition of l, it is coplanar with $l$, and since, by hypothesis, it neither is parallel to it nor coincides with it, must cut $l$ in a single point. $m$ and $l$ are coplanar, since they are parallel, and consequently the intersection of $p$ and $q$, which cuts them both, lies in the same plane as $l$ and $m$. Therefore the plane which $l$ determines together with this intersection—namely, $p$—is identical with the plane that $m$ determines together with this intersection—that is, $q$, and since we assumed that $p$ and $q$ were *two* planes, and not a single plane, we get a contradiction. This proves our theorem. As a consequence, if space is decently behaved, two ordinary generalized points A and B are collinear with a direction C when and only when C may be said to be made up of lines parallel to the line AB. In this case we may very naturally call C the direction of AB.

We are now in a position to define generalized lines. The generalized line AB is the class of all generalized points that are collinear with A and B, together with the two generalized points A and B themselves. In a prop-

erly behaved space generalized lines, like generalized points may be divided into two classes: ordinary generalized lines and irregular generalized lines. An ordinary generalized line is made up of all the ordinary generalized points which we should usually regard as lying in that line, together with the irregular generalized point which is the direction of the line. As a consequence, two parallel ordinary generalized lines always intersect in an irregular generalized point. An irregular generalized line is made up of all the irregular generalized points on these ordinary generalized lines which, we should naturally say, lie in a certain plane. This one may readily show. Every generalized line, therefore, contains irregular generalized points, and since even parallel generalized lines intersect—in irregular generalized points—it is clear that we must sooner or later find a way of distinguishing irregular generalized points from ordinary generalized points, in order that we may remove the former from our system, and obtain a set of generalized points which corresponds completely to the set of all the points in ordinary geometrical space; and hence, since we are still unaware just what the set of all the points in our ordinary geometrical space is, by accomplishing the task whose necessity has just been indicated, we shall obtain a set of entities which may be said to *be* the set of all the points in our ordinary geometrical space. To do this with the introduction of few or no new sorts of experience is, as we have already said, the task of our next lecture.

We have succeeded, then, in defining all the points and lines of space, together with certain additional entities which we have called *irregular* points and lines—they really include the points at infinity and lines at infinity, respectively, of projective geometry—in terms indirectly of our experience of the intersection of convex solids, but directly in terms of the points in a given convex region of space and of the linear segments intercepted by the surface of this

region. The method which we have used here in the solution of this latter problem is not original, but is due to Bonola, who makes use of it in an article that appeared about 1903 in the *Giornale di Matematice.* We have omitted the theorems which he proves there, however, and have only made use of his definitions and method, because it is only these that are relevant to the main purpose of our paper. The general notion of defining points as classes of lines dates back at least as far as Pasch, *Vorlesungen über der neueren Geometrie,* and has since been used by many mathematical writers, notably Schur and Whitehead.

## V

### Parallel Lines and their Empirical Basis

IN OUR previous lectures we have arrived at certain definitions which enable us to express the lines and points of our ordinary, every-day space, or at least entities that correspond to them in a one-one fashion, under a few quite natural hypotheses, in terms of our experience of the intersection of convex solids, where only such solids as come nearer to us than a certain maximum distance are experienced as intersecting at all. These definitions, moreover, yield us entities corresponding to *all* the points and lines of our every-day space. As we showed in our last two lectures, the reason that a definition of *all* the points and lines of space is such a necessary step in the definition of geometrical notions in terms of our experiences is that unless we have definitions of points and lines which will give us all the points and lines of space, an adequate definition of parallelism is impossible; and without an adequate definition of parallelism one cannot introduce measurement and such notions as depend on measurement (e. g., distance, angle, area, and volume) into geometry, notwithstanding the fact that the study of these is one of the most important portions of ordinary Euclidean geometry. Now that we have given definitions that cover all the points and lines of geometry, it might be thought that it would be a very easy task to define the relation between two lines which consists in their being parallel. Two lines, one would naturally say, are parallel if and only if they lie in the same plane, but have no point in common. This defini-

tion, however, is objectionable for a reason different from any which has led us to discard certain of the definitions which we have formerly made: the space of points and lines which we reached in our last lecture is not, it is true, too poor and niggardly in points and in lines, but—and this is almost as bad—it is too *rich* in points and in lines. It will be remembered that we showed in our last lecture how, under the hypothesis that our experience of the intersection of convex solids has such formal properties as the name we have given it suggests, the entities that we defined as "generalized points" will not, to put it crudely, all turn out to be the points of ordinary geometry, for to some of our generalized points there will correspond in our everyday space, not points, but such *directions* as east or north or up. We further showed how, under the same hypothesis any two of the things we defined as "generalized lines"—i. e., lines made up of generalized points, always intersect if they are in the same plane, even if they are parallel, since a generalized line contains its direction as if it were a point upon it, and two parallel lines always have the same direction—for we do not regard east and west, or up and down, or any other two opposite directions as distinct. Since coplanar generalized lines intersect, even if they are parallel, it is impossible to define two generalized lines as parallel when and only when they are coplanar and do not intersect. How, then, are we to define parallelism in terms solely of our experience of the intersection of convex solids, which we have chosen to regard as primitive in this course of lectures? Manifestly, the simplest way to do this is to find some intrinsic peculiarity of those generalized points which represent directions, so that we may recognize these intruders and remove them from our system. But before we can execute the sentence of death on these anarchistic invaders, we must find out which of our generalized points are the culprits and which are not. Remember, what we

are trying to do is to define all geometrical notions in terms of our experience of the intersection of convex solids *alone*. Either the diagrams we draw on the blackboard and the references we make to ordinary Euclidean geometry are only aids to intuition, to enable us to picture to ourselves what we are doing, and to keep us from getting tangled up in the prolixity and dullness that are inseparable from any purely abstract chain of definitions, or else these diagrams and geometrical phrases and chains of reasoning are just so many assurances that, *if* the experience of the intersection of convex solids with which we have started has such properties as one would naturally attribute to an experience of that name, the entities that we have called points and lines will have many properties belonging to the lines and points of ordinary geometry, and might be regarded as actually identical with the points and lines of ordinary geometry, without any very extravagant alteration of the usual meanings of these terms. Except in such merely illustrative and secondary ways, no use whatever of geometrical theorems, notions, or figures is to be made in this course of lectures. It is consequently not possible to us to adduce the usual geometrical notion of parallel lines for the purpose of distinguishing ordinary generalized points from directions, in any such way as to render this distinction an integral part of that geometry which we are trying to build up from the beginning, on the basis of experience alone. Until we reach a definition of parallel lines dependent upon our experience of the intersection of convex solids and upon no other notion, or else explicitly augment our list of those experiences which we take as primitive in such a manner as to be able to give a completely adequate definition of parallel lines—and we have done neither of these things so far—we are not entitled to define a generalized point as a direction when and only when it is made up of parallel lines of our first sort, even though we thus

defined directions in our last lecture. We have thus the problem before us: how are we to distinguish directions from other generalized points, either by means of a definition of the parallelism of the lines which we defined in our second lecture, which definition must be made entirely in terms of experience, or through some method not dependent on a previous definition of parallelism?

We shall develop in this lecture two alternative definitions of the undesirable irregular generalized points or directions. One of these definitions will involve no notion not already defined by us in terms of our experience of the intersection of convex solids, and will be to that extent superior to our other definition, which will demand the introduction of new ideas, entirely foreign to anything that has already been considered in this course of lectures. Notwithstanding this fact, the second method which we shall give for the definition of parallel lines will have certain marked advantages over the other one in that, for example, the notion which we define as parallelism by means of the latter is likely to depart further from our usual notion of parallelism than that which we obtain through the method that introduces notions not definable in terms of those already presented in this course. We shall first take up the method of definition of parallel lines which involves no new primitive experiences.

You all remember how our experience of the intersection of convex solids practically reduced itself to the actual intersection of other convex solids. These other solids we called a-solids, and we regarded the a-solid corresponding to a given convex solid as made up of the convex solid itself, together with what we called its *aura*—a certain layer surrounding it in such a manner that one a-solid is experienced as intersecting another when and only when the aurae of the two actually intersect one another. On several occasions we made the more or less natural assumption,

which, it is true, was nowhere *absolutely* essential to our reaoning, that the aurae of all convex solids, and all parts of the aura of any convex solid, are of the same uniform thickness throughout. Let us suppose in all that now follows that this assumption is correct, and that the uniform thickness of all the aurae of convex solids is $t$. This being the case, it is obvious that no a-solid can be smaller or equal to a sphere of radius $t$. It is further obvious that a-solids can be found which approximate as closely as you please in shape and size to spheres of radius $t$, supposing that $t$ exists, since convex solids may be constructed as small, and consequently as near to a point in size, as you please, whereas these spheres of radius $t$ may be regarded as if they were formed by the adjunction to a point of an aura of thickness $t$. Now, I wish to define the class of all the points, in the first sense in which we used that term, which lie inside or on the surface of any of these spheres of radius $t$ in terms of our experience of the interection of convex solids and of notions already derived from this experience alone. How am I to go about it, on the basis of what I have already said concerning such spheres?

Let it be remembered that when we defined a point as a class of convex solids, we so defined it that every convex solid which is experienced as passing through a point is a member of that point. As a consequence, the class of points which lie in the a-solid of a given convex solid—i. e., the class of all those points which are experienced as if they belonged to the convex solid, for the aura of a convex solid, which transforms it into an a-solid is that part of space in which it is experienced as lying, but does not lie—this class, I say, is precisely the class of all the points of which the convex solid is a member. It is a consequence of what we said in the last paragraph that if we take a sufficiently small convex solid, the class of all the points contained in its a-solid will approach as closely as we wish

to the class of points in some sphere of radius $t$—or, as we shall say, to the class of all the points in some *a-sphere*. On the other hand, an a-sphere is always smaller than an a-solid. Since we can construct a-solids approximating as closely as we wish to any given a-sphere, though they never become quite as small as the latter, it is possible to determine any given a-sphere by a sequence of a-solids approximating to it. By a judicious choice of the members of such a sequence, it is possible to make each approximation to a given a-sphere surround the next more accurate one. In such a case, those points which belong to the a-sphere will be precisely those which lie within *every one* of the a-solids belonging to the sequence. We are consequently in a position to give a definition of an a-sphere, *qua* set of points, in our first sense, without introducing any other notions than such as are definable in terms of our experience of the intersection of convex solids, provided we are able to give a definition of a sequence of a-solids leading to an a-sphere. It is obvious that a partial list of these properties which a sequence leading to an a-sphere must have includes (1) the property which consists in there being no a-solid contained in every member of the sequence (for then the members of the sequence would not approach as near as you please to an a-sphere, but rather to an a-solid), and (2) the property which consists in the fact that each member of the sequence is contained in the previous one. The phrase "is contained in" which is used in the above statements has the following interpretation: one a-solid is contained in another when and only when all those points which lie in the first of the a-solids in the manner defined above also lie in the second. Using the phrase, "is contained in," in this sense, what are the various sorts of sequences of a solids which are such that (1) there is no a-solid contained in every member of the sequence, and (2) each member of the sequence is contained in the previous

one? A little reflection will convince us that such sequences of a-solids approach and contain as the set of all the points common to all their members either (1) the set of all the points in an a-sphere, or (2) a cylinder of radius $t$, with hemispherical caps at its ends, or (3) a solid consisting of all the points whose distance from a given bit of a plane is not greater than $t$. It is easy to see that solids of classes (2) and (3) contain a-spheres, whereas no a-sphere contains another. We may therefore *define* a set of points, in our first sense, as the set of all the points in some a-sphere, when and only when it is made up of all the points common to all the members of a sequence of a-solids such that each term of the sequence contains the next, though there is no a-solid contained in every member of the sequence, provided that our set of points contains as a part no other set satisfying the conditions just formulated. To avoid certain exceptional possibilities which may arise when the a-spheres which we have just defined lie on the boundary of the region within which we experience the intersection of convex solids, we shall recast our definition so as to exclude all such sets of points as contain either but a single point or no point at all from the sets of points which we call a-spheres. If you follow this definition of an a-sphere step by step, you will see that no notion is involved in it other than that of our experience of the intersection of convex solids and notions which have already been defined in terms of this by various logical artifices.

Now that we have defined our a-spheres, how are we to make use of them in attaining the true goal of this chapter—the definition of the irregular generalized points? We are still a long way off from the solution of this latter question. Let it be noted, however, that we have defined, practically speaking, the class of all those spheres with radius $t$ which lie in a certain region of space. Furthermore, any two equal spheres determine uniquely a right

circular cylinder into which they both fit as a marble fits into a paper cylinder wrapped tightly about it, while any two lines lying wholly within the surface of such a cylinder are both parallel to the axis of this cylinder, and consequently to one another. Consequently, a generalized point will be a direction whenever it contains two members —i. e., two lines, in our first sense of the term—which both belong to a single cylindrical surface of the sort just indicated. It seems further likely that we can find a cylinder determined by two a-spheres pointing in any desired direction. Under this hypothesis, we may regard a generalized point as a direction when and only when it contains as members two distinct lines both situated within the surface of some cylinder determined by two a-spheres. As a consequence, if we are able to give a general definition of what it means to say that two lines, in our first sense, both lie within the surface of some cylinder determined by two a-spheres, we possess a completely adequate criterion by which we may distinguish directions from ordinary generalized points. What remains for us now is the definition of the surface of a cylinder determined by two a-spheres in terms of notions which we have already carried back to experience.

In our second lecture, we gave a definition of a linear segment made up of points, in our first sense of that term, and of the end points of such a segment. We shall say that a point $p$ is *properly between* two other points, $q$ and $r$, if $p$ is distinct from both $q$ and $r$, but lies on a linear segment of which they are the end-points. If A and B are two classes of points, we shall say that the region between A and B is the class of all of those points between some member of A and some member of B. Since a-spheres are so defined as classes of points as to include the points on their surfaces, the region between two a-spheres is the class of all points inside (not on the surface of) the two spheres, together

with the class of all points *inside or on the surface of* the right circular cylinder stretching from a great circle of one a-sphere to a great circle of the other. Now, we wish to be able to define the surface of this cylinder, and, in general, the surface of the region between any two classes of points. But what intrinsic difference is there between the surface of a region and that part of it which is not surface? One would naively say that the surface of a region is that part of it which one can reach without digging down into it. Let us analyse this notion and see what the essence of it really is. To say that one can only reach a certain point of a region by digging down into it really means, to put it crudely, that if we probe into the region with some straight thing, such as a wire, the wire must touch the region some time *before* it reaches the point in question. Stated in precise language, this reads: if a linear segment contains a point in a region, but not on its surface, it contains at least one other point in the region besides. Since the surface of the region consists of those points of it which we can reach *without* digging down into the region, a strict definition of the surface of a region reads: the surface of a region R is the class of all those points which belong to R and also belong to some linear segment, taken as including its end-points, which has no other point in common with R. It will be found that this definition is in complete accord with our usual notions of the properties which the surface of a region should have. Let it be noticed, too, that we have so framed our definitions of the region between two solids that the surface of the region between two a-spheres consists only of the portion of a right circular cylinder lying between a great circle on one a-sphere and a great circle on the other, and does not contain the surface of the two projecting hemispherical caps which must be adjoined to this cylinder to make the complete region between the two a-spheres—for the hemispherical

caps do not include their outer surface, as no point of this outer surface is situated between some point of one a-sphere and some point of the other, so that any linear segment which contains a single point of one of these caps contains other points of the cap as well.

Now that we have arrived at the definition of the surface of a cylinder stretching from a great circle of one a-sphere to a great circle of another, we are in a position to define a line lying in this cylinder, or, as mathematicians say, an *element* of this cylinder. We shall say that a straight line, in our first sense, is an element of one of the cylinders which we have just defined if and only if it contains a linear segment in common with the cylinder, for then it is obviously a prolongation of a piece of line lying entirely within the cylinder. As we have already said, all that remains for us to do if we desire a definition of irregular generalized points in terms of our experience of the intersection of convex solids is to define a direction as a generalized point which contains two distinct elements of some one of the cylinders that we have defined in terms of a-spheres. We can then go on and define two *generalized* lines as parallel when and only when they both contain some particular direction, or irregular generalized point, in the sense in which we have just now defined such entities. Although this definition of the parallelism of two generalized lines is made in terms of directions, which are themselves defined in terms of the parallel elements of a cylinder determined by two a-spheres, we cannot eliminate the intermediate step of defining directions, first, on account of the fact that the elements of a cylinder such as we have described are lines of our first sort, while we desire to define the parallelism of generalized lines, and secondly, because only such lines as are not further from one another than twice the thickness of the aural layer of a convex solid

can both be elements of one of our cylinders, even though they be generalized lines.

Another point to notice concerning the definition of parallelism that we have just given is that the a-spheres which play so large a part in it are really nothing but the *minima sensibilia* of the philosophers, for an a-sphere is simply the region which a point seems to fill, to put it crudely. Of course, we do not mean to say that a-spheres are actually perceived; they may or may not be, but whichever is the case, they mark, like the *minima sensibilia* of Hume, some sort of a lower boundary of the perceivable, with respect to its spatial extension. The assumption which we have made that a-spheres are equal—that is, that the aural layer of all a-spheres and all parts of the aural layer of each a-sphere are of the same thickness—is, then, practically the same assumption as that which the philosophers of the English school made when they degraded the magnitude of a spatial region as determined by the number of *minima sensibilia* or "points" in it, which amounted to the same thing as supposing that all *minima sensibilia* were equal. Our formulation of this view, however, avoids many of the crudities of the orthodox Humian position. We have not avoided the difficulty, however, that it does not seem quite likely that the acuteness of our discrimination between bodies which do and bodies which do not intersect is the same throughout space. As a consequence, it would appear, contrary to what we have assumed, that the *minima sensibilia* remote from us and towards the boundary of that part of space which is open to our experience at all are larger than those more centrally situated, for the cruder our experience is, the larger must a thing be if it is to be perceived. This fact would make our "cylinders" determined by two a-spheres more or less conical in form, so that it would be possible for one of them to contain two non-parallel lines, or even not to contain any

two parallel lines at all; and consequently our "cylinders" would be of no avail to us in the discrimination of irregular generalized points from ordinary generalized points. Therefore, though the definition of directions and that of parallelism which we have given define certain things and relations in terms of our experience of the intersection of convex solids alone, and in so far satisfy the requirements that must be satisfied by all our definitions, we have not any sufficient reason, to say the least, for supposing that the directions and parallelism that we have just defined represent precisely the directions and parallelism which we meet in the space of ordinary geometry. Since this is the case, an alternative definition of an irregular generalized point is desirable, as is also a definition of parallelism dependent thereon, and we shall try so to frame these new definitions that they shall not be entirely dependent for their applicability to the space of ordinary geometry on the assumption that all aurae of convex solids are of the same uniform thickness.

The first thing to notice is that any method which gives us three distinct directions that are not all directions of lines in any one plane—such directions as east, north, and up, for intance, satisfy this condition—that any method which does this, I say, yields us all the directions in space. We have already seen that any three directions lie in a single generalized line, in our every-day space, when and only when they are all, like east, north-east, and north, directions of lines in some single plane. This being the case, given any two distinct lines made up entirely of irregular generalized points and any particular irregular generalized point whatever, it is possible to construct a generalized line passing through the generalized point in question and cutting our two given irregular generalized lines in two distinct points; for, given any two non-parallel planes in ordinary space, and any direction whatever, one

can construct a plane containing that direction—that is, parallel to any line pointing in that direction—and either cutting our two given planes in two distinct lines which point in different directions or else parallel to one of our two given planes, so that it contains every direction belonging to that one of our given planes. The reason why we have insisted that our two given planes shall not be parallel is that two parallel planes represent the same generalized line, for they contain the same directions. The facts which we have just stated, together with the fact that, in our ordinary space, only a direction can be collinear with two directions, entitle us to define the set of all irregular generalized points or directions in terms of any already known non-collinear triad of directions in the following manner: if A, B, and C are three known directions which are not all collinear, then a direction will be a generalized point which is collinear with a generalized point on AB and a distinct one on AC.

How are we to determine our A, B, and C, however?

Here we shall for the first time introduce into our definitions a notion not defined in terms of our experience of the intersection of convex solids. Let us suppose that we have somehow or other, by means into which we shall not now inquire, obtained the knowledge, not of the general notion of a sphere, but of four given sets of points, in our first sense, which we shall call unit spheres, whose centers are not all coplanar. What is required here can easily be given by experience, as it is not a general criterion of sphericity which we postulate, but an empirical acquaintance with a concrete set of entities. Let these four spheres be called X, Y, U, and V, respectively. The definitions of cylinders, their surfaces, and directions determined by them, which we gave in connection with our first theory of parallelism had no essential dependence on the fact that the spheres to which we applied them were a-spheres

—they can be applied without any alteration to the determination of an irregular generalized point in terms of any two equal spherical collections of points, in our first sense. As a consequence, since X, Y, U, and V, being all unit-spheres, are all equal, the cylinders XY, XU, and XV all determine irregular generalized points. These three points are not all collinear, as each is the direction of the axis of the corresponding cylinder, and these three axes are, by hypothesis, not all coplanar. As a consequence, we are able to define the class of all irregular generalized points in terms of the three directions, XY, XU, and XV, and therefore ultimately in terms of the four points X, Y, U, and V, and our experience of the intersection of convex solids, and of no other notion. We can now go on, as before, and define two lines as parallel when and only when they both contain a direction in common. This definition of parallelism does not, like our first definition, presuppose some such probably false assumption as that of the uniform thickness throughout all space of the aural layer of a convex solid, and moreover, as we shall see later, lends itself more readily than our first definition of parallelism to the introduction of metrical considerations into geometry, but it labors under the grave disadvantage that it does not depend exclusively on our experience of the intersection of convex solids, as our former definition does, but demands in addition the selection of four distinct convex solids from all those in the universe, not merely as spheres, but as the particular set of equal, non-coplanar spheres on which our whole subsequent theory of measurement will depend; while we are given in experience no four such spheres, singled out from among all convex solids. Thus neither of our two definitions of parallelism is entirely satisfactory. That a much better definition of parallelism than either, combining their advantages and eliminating their shortcomings, can be obtained without any very great difficulty,

I do not doubt in the least, but I have not been able to formulate such a definition up to the present. You all see the philosophical interest of the solution of this problem: on it depends a satisfactory knowledge of what space really is, and of what the true meaning of geometrical propositions is, since many of the most important geometrical propositions deal either with parallelism, or with the measurement of distances, angles, etc., all of which depend in an ordinary Euclidean space upon parallelism.

To conclude: we have developed two distinct definitions of parallelism. Both of them start from the notion of the cylinder enwrapping two spheres. One of them involves ultimately no other concrete experience than that of the intersection of convex solids, but is not completely satisfactory, since, if we accept it, we must make an assumption concerning the nature of this experience which the experience probably fails to satisfy, in order to secure that the relation we call parallelism is really essentially the same as the relation usually known by that name. On the other hand, we have given a definition of parallelism which avoids this difficulty, but involves other experiences than that of the intersection of convex solids, namely, those of a certain set of four non-coplanar equal unit spheres. The present lecture covers the weakest portion of the theory which we are developing in this course, but there is every reason to believe that its weakness is only temporary, being due rather to the fact that the problems discussed have hitherto been little investigated than to the inherent nature of the subject.

In our next lecture we shall begin the investigation of how we are able to develop a theory of measurement on the basis of either of the definitions that we have given of parallel lines.

## VI. The Logic of Distances

IN OUR last lecture, we arrived at two alternative definitions of parallel generalized lines. On the basis of either of these definitions we are able to define what is ordinarily called by mathematicians a vector. I can best illustrate what a vector is by introducing a few familiar examples. In a region so small that the sphericity of the earth is not noticeable, the relations, "ten feet to the north of," "a mile south-west of," "two yards up," etc., are, to all intents and purposes, vectors. A vector, that is, is the relation which holds between one point and another when they are separated by a given distance in a given direction. Let it be noticed, by the way, that a vector leading from A to B does not, in general, lead from B to A—for example, if A is ten feet east of B, then B is not ten feet east of A, but ten feet *west* of A.

One property of vectors in ordinary geometry—the property, indeed, which we shall use in defining them—is that if A and B are separated by a given vector, if C and D are separated by the same vector, and if the points, A, B, C, and D, do not all lie on a single line, then the line AC is parallel to the line BD. It is conversely true that if AC is parallel to BD, and if, further, AB is parallel to BC, the vector which separates A from B also separates C from D. These facts follow from the very elementary geometrical theorems that if a quadrilateral has a given pair of opposite sides equal and parallel, the other two sides are parallel, and that if each side of a quadrilateral is parallel to the opposite side, ech side is equal to the opposite side. From these facts and the fact that a vector always points in a

given direction, it is easy to deduce the conclusion that to say that the vector that stretches from A to B is the same as the vector that stretches from C to D is precisely equivalent to the statement that AB is parallel to CD and that AC is parallel to BD, provided that A, B, C, and D are not all collinear. Furthermore, if A, B, C, and D are all collinear, and the vector from A to B is the same as the vector from C to D, then there must be some pair of points, E and F, which do not lie on AB, such that the vector from E to F is identical both with that from A to B and with that from C to D, as there obviously exist on any line pointing in a given direction all possible vectors in that direction. We may, therefore, *define* the vector separating A and B (which we shall write (AB) ) as the relation which a point C bears to another point D when and only when either (1) AC is parallel with BD and AB is parallel with CD, or (2) A, B, C, and D are all on some single straight line, and there are two points, E and F, which are not on this line, and which are such that A, B, E, and F, on the one hand, and C, D, E, and F, on the other satisfy the condition that we laid down in one for A, B, C, and D: that is, if AB is parallel to EF, if AE is parallel to BF, and if CE is parallel to DF. Furthermore, a relation is defined as a vector in general if there are two points, A and B, such that the relation is the vector (AB). It will be noticed that this definition, like all those which have already been accepted by us, involves no notions which have not either been taken explicitly as primitive experiences by us, or else been defined in terms of these primitive experiences alone. When we have given, according to which of our definitions of parallelism we choose, either our experience of the intersection of convex solids alone, or this experience together with four selected solids which we choose as unit spheres, it is unambiguously determined whether any given thing is or is not a vector. I want you all to notice that

we have not presupposed in any manner any notion already involving measurement in the definition of a vector which we have just given.

The next notion which we have to define is that of all vectors lying within a given region. Suppose that $\alpha$ is the class of all the points in a certain region of space. Let us consider only such vectors as separate point lying in $\alpha$ from points lying in $\alpha$. Furthermore, let us consider even these vectors as relations which hold between points in $\alpha$ only. The relations thus obtained will be called the vectors-in-$\alpha$. To give a concrete illustration of what I mean by the vectors-in-$\alpha$, suppose that we give to our vectors the geographical interpretation which we gave to them in a previous illustration of the notion of vector, but that we limit our discussion to vectors on the surface of the earth—to vectors in such directions, namely, as are represented on a compass-card—excluding those, for instance, that poinit vertically upward, or 45° downward and north-east, etc. Let $\alpha$ be the class of all the points on the surface of some island in the ocean. We wish to consider only such vectors as lead from points on the island to points on the island. These vectors will be the class of vectors on the island, provided that we only consider them in so far as they join points on the island to one another. Suppose, for example, that our island is circular and one mile in radius. "Three miles to the north of" will not, then, determine a vector-in-the-island, for one will be unable to find any two points in the island such that one is three miles to the north of the other. The relation, "one mile to the north of," however, will determine a vector lying in the island, for it will be possible to select two points on the island such that one is one mile to the north of the other. The *vector-in-the-island* determined by the relation "one mile to the north of" will differ from the ordinary vector "one mile to the north of" in that, if A and B are not both points in the

island, while A may be separated from B by the ordinary vector, "one mile to the north of"—i. e., A may be one mile to the north of B—it is impossible for A to be separated from B by the vector-in-the-island determined by the relation " one mile to the north of." This is the sole difference between the ordinary vector "one mile to the north of" and the vector-in-the-island known by the same name. The ordinary vector "one mile to the north of" will be called in our subsequent discussion the *extension* of the vector-in-the-island of the same name, and in general we shall call the ordinary vector R, from which the vector-in-$\alpha$ S is obtained by considering R only in so far as it relates members of $\alpha$ to members of $\alpha$, the *extension* of S.

The next point which we shall take up is what it means to say that one vector R is $n$ times as great as another vector S. Suppose that I go from a point A to a point B separated from A by the vector S. Suppose that from B, I go still further on to a point C, which is separated from B by the same vector S. Further, suppose that I might have gone directly from A to C by the vector R, or to put it a little differently, that A is separated from C by the vector R. Then we should naturally say that R is twice as great as S: that is, if by going $n$ miles to the north from A to B, and by going $n$ miles to the north from B to C, we find that we have gone $p$ miles to the north in going from A to C, we should naturally say that $p$ miles is twice as great a distance as $n$ miles. In a precisely similar manner, we can define a vector R as 3, 4, 5, 6, . . . ., or $k$ times as great as a vector S, if by taking 3, 4, 5, 6, . . . ., or $k$ steps of the vector S we can sometimes take one step of the vector R. Let it be noticed that I only say that *sometimes* we can take one step of R by taking $k$ steps of S, and not that this is always possible. In ordinary geometry, it is true that if one vector can sometimes be obtained by repeating another vector $k$ times end on end, it can always be so obtained.

We do not wish, however, to have either the intrinsic adequacy or the generality of application of our definitions depend on the axioms or theorems of ordinary geometry, and it would be easy, by substituting other relations and entities than such as we should naturally call the relation of experienced intersection among convex solids or spheres, respectively, for our experience of the intersection of convex solids and for the four selected spheres in terms of which, by our second definition, we determined the class of all the irregular generalized points—it would be easy, I say, to obtain systems in which, for some values of A and B, one could go from A to B either by one R-step or by $k$ S-steps, while for other values of A and B, it might be impossible to go from A to B by a single R-step, but still possible to go from A to B in $k$ S-steps. Here, of course, we assume R and S to be vectors, in the sense in which we have already defined vectors, and since our definition of one vector as a multiple of another still applies, we shall say even in this case that S is one $k$th of R. We shall further define a multiple of a vector-in-a-region in a manner precisely parallel to that in which we have just defined a multiple of a general vector: that is, we shall say that if R and S are both vectors-in-$\alpha$, and it is sometimes possible to accomplish a journey consisting of one step of R by $k$ steps of S, we shall say that R is $k$ times as great as S. We have thus given a definition of the relation which one vector-in-$\alpha$ bears to another $k$ times as large, in terms only of our original experience of the intersection of convex solids, and perhaps of certain selected convex solids.

We shall now define what it means to say that one vector-in-a-given-region bears a ratio to another. A vector-in-$\alpha$ R bears the ratio $m/n$ to another vector-in-$\alpha$ S if there is a vector-in-$\alpha$ T such that R is $m$ times as large as T and such that S is $n$ times as large as T. Thus, for example, the relation, "two miles to the north of," on an

island is, by our definition, two-thirds as great as the relation, "three miles to the north of," on the same island, since there is a certain relation—namely, "one mile to the north of" on the same island—which is half as large as the former relation or vector and a third as large as the second. This definition now enables us to compare different vectors-in-a-region with respect to their magnitude. Be it noted, however, that these vectors which we compare must be vectors in the same direction, for it is impossible, for example, for us to go ten miles to the east by taking any number of successive one-mile steps to the north, nor is there any step which, when repeated a certain integral number of times, will give you a one-mile step to the north, and which, when repeated another integral number of times, will yield a ten-mile step to the east. Furthermore, we are not yet in a position to compare *any* two vectors in a given direction with one another with respect to magnitude, for two distances along one and the same line may be incommensurable. Suppose that I draw on this blackboard a line one foot long vertically upward from some selected point O, and another line of the same length running horizontally to the left from the same point. Connect the two free extremities of the linear segments so obtained by a linear segment *l*. Lay off a linear segment as long as *l* stretching upward from O. It is a simple matter to give a strict mathematical proof that there will be no distance which, when multiplied by some integer, will be equal to this latter distance, while it will equal one foot, when multiplied by some other integer. As a consequence, our definition of the ratio of two vectors will not enable us to ascertain what ratio our one-foot vector upwards from O bears to our vector in the same direction of length equal to *l*, notwithstanding the fact that they are vectors in the same direction. What we have just pointed out concerning vectors in a blackboard holds true, of course, for vectors in any region

whatsoever: our criterion for determining the ratio of two vectors in the same direction does not enable us to compare the magnitudes of incommensurable ratios directly.

We wish, however, to be able to define the quotient of any vector—at least in a given region—by any other vector whatsoever. We wish to be able to regard a vector as a *distance* alone, regardless of its direction. We wish to find some way of comparing the magnitudes of incommensurable vectors. These things must be carried out on the basis of such experiences and notions alone as we have explicitly taken as primitive, if we are to obtain in our treatment of geometry any satisfactory theory of measurement. How are we to accomplish all this?

I shall first give a rough sketch of the method by which we shall obtain this desired result, and then I shall take this method up step by step and give precise definitions of everything I shall do. Roughly speaking, my method is to take some set of generalized points which may be regarded as a sphere, and to find a way of measuring every vector in space in terms of its diameter. As there is a diameter of our sphere in the same direction as any vector we please in space, one can see that the methods which we have already introduced for the comparison of the magnitudes of vectors in the same direction will be applicable in this instance. We shall consequently compare vectors in different directions by referring them to the diameter of our sphere as a standard, for that diameter is of a constant and determinate length, which is independent of the direction in which this diameter is taken. However, in our precise formulation of this method of comparing distances in different directions, there will appear no explicit reference to any diameters of our standard sphere. Our definitions will not depend for their intrinsic logical value as definitions upon any particular assumptions to be made concerning the set of generalized points which we take as our sphere—

we shall not even assume that it possesses anything such as one would naturally call a diameter. Such notions expressing peculiar properties of spheres as that of the center of a sphere or that of the radius of a sphere will likewise fail to make their appearance in the ensuing discussion.

The only spheres which we have already considered *qua* sets of points are sets of points in the first sense which we have given to the word "point" in this course of lectures. The spheres which we wish to consider now are sets of *generalized* points, for we wish to be able to consider our points as termini of vectors, and vectors, since they depend upon parallel lines, have been defined in the space of generalized points, and it was only in this space that we were able to arrive at the notion of parallel lines. We wish to avoid in our present lecture the necessity of supposing arbitrarily that any more sets of points are spheres than it is abolutely necessary for us to so consider. As a consequence of this, a definition of the sphere of generalized points corresponding to a sphere of points in our first sense is a desideratum. It will be remembered that we have already given a definition of the correspondence between a generalized point and a point, in our first sense, in terms of the experience of the intersection of convex solids alone. It will be further remembered that we had no reason to regard this correspondence as one-one except under the hypothesis that our experience of the intersection of convex solids has such properties as we should naturally attribute to it. Be this as it may, I wish to call attention to the fact, which is obvious upon a very slight reflection, that, given a set of points in our first sense, the set of all the generalized points which correspond to any of its members, whether univocally or not, is uniquely determined by the set of points in our first sense. By applying this fact to spheres, it is easy to see that, given any sphere of points in our first sense, there is always a certain single set of

generalized points which we may define as "the same sphere" in the space of generalized points. This identification of these two spheres presupposes no other primitive notion than that of our experience of the intersection of convex solids.

Let us suppose that we have already singled out some sphere of points, in our first sense, and that we have obtained from it the corresponding sphere of generalized points. Let us discuss the class of all vectors-in-this-sphere—that is, by our definition, the class of all vectors in space, in so far as they connect points in this sphere. Since the sphere has a finite radius, if R is a vector-in-the-sphere, we cannot multiply R by a coefficient as large as we please, and yet have the resulting vector remain within the sphere. If we are on an island ten miles in diameter, it is clear that we can somehow manage to take one step of four miles to the north, or even two successive steps of four miles to the north, but it is impossible to make a journey in the island consisting of three consecutive steps of four miles to the north. For the same sort of reason as that which makes this latter task impossible—it involves a straight journey of twelve miles on an island only ten miles across—it is impossible, in general, to multiply a vector in a given spherical region by an arbitrarily large coefficient, and to obtain a vector in the same region as a result.

The question now arises, granted that it is true that a vector in a finite spherical region cannot be multiplied by an arbitrarily large numerical coefficient, just how is one to determine by how large a coefficient it may be multiplied, and still remain in the region under consideration? In particular, what relation does this coefficient bear to what one would ordinarily call the magnitude of the vector in question? The answer is fairly obvious if it is ordinary Euclidean geometry with which we are dealing, and if the

"sphere" inside of which our vectors are confined is actually a sphere. It may be seen on inspection that, by our very definition of a vector, a vector not limited to a given region can be laid off on any line which points in the direction of the vector. Furthermore, not only can our vector always be laid off on such a line, but if $l$ is such a line, and if A is any point on $l$, there is some point B on $l$ which is separated from A by the vector in question. For example, if $l$ is any line going north and south, and A is any point on this line, then if we choose any such vector as "five miles south of," there is a point on $l$ which is separated by this vector from A—that is, which is five miles south of A. Furthermore, it is not only obvious that, as a consequence of this, every vector can be laid out on some line passing through the center of our sphere—for a line can be drawn in any desired direction through any point you please—but it is also true that every vector-in-our-sphere can be laid off on some diameter of the sphere. This is the same as saying, for instance, that if we are on a circular island, and can somewhere make a journey of ten miles to the north without leaving the island, we certainly can take such a journey along a diameter of the island. All this is the consequence of the familiar fact that a diameter is the longest linear segment which can be drawn within a circle or a sphere. From this principle it results that if R is a vector inside a given sphere, all vectors inside the sphere can be laid off on the diameter of the sphere pointing in the same direction as R. If the fraction which we have just defined as the ratio of two vectors be what we should naturally consider their ratio to be—and we have already shown that this is actually the case if our primitive notions live up to their names—it is obvious that what one would ordinarily call the ratio of the diameter of our sphere to any vector which we wish to measure is at least as great as the ratio of *any* vector on the diameter,

in the appropriate direction to the vector which we wish to measure, and consequently, from what we have just seen, it is as great as the ratio borne to the vector which we desire to measure by *any vector whatsoever in the sphere.*

It is an easy matter to show that the notion of ratio which we have defined among the vectors in our sphere agrees precisely, as we have already said, with our usual notions of ratio, under the hypothesis that our fundamental experiences have not been misnamed. We are therefore justified in making use of the ordinary geometrical properties of ratio to find out what the magnitude of a vector-in-a-sphere has to do with the extent to which it can be multiplied by a ratio. Now, it is a commonplace of geometry that if we have given to us two linear segments, say AB and CD, either they are commensurable—that is to say, they bear a ratio to one another which is the quotient of one integer by another—or, given any distance *d,* no matter how small, it is possible to find on AB a point E, such that AE is commensurable with CD, while EB is less than *d.* The proof of this is at once simple and instructive, so perhaps I may be permitted to give it here. All of you know that when two quantities, such as distances, bear no ratio to one another of the form $m/n$, where $m$ and $n$ are integers, but are yet comparable with respect to their magnitudes, we say that the ratio of one quantity to the other is irrational, and we represent it by a non-terminating decimal fraction, such as 3.141592.., which is $\pi$, or the ratio of the circumference of a circle to its diameter. Now, what does it mean, for example, to say that the circumference of a circle is 3.141592..... times as large as its diameter? It means that if we take a distance 31/10 as large as the diameter of a circle, we shall fall less than 1/10 of the diameter short of the circumference, that if we take a distance 314/100 as large as the diameter, we shall fall less than 1/100 of the diameter short of the circum-

ference, that if we take a distance 3141/1000 of the diameter, we shall fall less than 1/1000 of the diameter short of the circumference, and so on indefinitely We can thus define a distance which is commensurable with the diameter of a circle, and yet differs from its circumference by less than $1/10^k$ of its diameter, for any assigned positive integral value of $k$. It is obvious that we can make $1/10^k$ as small as we please by making $k$ sufficiently large. By using precisely this line of reasoning, we may prove our thesis: that, given any two incommensurable linear segments, AB and CD, and any distance $d$ as small as you please, there is a point E on the segment AB such that AE is commensurable with CD—that is, the vector AE bears a ratio, in the sense already defined in terms of our fundamental notions, to the vector CD—while the distance BE is less than $d$. As a consequence, it is possible to lay off on some diameter of our standard sphere vectors which are commensurable with any given vector in the sphere, say R, and which, if they do not completely fill up the diameter, leave a remainder which can be made smaller than any assignable vector. Therefore, it is natural for us to say that, although the ratios which other vectors bear to a given vector R can never exceed what we should properly call the ratio of the diameter of the sphere to the given vector R, there is no smaller ratio which is not exceeded by some of the ratios which other vectors in the sphere bear to R. That is, if we call the class of all the ratios which other vectors in the sphere bear to R by the name $\alpha$, the ratio which the diameter of our sphere bears to our vector R will be larger than or as large as any member of $\alpha$, yet there will be ratios which belong to $\alpha$ yet differ from this latter ratio which the diameter bears to R by less than $\varepsilon$, when $\varepsilon$ is any assigned quantity, however small.

In the preceding paragraph, we came across a notion which is of the highest importance for the determination

of the magnitude of a vector. This notion is that of a number $n$ which is larger than or as large as all the members of a given class K of numbers, yet is such that there are members of K which differ from $n$ in magnitude by as little as you please. It is obvious that there is only one value of $n$ which can have this property, for a given class K, since if $m$ also had the property in question, it would, of necessity, be either larger or smaller than $n$. If larger, since some members of K would differ from it by less than $\varepsilon$, however small $\varepsilon$ might be, they would differ from it by less than $m-n$, and hence, contrary to our hypothesis, would be greater than $n$. If $m$ is less than $n$, a precisely similar contradiction arises. As a result, the ratio which a vector R bears to the diameter of our sphere of measurement is uniquely determined by the class of all the ratios which other vectors in our sphere bear to it. Since this is the case, there is no objection to our *defining* the notion—previously undefined—of the ratio which the diameter of our sphere bears to the vector-in-the-sphere R as the number which is greater than all the ratios which vectors-in-the-sphere bear to R, but to which these latter ratios approach as near as you please. This definition will involve no notions not already incorporated into our system, and will be sufficient to secure the unicity of the ratio which the diameter of our sphere bears to our given vector R. If there are ratios which other vectors bear to R and these are all less than some fixed finite number, it is possible to prove by means of our definition alone that there is some number which represents the ratio which the diameter of our sphere bears to our given vector, although this proof involves certain elementary mathematical considerations, drawn from the modern theory of aggregates, which would be a little too intricate for us to consider here.

In ordinary Euclidean space, unless R is a vector that connects a point to itself, all the ratios which other vectors-

in-our-sphere bear to R will be less than some fixed finite number, so that the ratio which the diameter of our sphere bears to R is actually determined as some particular number, if there are any vectors in our sphere that bear ratios to R. Now, it follows from our definitions alone that R bears the ratio 1 to itself, so that there are always vectors which bear the ratio 1, at least, to R. From this it follows, first, that every vector which cannot be repeated an indefinite number of times end on end and still remain within our sphere of measurement determines some single number that represents the ratio of the diameter of the sphere to R, and second, that this latter number is always greater than or equal to one, since it is greater than or equal to any ratio which R bears to another vector-in-the-sphere, and one of these ratios is equal to one. Where a vector R can be repeated end on end an indefinite number of times, and there are vectors smaller than the diameter of our sphere—i. e., vectors-in-our-sphere—that bear as large a ratio as you please to R, the natural thing to say would seem to be that the ratio of the diameter of our sphere to R is infinite. In a space living up to the axioms and theorems of ordinary geometry, if our sphere is a genuine sphere, this latter situation can only arise when R is the vector that connects a point with itself. It is obvious that since a step of this vector leaves you just where you were, any number of consecutive steps of this vector will likewise leave you just where you were, and will not take you outside of our sphere of measurement.

Let us consider for a while the ratio which a given vector-in-our-sphere bears to the diameter of our sphere. Every vector-in-our-sphere bears to its diameter a ratio which may naturally be regarded as the reciprocal of the ratio which the diameter of our sphere bears to the vector in question, and which we may *define* as this reciprocal as the notion of this ratio has not been already defined. In

this context, we shall regard zero as the reciprocal of $\infty$. We shall, for short, call the ratio which the vector-in-our-sphere R bears to the diameter of this sphere the *index* of R with respect to the sphere of measurement in question. We may naturally regard the index of R in a certain sphere as the expression of the magnitude of R in terms of the diameter of our standard sphere as a unit. It follows from the fact that the ratio of the diameter of our standard sphere to any vector-in-the-sphere cannot be less than one, that the index of any vector-in-our-sphere, being the reciprocal of the ratio which the diameter bears to it, cannot be greater than one. This is as we should expect—no vector-in-a-sphere can exceed in magnitude the diameter of the sphere. Further, the index of a vector need not be a rational number, as may be seen without much difficulty by a mathematical analysis of the stages through which we have gone in defining it, for its reciprocal, the ratio which the diameter of our standard sphere bears to the vector-in-the sphere in question is defined, not as a ratio, but as a limit of ratios, and a limit of ratios may be an irrational number. The importance of this fact arises from the fact that it makes it possible for us to measure by their indices not only such vectors as are commensurable with the diameter of our sphere of measurement, but also vectors which are incommensurable with this diameter. This measurement of incommensurable distances by a common unit would have been impossible if we had tried to found a theory of measurement directly upon the notion of ratios among vectors, without the introduction of our standard sphere. Another fact which makes us introduce the standard sphere into our theory of measurement is that it

enables us to compare vectors in different directions, as we have already seen, and makes it possible for us to consider them as mere distances, and not as vectors—i. e., *directed* distances. The index of a vector represents it in terms of the diameter of the sphere in its direction as a unit. However, the length of one diameter of a sphere is the same as the length of another diameter of the same sphere, no matter in what directions the two diameters of the sphere point. As a consequence of this, the index of a vector expresses its length in terms of a unit which is independent of the direction of the vector. We are thus enabled to say that a vector-in-our-sphere in one direction is as long as, or half as long as, or twice as long as, etc., a vector in another direction, according to whether the index of the first vector is equal to, or half as great as, or twice as great as the index of the second, respectively.

To conclude, we have defined in this lecture the notions of a vector, and of a vector-in-a-region. We saw that these could be regarded as magnitudes, and we defined ratios among them in terms solely of notions that we have already taken as fundamental—that is, in terms only of the experience of the intersection of convex solids, or in terms of this, together with our experience of four selected spheres, according to the one of our two alternative definitions of parallelism that we choose. Then, given any region whatever, we saw how we could define a system of measurement in terms of it which would always enable us to measure all the vectors-inside-the-region in terms of a common unit, and which, in case our space should satisfy the axioms of ordinary Euclidean geometry, and in case our selected standard region should be an ordinary sphere, would give

us an ordinary, every-day Euclidean system of measurement. In our next lecture we shall consider how this theory of measurement may be extended so as to cover, not only all distances within a certain sphere, but all distances in space, and we shall consider on what principle our standard sphere of measurement shall be selected.

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# EINSTEIN'S THEORY OF RELATIVITY CONSIDERED FROM THE EPISTEMOLOGICAL STANDPOINT

## III

### The Philosophical Concept of Truth and the Theory of Relativity*

THE general principle of the relativity of knowledge received its first complete systematic working out in the history of ancient skepticism. Here it possessed, according to the fundamental character of skepticism, an exclusively negative meaning; it signified the limit in principle which is set to all knowledge and by which it is separated once for all from the definitive apprehension of the truth as "absolute." Among the skeptical "tropes" intended to show the uncertainty of sensuous and conceptual knowledge, the "trope" of πρός τι stands in the first place. To know the object, our knowledge would, above all, have to be in a position to grasp it in its pure "in itself" and to separate it from all the determinations, which only belong to it relatively to us and other things. But this separation is impossible, not only actually, but in principle. For what is actually given to us only under certain definite conditions can never be made out logically as what it is in itself and under abstraction of precisely these conditions. Thus, in what we call the perception of a thing, we can never separate what belongs to the objective thing from what be-

* Translated by W. C. and M. C. Swabey.

longs to the subjective perception and contrast the two as independent factors. The form of subjective organization enters as a necessary element into all our so-called objective knowledge of things and properties. The "thing" appears, accordingly, not only differently to the various senses but it is limitlessly variable for the same organ according to the time and varying conditions of perception. For its whole character depends on the relations under which it is presented to us. No content is given us in experience unmixed with others in a purely self-identical character, but what is given us is always only a general combination of impressions. It is not one or the other, "this" or "that" definite quality, but only the reciprocal relation of the one to the other and the other to the one that is here known, indeed that is alone knowable.

Modern science has overcome the objections of skepticism to the possibility of knowledge, not by contesting their content, but by drawing from them a wholly different, indeed, opposite logical consequence. Modern science also assumes the reduction of what is taken in the naïve view of the world, as fixed and absolute "properties" of things to a system of mere relations. "With regard to the properties of the objects of the outer world," we read in, *e. g.*, Helmholtz's *Handbuch der physiologischen Optik*, "it is easy to see that all the properties we can ascribe to them; signify only the effects they produce either on our senses or on other natural objects. Color, sound, taste, smell, temperature, smoothness, solidity belong to the first class; they signify effects on our sense organs. The chemical properties are likewise related to reactions, *i. e.*, effects, which the natural body in question exerts on others. It is thus with the other physical properties of bodies, the optical, the electrical, the magnetic. Everywhere we are concerned with the mutual relations of bodies to each other, with effects which depend on the forces different bodies

exert on each other. . . . From this it follows that in fact, the properties of the objects of nature do not signify, in spite of their name, anything proper to the particular objects in and for themselves, but always a relation to a second object (including our sense organs). The type of effect must naturally always depend on the peculiarities of the effecting body as well as on those of the body on which the effect is exerted. . . . To question whether cinnabar is really red as we see it, or whether this is only an illusion of the senses, is therefore meaningless. The sensation of red is the normal reaction of normally constituted eyes to the light reflected from cinnabar. A color-blind person will see the cinnabar black or dark grey; this also is the correct reaction of his peculiarly constituted eye. . . . In itself, the one sensation is not more correct and not more false than the other. . . ." (30, p. 588f.) The old skeptical "trope," the argument of the πρός τι here stands before us again in all distinctness. But renunciation of the absoluteness of things involves no longer renunciation of the objectivity of knowledge. For the truly objective element in modern knowledge of nature is not so much things as laws. Change in the elements of experience and the fact that no one of them is given in itself, but is always given with reference to something else, constitute no objection to the possibility of objectively real knowledge in so far as the laws establish precisely these relations themselves. The constancy and absoluteness of the elements is sacrificed to gain the permanency and necessity of laws. If we have gained the latter, we no longer need the former. For the objection of skepticism, that we can never know the absolute properties of things, is met by science in that it defines the concept of property in such a way that the latter involves in itself the concept of relation. Doubt is overcome by being outdone. When it is seen that "blue" can mean absolutely nothing save a relation to a seeing eye,

that "heavy" means nothing save a relation of reciprocal acceleration and that in general all "having" of properties can be resolved purely and simply into a "being-related" of the elements of experience, then the longing to grasp the ultimate absolute qualities of things, secretly at the basis of skepticism, loses its meaning. Skepticism is refuted, not by showing a way to a possible fulfillment of its demands, but by understanding and thus rendering ineffective the dogmatic import of these demands themselves.

In this transformation of the general ideal of knowledge, modern science and modern logic are both involved; the development of the one is in closest connection with that of the other. Ancient logic is entirely founded on the relation of "subject" and "predicate," on the relation of the given concept to its also given and final properties. It seeks finally to grasp the absolute and essential properties of absolute self-existent substances. Modern logic, on the contrary, in the course of its development, comes more and more to abandon this ideal and to be made into a pure doctrine of form and relation. The possibility of all determinate character of the content of knowledge is grounded, for it, in the laws of these forms, which are not reducible to mere relations of subsumption but include equally all the different possible types of relational construction and connection of elements of thought. But here doubt must begin in a new and deeper sense. If knowledge of things is understood as knowledge of laws and if an attempt is made to ground the former in the latter and to protect it from the attacks of skepticism, then what guarantees the objectivity, the truth and universality of the knowledge of laws? Do we have, in the strict sense, knowledge of laws or does all that we can gain in the most favorable case resolve itself into knowledge of particular cases? Here as we see, the problem of skepticism is reversed on the basis of the modern conception of law. What perplexed the ancient skep-

tic, who sought the substance of things, was the limitless relativity of all phenomena; it was the fact that phenomena would not remain fixed individual data, but were reduced for knowledge ever again into mere relations and relations of relations. But for the modern skeptic, to whom the objective truth, in so far as it is attainable, means the one all-inclusive and necessary law of all process, the basis of doubt lies in the fact that reality is never given us in this universal intellectual form, but is always divided and broken up into mere punctual particularities. We grasp only a here and a now, only a particularity isolated in space and time, and it is not to be seen how we could ever pass from this perception of the individual to a view of the objective form of the whole. No more than the continuum can be built up and generated by the summation of mere unextended points can a truly objective and necessary law be gained and deduced by the simple aggregation of however many particular cases. This is the form of Hume's skepticism, which is characteristically distinguished from the ancient. While the ancient skeptic could not reach the absolute substance because of the relativities in which the phenomenal world involved him, the modern skeptic fails to reach laws as universal relations because of the absolute particularities of sensation. While in the former it is the certainty of things that is questionable, in the latter it is the certainty of causal connections. The connections of processes become an illusion; what remains is only their particular atoms, the immediate data of sensation, in which all knowledge of "facts," of "matter of fact" ultimately consists.

If it is possible to overcome this essentially more radical form of skepticism also, it can only be by there being shown in it too a concealed dogmatic assumption, which lies implicitly at its basis. And this assumption consists in fact in its concept of empirical "givenness" itself. This given-

ness of "bare" impressions in which abstraction is made in principle from all elements of form and connection, proves to sharper analysis to be a fiction. When this is understood, doubt is directed, not on the possibility of knowledge, but on the possibility of the logical measuring-rod with which knowledge is measured here. Instead of the criterion of the "impression" making the universal formal relations of knowledge and its axioms questionable, the validity of this criterion must be contested on the basis of these relations. The only refuge from radical doubt lies in its being not set aside but intensified, in our learning to question, as ultimate elements of knowledge known in themselves, not only "things" and "laws" but especially sensations. The skepticism of Hume left the "simple" sensation as a completely unproblematic certainty, as a simple and unquestionable expression of "reality." While antique skepticism rested completely on the tacit assumption of absolute things, that of Hume rests on the assumption of absolute sensations. The hypostasization in the one case concerns "outer" being, in the other, "inner" being, but its general form is the same. And only by this hypostasization does the doctrine of the relativity of knowledge gain its skeptical character. Doubt does not result directly from the content of this doctrine, but, on the contrary, it depends on the fact that the doctrine is not truly and consisently thought through. As long as thought contents itself with developing, with reference to phenomena and according to demands of its own form, its logical axioms, and truth as a system of pure relations, it moves within its own circle with complete certainty. But when it affirms an absolute, whether of outer or inner experience, it is forced skeptically to annihilate itself with reference to this absolute. It strikes this absolute of things or of sensations again and again as if against the wall of the cell in which it is enclosed. Relativity, which is, fundamentally,

its immanent force, becomes its immanent limit. It is no longer the principle, which renders possible and governs the positive advance of knowledge, but is merely a necessary instrument of thought, which by that fact confesses itself not adequate to being the absolute object and the absolute truth.

This relation is indeed changed when we contrast to both the dogmatic and the skeptical concept of truth, which are united by a common root, the idealistic concept of truth. For the latter does not measure the truth of fundamental cognitions by transcendent objects, but it grounds conversely the meaning of the concept of the object on the meaning of the concept of truth. Only the idealistic concept of truth overcomes finally the conception which makes knowledge a copying, whether of absolute things or of immediately given "impressions." The "truth" of knowledge changes from a mere pictorial to a pure functional expression. In the history of modern philosophy and logic, this change is first represented in complete clarity by Leibniz, although in his case, the new thought appears in the setting of a metaphysical system, in the language of the monadological scheme of the world. Each monad is, with all its contents, a completely enclosed world, which copies or mirrors no outer being but merely includes and governs by its own law the whole of its presentations; but these different individual worlds express, nevertheless, a common universe and a common truth. This community, however, does not come about by these different pictures of the world being related to each other as copies of a common "original" but by the fact that they correspond functionally to each other in their inner relations and in the general form of their structure. For one fact, according to Leibniz, expresses another when there exists between what can be said of the one and of the other a constant and regular relation. Thus a perspective pro-

jection expresses its appropriate geometrical figure, an algebraic equation expresses a definite figure, a drawn model a machine; not as if there existed between them any sort of factual likeness or similarity, but in the sense that the relations of the one structure correspond to those of the other in a definite conceptual fashion. (43, VII, 263f, 44, II, 233; *cf.* 7, II, 167.) This Leibnizian concept of truth was taken up and developed by Kant who sought to free it from all the unproved metaphysical assumptions that were contained in it. In this way he gained his own interpretation of the critical concept of the object, in which the relativity of knowledge was affirmed in a far more inclusive meaning than in ancient or modern skepticism, but in which also this relativity was given a new positive meaning. The theory of relativity of modern physics can be brought without difficulty under this meaning, for, in a general epistemological regard, it is characterized by the fact that in it, more clearly and more consciously than ever before, the advance is made from the copy theory of knowledge to the functional theory. As long as physics retained the postulate of absolute space, the question still had a definite meaning as to which of the various paths of a moving body that result when we regard it from different systems of reference, represents the real and "true" motion; thus a higher objective truth had to be claimed for certain spatial and temporal values, obtained from the standpoint of certain selected systems, than for others. The theory of relativity ceases to make this exception; not that it would abandon the determinateness of natural process, but because it has at its disposal new intellectual means of satisfying this demand. The infinite multiplicity of possible systems is not identical with the infinite indeterminateness of the values to be gained in them —in so far as all these systems are to be related and connected with each other by a common rule. In this respect,

the principle of relativity of physics has scarcely more in common with "relativistic positivism," to which it has been compared, than the name. When there is seen in the former a renewal of ancient sophistical doctrines, a confirmation of the Protagorean doctrine that man is the "measure of all things," its essential achievement is mistaken.[10] The physical theory of relativity teaches not that what appears to each person is true to him, but, on the contrary, it warns against taking appearances, which hold only from a particular system, as the truth in the sense of science, *i. e.*, as an expression of an inclusive and final law of experience. The latter is gained neither by the observations and measurements of a particular system nor by those of however many such systems, but only by the reciprocal coördination of the results of all possible systems. The general theory of relativity purports to show how we can gain assertions concerning all of these, how we can rise above the fragmentariness of the individual views to a total view of natural processes. (*Cf.* above.) It abandons the attempt to characterize the "object" of physics by any sort of pictorial properties, such as can be revealed in presentation, and characterizes it exclusively by the unity of the laws of nature. When, for example, it teaches that a body regarded from one system possesses spherical form and, regarded from another system, in motion relatively to the first, appears as an ellipsoid of rotation, the question can no longer be raised as to which of the two optical images here given is like the absolute form of the object, but it can and must be demanded that the multiplicity and diversity of the sensuous data here appearing can be united into a universal concept of experience. Nothing more is demanded of the critical concept of truth and the object. According to the critical view, the object is no absolute model to which our sensuous pres-

[10] *Cf.* Petzoldt (61).

entations more or less correspond as copies, but it is a "concept, with reference to which presentations have synthetic unity." This concept the theory of relativity no longer represents in the form of a picture but as a physical theory, in the form of equations and systems of equations, which are co-variant with reference to arbitrary substitutions. The "relativization," which is thus accomplished, is itself of a purely logical and mathematical sort. By it the object of physics is indeed determined as the "object in the phenomenal world"; but this phenomenal world no longer possesses any subjective arbitrariness and contingency. For the ideality of the forms and conditions of knowledge, on which physics rests as a science, both assures and grounds the empirical reality of all that is established by it as a "fact" and in the name of objective validity.

## IV. Matter, Ether and Space

IN the structure of physics we must, it seems, distinguish two different classes of concepts from each other. One group of concepts concerns only the form of order as such, the other the content that enters into this form; the first determines the fundamental schema which physics uses, the other concerns the particular properties of the real by which the physical object is characterized. With regard to the pure formal concepts, they appear to persist as relatively fixed unities in spite of all changes of physical ideals in detail. In all the diversity and conflict of the systematic concepts of physics, space and time are distinguished as the ultimate, agreeing unities. They seem, in this sense, also, to constitute the real *a priori* for any physics and the presupposition of its possibility as a science. But the first step from these bare possibilities to reality, which is a matter not of the spatio-temporal form, but of the *somewhat* that is thought to be somehow "given" *in* space and *in* time, seems to force us beyond the circle of the *a priori*. Kant indeed, in the *Metaphysischen Anfangsgründen der Naturwissenschaft*, attempted an *a priori* deduction and construction of the concept of "matter" as a necessary concept of physics; but it is easy to see that this deduction does not stand on the same plane and cannot claim

the same force as the Transcendental Aesthetic or the Analytic of the Pure Understanding. He himself believed that he possessed in these deductions a philosophical grounding of the presuppositions of the science of Newton; today we recognize to an increasing extent that what he so regarded was in fact nothing but a philosophical circumlocution for precisely these presuppositions. As a fundamental definition of the physical concept of the object, the classical system of mechanics is only one structure, by the side of which there are others. Heinrich Hertz, in his new grounding of the mechanical principles, distinguished three such structures: the first is given in the Newtonian system, which is founded on the concepts of space, of time, of force, and of mass, as given presentations; the second leaves the presuppositions of space, time and mass unchanged, but substitutes for the concept of force as the mechanical "cause of acceleration" the universal concept of energy, which is divided into two different forms, potential and kinetic energy. Here, too, we have four mutually independent concepts, whose relations to each other are to constitute the content of mechanics. Hertz's own formulation of mechanics offers a third structure in which the concept of force or of energy as an independent idea is set aside and the construction of mechanics is accomplished by only three independent fundamental ideas, space, time and mass. The circle of possibilities would thus have seemed completely surveyed—had not the theory of relativity once more given a new interpretation to the mutual relation between the pure formal concepts and the physical concepts of the object and substance, and thus transformed the problem not only in content but in principle.

The concept of "nature," the gaining of which is the real methodic problem of physics, leaves room, as the history of physical thought shows, for a dualism of presuppositions, which as such seems necesary and unavoidable. Even in the first logical beginnings of genuine natural science, which are found in Greek thought, this dualism appears in full distinctness and clarity. Antique atomism, which is the first classical example of a conceptual and scientific picture of the world, can only describe and unify the "being" of nature by building it up out of two heterogeneous elements. Its view of nature is founded on the opposition of the "full" and the "void." The two, the full and the void, prove necessary elements for the constitution of the object of physics. To the being of the atom and matter as the παμπλῆρες ὄν, there is opposed by Democritus the not-being, the μὴ ὄν of empty space; but this being and this not-being possessed for him, however, uncontested physical truth and thus indubitable physical reality. The reality of motion was only intelligibe by virtue of this dual presupposition; motion would disappear if we did not both distinguish empty space from the material filling of space and conceive the two as in inseparable mutual relation, as fundamental elements in all natural processes. At the beginning of modern times, Descartes attempted philosophically to overcome this duality in the foundations of physical thought. Proceeding from the thought of the unity of *consciousness,* he postulated also a new unity of *nature.* And this seemed to him only attainable by abandoning the opposition of the "full" and the "void," of matter and extension. The physical being of the body and the geometrical being of extension constitute one and the same object: the "sub-

stance" of a body is reduced to its spatial and geometrical determinations. Thus a new approach to physics, methodologically deeper and more fruitful, was found, the concrete realization of which, however, could not be accomplished by Descartes' physics. When Newton fought the hypothetical and speculative premises of the Cartesian physics, he also abandoned this approach. His picture of the world was rooted in the dualistic view, which was even intensified in it and which set its seal on his universal law of nature and the cosmos. On the one side, there stands space as a universal receptacle and vessel; on the other, bodies, inert and heavy masses, which enter into it and determine their reciprocal position in it on the basis of a universal dynamic law. The "quantity of matter," on the one hand, the purely spatial "distance" of the particular masses from each other, on the other, give the universal physical law of action, according to which the cosmos is constructed. Newton as a physicist always declined to ask for a further "why," for a reason for this rule. It was for him the unitary mathematical formula, which included all empirical process under it and thus perfectly satisfied the task of the exact knowledge of nature. That this formula concealed—in the expression for the cosmic *masses* and in the expression for their *distance*—two wholly different elements seemed a circumstance that no longer concerned the physicist but only the metaphysician and the speculative philosopher of nature. The proposition *"hypotheses non fingo"* cuts off any further investigation in this direction. For Newton as for Democritus, matter and space, the full and the void, form for us the ultimate but mutually irreducible elements of the physical world, the fundamental building-stones of all reality, because as equally justified and equally necessary factors, they enter into the highest law of motion taught us by experience.

If we contrast this view with the picture of the world of modern and most modern physics, there results the surprising fact that the latter seems to be again on the road to Descartes, not indeed in content, but certainly in method. It too strives from various sides toward a view in which the dualism of "space" and "matter" is cancelled, in which the two no longer occur as different classes of physical object-concepts. There now appears in the concept of the *"field"* a new mediating concept between "matter" and "empty space"; and this it is which henceforth appears with increasing definiteness as the genuine expression of the physically real since it is the perfect expression of the physical law of action. In this concept of the field, the typical manner of thought of modern physics has gained, from the epistemological standpoint, its sharpest and most distinct expression. There now takes place, starting from electrodynamics, a progressive transformation of the concept of matter. Already with Faraday, who constructed matter out of "lines of force," there is expressed the view that the field of force cannot depend on matter, but that, on the contrary, what we call matter is nothing else than specially disinguished places of this field.[11] In the progress of electrodynamics, this view is confirmed and assumes ever more radical expression. The doctrine is carried through more and more of a pure "field-physics," which recognizes neither bare undifferentiated space by itself nor matter by itself subsequently entering into this finished space, but which takes as a basis the intuition of a spatial manifold determined by a certain law and qualified and differentiated according to it. Thus, *e. g.*, there was established by Mie a more general form of electrodynamics on the basis of which it seemed possible to construct matter out of the field. The concept of a substance existing along with the electromagnetic field seemed unnecessary in this

[11] On Faraday, *cf.* Buek (4, esp. p. 41ff.); *cf.* also Weyl (83, p. 142).

approach; according to the new conception, the field no longer requires for its existence matter as its bearer, but matter is considered and treated, on the contrary, as an "outgrowth of the field." It is the last consequence of this manner of thought that is drawn by the theory of relativity. For it, too, the real difference finally disappears between an "empty" space and a space-filling substance, whether one calls this matter or ether, since it includes both moments in one and the same act of *methodic determination.* The "riddle of weight" is revealed to us, according to the fundamental thought of Einstein's theory of gravitation, in the consideration and analysis of the inner relations of measurement of the four dimensional space-time manifold. For the ten functions $g_{\mu\nu}$, which occur in the determination of the linear elements of the general theory of relativity $ds^2 = \sum_1^4 g_{\mu\nu} dx_\mu dx_\nu$ $(\mu, \nu = 1, 2, 3, 4)$, represent also the ten components of the gravitation potential of Einstein's theory. It is thus the same determinations, which, on the one hand, designate and express the metrical properties of the four-dimensional space and, on the other, the physical properties of the field of gravitation. The spatio-temporal variability of the magnitudes $g_{\mu\nu}$ and the occurrence of such a field prove to be equivalent assumptions differing only in expression. Thus it is shown most distinctly that the new physical view proceeds neither from the assumption of a "space in itself," nor of "matter" nor of "force in itself"—that it no longer recognizes space, force and matter as physical objects separated from each other, but that for it exists only the unity of certain *functional relations,* which are differently designated according to the system of reference in which we express them. All dynamics tends more and more to be resolved into pure metrics, a process in which indeed the concept of metrics

undergoes, in contrast with classical geometry, an extraordinary broadening and generalization whereby the measurements of Euclidean geometry appear as only a special case within the total system of possible measurements in general. "The world," as is said by Weyl, in whose account of the general theory of relativity one can trace and survey this development most clearly," is a (3+1)=dimensional metrical manifold; all physical phenomena are expressions of world metrics. . . . The dream of Descartes of a purely geometrical physics seems to be about to be fulfilled in a wonderful way, which could not have been foreseen by him." (83, p. 244; *cf.* p. 85ff., 170 ff.)

Just as the dualism of matter and space is superseded here by a unitary physical conception, so the opposition between "matter" and "force" is to be overcome by the principle and law of the new physics. Since Newton, as a physicist, established this opposition between the "inert masses" and the forces that affect them in the *Philosophiae Naturalis Principia Mathematica,* attempts, indeed, have not been lacking to overcome it from the philosophical and speculative side. Leibniz led the way here: but although, in his metaphysics, he wholly resolved substance into force, he retained in the construction of his mechanics, the duality of an "active" and a "passive" force, whereby matter is subsumed under the concept of the latter. The essence of matter consists in the dynamic principle immanent in it; but this expresses itself, on the one hand, in activity and striving for change, on the other hand, in the resistance which a body opposes, according to its nature, to change coming upon it from without.[12] As for Newton, the opposition in fundamental concepts, which he assumes, threatens finally to destroy the unity of the physical structure of his world; he can only retain this unity by introducing at

[12] *Cf.* (44), I, 204, 267ff., 332 II, 290ff., 303.

a certain place a *metaphysical* factor. The principle of the conservation of *vis viva* is disputed by him because all bodies consist of "absolutely hard" atoms, and in the rebounding of such atoms, mechanical energy must be lost; the sum total of force is in a continuous decrease, so that for its preservation the world needs from time to time a new divine impulse. (58, p. 322ff.) Kant attempted in a youthful work, the *Monadologia Physica* of the year 1756, a reconciliation and mediation between the principles of the Leibnizian philosophy and those of Newtonian mechanics; and in the *Metaphysischen Anfangsgründen der Naturwissenschaft* he returns to the attempted purely dynamic deduction and construction of matter. The "essense" of matter *i. e.,* its pure concept for experience, according to which it is nothing else than a totality of external relations, is resolved into a pure interaction of forces acting at a distance; but since these forces themselves occur in a double form, as attracting and repelling forces, the dualism is not fundamentally overcome, but is only shifted back into the concept of force itself.

Modern physics has sought, from essentially different standpoints and motives, to overcome the old opposition between matter and force, which seemed sanctioned and made eternal in the classical system of mechanics. Heinrich Hertz's *Prinzipien der Mechanik* takes the opposite course to that of previous philosophical speculation by placing the sought unity in the concept of mass, instead of in the concept of force. Along with the fundamental concepts of space and time, only the concept of mass enters into the systematic construction of mechanics. The carrying out of this view presupposes, indeed, that we do not remain with gross perceptible mass and gross perceptible motion, but supplement the sensuously given elements, which by themselves do not constitute a lawful world, by

assuming certain "concealed" masses and "concealed" motions. This supplementation takes place when it is shown to be necessary for the description and calculation of phenomena, and without arousing suspicion since mass is conceived by Hertz from the beginning merely as a definite factor of calculation. It is intended to express nothing but certain coördination of space and time values: "a particle of mass," as Hertz defines it, "is a property by which we coördinate unambiguously a certain point of space to a certain point of time (and)[13] a certain point of space to every other time." (31, p. 29ff., 54.) Another attempt was made by general energetics to reach a unified foundation for physics and with it for mechanics. Inert mass appears here merely as a definite factor of energy, as the capacity-factor of the energy of motion, which with certain other capacity-factors shares with the different types of energy, *e. g.*, electricity, the empirical property of quantitative conservation. Energetics refuses to grant this law of conservation a special place and to recognize matter as a particular substance along with energy. (*Cf.* 60, p, 282ff.) But precisely in this we see very distinctly what is logically unsatisfactory, which consists in that the principle of conservation refers to wholly different moments between which an inner connection is not to be seen.

The theory of relativity brings important clarification here too in that it combines the two principles of conservation: that of the conservation of energy and that of the conservation of mass into a single principle. This result it gains by applying its characteristic manner of thought; it is led to this result by general considerations on the conditions of measurement. The demand of the theory of relativity (at first of the special theory) is that the law of the conservation of energy be valid not only with reference

[13] Trans.

to any system of coördinates K but also with reference to any other in uniform rectilinear motion relatively to it; it results from this presupposition, however, combined with the fundamental equations of Maxwell's electrodynamics that when a body in motion takes up energy $E_0$ in the form of radiation its inert mass increases by a definite amount $\left(\frac{E}{c^2}\right)$. The mass of a body is thus a measure of its content of energy; if the energy content alters a definite amount then its mass alters proportionately.[14] Its independent constancy is thus only an appearance; it holds good only in so far as the system takes up and gives off no energy. In the modern electron theory, it follows from the well-known investigation of Kaufmann that the "mass" of an electron is not unchangeable, but that it rapidly increases with the velocity of the electron as soon as the latter approaches the velocity of light. While previously a distinction had been made between a "real" and a "fictitious" mass of electrons, *i. e.*, between an inertia, which came from its ponderable mass, and another, which they possessed solely because of their motion and their electric charge, in so far as this opposed a certain resistance to every change of velocity; it now turns out that the alleged ponderable mass of the electrons is to be taken as strictly=O.

The inertia of matter thus seems completely replaced by the inertia of energy; the electron—and thus the material atom as a system of electrons—possesses no material but only "electromagnetic" mass. What was previously regarded as the truly fundamental property of matter, as its substantial kernel, is resolved into the equations of the electro-magnetic field. The theory of relativity goes further in the same direction; but it reveals in this too its peculiar *nuance* and character. This comes out especially in the process by which it gains one of its fundamental propositions: in the establishment of the equivalence of

[14] Einstein (16a) and Planck (64 and 65).

phenomena of inertia and weight. Here it is at first merely a calculation, a consideration of the same phenomena from different systems of reference, which points the way. We can, as it shows, regard one and the same phenomenon now as a pure inertial movement and now as a movement under the influence of a field of gravitation according to the standpoint we choose. The equivalence of *judgment,* here indicated, grounds for Einstein the *physical identity* of phenomena of inertia and weight. If certain accelerated motions occur for an observer within his sphere of observation, he can interpret them either by ascribing them to the effects of a field of gravitation or conceive the system of reference from which he makes his measurements as in a certain acceleration. The two assumptions accomplish precisely the same in the description of the facts and can thus be applied without distinction. We can—as Einstein expresses it—produce a field of gravitation by a mere change of the system of coördinates. (17, p. 10; *cf.* 18, p. 45ff.) Hence, it follows that to attain a universal theory of gravitation we need only assume such a shift of the system of reference and establish its consequences by calculation. It suffices that in purely ideal fashion we place ourselves at another standpoint to be able to deduce certain physical consequences from this change of standpoints. What was previously done in the Newtonian theory of gravitation by the dynamics of forces is done by pure kinematics in Einstein's theory, *i. e.,* by the consideration of different systems of reference moving relatively to each other.

In emphasizing this ideal element in Einstein's theory of gravitation, the *empirical* assumption on which it rests must naturally not be forgotten. That we change in thought, by the mere introduction of a new system of reference, a field of inertia into a field of gravitation, and a field of gravitation of special structure into a field of iner-

tia, rests on the empirical equality of inert and gravitating masses of bodies, as was established with extraordinary exactitude by the investigation of Eötvös to which Einstein refers. Only the fact that gravitation imparts to all bodies found at the same place in the field of gravitation, the same amount of acceleration, and that thus it is for any definite body the same constant, *i. e.*, mass, which determines its inertial effects and its gravitational effects, renders possible that transformation of the one into the other, from which the Einstein theory starts.[15] But it is especially interesting and important from a general methodological standpoint that this fundamental fact is given a completely different interpretation than in the Newtonian mechanics. What Einstein urges against the latter is that it *registered* the phenomenon of the equivalence of gravitating and inert masses, but did not interpret it. (18, p. 44.) What was established as a fact by Newton is now to be understood from principles. In this problem one can trace how gradualy the question as to the "essence" of matter and of gravitation is superseded by another epistemological formulation of the question, which finds the "essence" of a physical process expressed wholly in its quantitative relations and its numerical constants. Newton never ceased to reject the question as to essence, which met him ever again, and the phrase that physics has to do merely with the "description of phenomena" was first formulated in his school and is an expression of his method.[16] But so little was he able to escape this question that he expressly urged that universal attraction was not itself grounded in the essence of body, but that it came to it as something new and alien. Weight is, as he emphasizes, indeed a universal but not an essential property of matter. (59, Vol. III, p. 4.) What this distinction be-

[15] For more detail, *cf.* Freundlich (24), pp. 28 and 60f. and Schlick (79). p. 27ff; *cf.* Einstein (18), p. 45ff.

[16] Keill, *Introductio ad veram Physicam* (1702), (36); *cf.* 7, II, 404ff.

tween the universal and the essential means from the standpoint of the physicist, who has to do merely with the laws of phenomena, and thus with the universality of the rule to which they are subjected, is here left in the dark. Here lies a difficulty, which has been felt again and again in the tedious controversy of physicists and philosophers on the actuality and possibility of force acting at a distance. Kant, in his *Metaphysischen Anfangsgründen der Naturwissenschaft,* urges against Newton that, without the assumption that all matter merely by virtue of its essential properties exercises the action we call gravitation, the proposition that the universal attraction of bodies is proportional to their inert mass, would be a totally contingent and mysterious fact. (35, IV, p. 421.) In its solution of this problem the general theory of relativity has followed the path prescribed by the peculiarity of the physical method. The numerical *proportion,* which is universally found between inert and heavy masses becomes the expression of physical *equivalence,* of the essential likeness of the two. The theory of relativity concludes that it is the *same* quality of the body, which is expressed according to circumstances as "inertia" or as "weight." We have here in principle the same procedure before us, which, *e. g.*, in the electromagnetic theory of light led to insight into the "identity" of light waves and electrical waves. For this identity too means nothing else and nothing more mysterious than that we can represent and master the phenomena of light and the phenomena of dielectric polarization by the same *equations* and that the same numerical value results for the velocity of light and for that of dielectric polarization. This equality of values means to the physicist likeness in essence —since for him essence is defined in terms of exact determinations of measure and magnitude. In the advance to this insight, there may be traced historically a definite series of steps, a culmination of physical theories. The

physics of the eighteenth century was in general rooted in a substantialistic view. In the fundamental investigations of Sadi Carnot on thermodynamics heat was still regarded as a material, and the assumption seemed unavoidable, in understanding electricity and magnetism, of a particular electric and magnetic "matter." Since the middle of the nineteenth century, however, there appears in place of this "physics of materials," ever more definitely and distinctly the physics that has been called the "physics of principles." Here a start is not made from the hypothetical existence of certain materials and agents, but from certain universal relations, which are regarded as the criteria for the interpretation of particular phenomena. The general theory of relativity stands methodologically at the end of this series, since it collects all particular systematic principles into the unity of a supreme postulate, in the postulate not of the constancy of things, but of the invariance of certain magnitudes and laws with regard to all transformations of the system of reference.

The same evolution, that is characteristic of physical conceptual construction in general, is seen when we go from the concept of matter to the second fundamental concept of modern physics, to the concept of the ether.[17] The idea of the ether, as the bearer of optical and magnetic effects was at first conceived in the greatest possible analogy and affinity with our presentations of empirically given materials and things. A sensuous description of its fundamental properties was sought by comparing it now with a perfectly incompressible fluid, now with a perfectly elastic body. But the more one attempted to work these pictures out in detail, the more distinctly was it seen that they demanded the impossible of our faculty of presentation, that

[17] Here I do not go into details in the development of the hypothesis of the ether; they have been expounded from the standpoint of epistemology by *e. g.*, Aloys Müller (55, p. 90ff.) and Erich Becher (2, p. 232 ff.). On the following *cf. Substanzbegriff und Funktionsbegriff* (8, p. 215ff.).

they demanded the unification of absolutely conflicting properties. Thus modern physics was more and more forced to abandon in principle this sort of sensuous description and illustration. But the difficulty was unchanged also when one asked, not concerning any concrete properties of the ether, but merely concerning the abstract laws of its motion. The attempt to construct a mechanics of the ether led little by little to the sacrifice of all the fundamental principles of classical mechanics; it was seen that, really to carry it through, one would have to give up not only the principle of the equality of action and reaction, but the principle of impenetrability in which, *e. g.*, Euler saw the kernel and inclusive expression of all mechanical laws. Ether was and remained accordingly, in an expression of Planck, the "child of sorrow of the mechanical theory"; the assumption of the exact validity of the Maxwell-Hertzian differential equations for electrodynamic processes in the pure ether excludes the possibility of their mechanical explanation.[18] An escape from this antinomy could only be reached by reversing the treatment. Instead of asking about the properties or constitution of the ether as a real thing, the question must be raised as to by what right here in general one seeks for a particular substance with particular material properties and a definite mechanical constitution. What if all the difficulties of the answer are based on the question itself, there being in it no clear and definite physical meaning? That is, in fact, the new position which the theory of relativity takes to the question of the ether. According to the outcome of Michelson's investigation and the principle of the constancy of the propagation of light, each observer has the right to regard his system as "motionless in the ether"; one must thus ascribe to the ether simultaneous rest with reference to wholly different sys-

[18] *Cf.* Planck (67), p. 64ff. Lenard (45*a* and *b*), especially declares for the possibility and necessity of a "mechanics of the ether."

tems of coördinates K, K′, K″, which are in uniform translatory motion relatively to each other. That, however, is an obvious contradiction and it forces us to abandon the thought of the ether as a somehow moving or motionless "substance," as a thing with a certain "state of motion." Physics, instead of imagining some sort of hypothetical substratum of phenomena and losing itself in consideration of the nature of this substratum, is satisfied, as it becomes a pure "physics of fields," with the body of field-equations themselves and their experimentally verifiable validity. "One cannot define," says *e. g.*, Lucien Poincaré, "ether by material properties without committing a real fallacy, and to characterize it by other properties than those, the direct and exact knowledge of which is produced for us by experiment, is an entirely useless labor condemned to sterility from the beginning. The ether is defined when we know the two fields, which can exist in it, the electric and magnetic fields, in their magnitude and direction at each point. The two fields can change; by custom we speak of a motion propagated in the ether; but the phenomenon accessible to experiment is the propagation of these changes." (75, p. 251.) Here we again face one of those triumphs of the critical and functional concept over the naïve notion of things and substances, such as are found more and more in the history of exact science. The physical rôle of the ether is played as soon as a type of exposition is found for the electrodynamic laws into which it does not enter as a condition. "The theory of relativity," remarks one of its representatives, "rests on an entirely new understanding of the propagation of electromagnetic effects in empty space; they are not carried by a medium, but neither do they take place by unmediated action at a distance. But the electromagnetic field in empty space is a thing possessing self-existent physical reality independently of all substance. Indeed, one must first accustom himself to this

idea; but perhaps this habituation will be made easier by the remark that the physical properties of this field, which are given most adequate expression in Maxwell's equations, are much more perfectly and exactly known than the properties of any substance." (Laue, 41, p. 112.) Habituation with regard to a "thing independent of any substance" can indeed be as little attributed to common human understanding as to the epistemologically trained understanding; for precisely to the latter does substance mean the *category* on the application of which rests all possibility of positing "things." But it is obvious that we have here only an inexactitude of expression and that the "independent physical reality" of the electromagnetic field can mean nothing but the reality of the relations holding within it which are expressed in the equations of Maxwell and Hertz. Since they are for us the ultimate attainable object of physical knowledge, they are set up as the ultimate attainable reality for us. The idea of the ether as an inexperiencable substance is excluded by the theory of relativity in order to give conceptual expression merely to the pure properties of empirical knowledge.

For this purpose, however, according to the theory of relativity, we do not need the fixed and rigid reference body, to which classical mechanics was ultimately referred. The general theory of relativity no longer measures with the rigid bodies of Euclidean geometry and classical mechanics, but it proceeds from a new and more inclusive standpoint in its determination of the unversal linear element *ds*. In place of the rigid rod which is assumed to retain the same unchanging length for all times and places and under all particular conditions of measurement there now appear the curved coördinates of Gauss. If any point P of the space-time continuum is determined by the four parameters $x_1$, $x_2$, $x_3$, $x_4$, then for it and an infinitely

close point P′ there is a certain "distance" *ds*, which is expressed by the formula:

$$ds^2 = g_{11}\,dx_1^{\,2} + g_{22}\,dx_2^{\,2} + g_{33}\,dx_3^{\,2} + g_{44}\,dx_4^{\,2} + 2g_{12}\,dx_1\,dx_2 + 2g_{13}\,dx_1\,dx_3 + \ldots$$

in which the magnitudes $g_{11}, g_{22} \ldots . g_{44}$ have values, which vary with the place in the continuum. In this general expression, the formula for the linear element of the Euclidean continuum is contained as a special case. We need not here go into details of this determination.[19]—its essential result, however, is that measurements in general different from each other result for each place in the space-time continuum. Each point is referred, not to a rigid and fixed system of reference outside of it, but to a certain extent only to itself and to infinitely close points. Thus all measurements become infinitely fluid as compared with the rigid straight lines of Euclidean geometry, which are freely movable in space without change of form; and yet, on the other hand, all these infinitely various determinations are collected into a truly universal and unitary system. We now apply, instead of given and finite reference bodies, only "reference mollusks" as Einstein calls them; but the conceptual system of all these "mollusks" satisfies the demand for an exact decription of natural processes. For the universal principle of relativity demands that all these systems can be applied as reference bodies with equal right and with the same consequences in the formulation of the universal laws of nature; the form of the law is to be completely independent of the choice of the mollusk. (18, p. 67.) Here is expressed again the characteristic procedure of the general theory of relativity; while it destroys the *thing-form* of the finite and rigid reference body it would thereby only press forward to a higher form of object, to the true *systematic form* of nature and its laws. Only by heightening and outdoing the difficulties which resulted even for classical

[19] *Cf.* Einstein (17 and 18, pp. 59ff.); *cf.* below VI.

mechanics from the fact of the relativity of all motions, does it hope to find an escape in principle from these difficulties. "The clearer our concepts of space and time become," as was said in the outline of mechanics, which Maxwell has given in his short work, *Matter and Motion*, "the more do we see that everything to which our dynamic doctrines refer, belongs in a single system. At first we might think that we, as conscious beings, must have as necessary elements of our knowledge, an absolute knowledge of the place, in which we find ourselves, and of the direction in which we move. But this opinion, which was undoubtedly that of many sages of antiquity, disappears more and more from the idea of the physicist. In space, there are no milestones; one part of space is precisely like any other part, so that we cannot know where we are. We find ourselves in a waveless sea without stars, without compass and sun, without wind and tide, and cannot say in what direction we move. We have no log that we can cast out to make a calculation; we can indeed determine the degree of our motion in comparison with neighboring bodies, but we do not know what the motion in space of these bodies is." (51, p. 92f.). From this mood of *"ignorabimus,"* into which physics was sinking more and more, only a theory could free it which grasped the problem at its root; and, instead of modifying the previous solutions, transformed fundamentally the formulation of the question. The question of absolute space and absolute motion could receive only the solution which had been given to the problem of the perpetual *mobile* and the squaring of the circle. It had to be made over from a mere negative expression into a positive expression, to be changed from a limitation of physical knowledge to a principle of such knowledge, if the true philosophic import, which was concealed in it, was to be revealed.

## V

## The Concepts of Space and Time of Critical Idealism and the Theory of Relativity

WE have hitherto sought primarily to understand the special and general theory of relativity on its physical side. In fact, this is the standpoint from which it must be judged and one does it poor service if one seeks precipitately to interpret its results in purely "philosophical" or indeed in speculative and metaphysical terms. The theory contains not one concept, which is not deducible from the intellectual means of mathematics and physics and perfectly representable in them. It only seeks to gain full consciousness of precisely these intellectual means by seeking not only to represent the result of physical measurement, but to gain fundamental clarity concerning the form of any physical measurement and its conditions.

Thereby it seems indeed to come into the immediate neighborhood of the critical and transcendental theory, which is directed on the "possibility of experience"; but it is nevertheless different from it in its general tendency. For, in the language of this transcendental criticism, the doctrine of space and time developed by the theory of relativity is a doctrine of empirical space and empirical time, not of pure space and pure time. As far as concerns this point, there is scarcely possible a difference of opinion; and, in fact, all critics, who have compared the Kantian and the Einstein-Minkowski theories of space and time seem to have reached essentially the same result.[20] From the

[20] *Cf.* esp. Natorp, (56, p. 392ff.). Hönigswald (33, p. 88ff.). Frischeisen-Köhler (26, p. 323ff.) and more recently Sellien (81, p. 14ff.).

standpoint of a strict empiricism, one could attempt to dispute the possibility of a doctrine of "pure space" and of "pure time"; but the conclusion cannot be avoided that in so far as such a doctrine is justified, it must be independent of all results of concrete measurement and of the particular conditions, which prevail in the latter. If the concepts of pure space and pure time have in general any definite justified meaning. to use a phrase of the theory of relativity, then this meaning must be invariant with regard to all transformations of the doctrine of the empirical measurement of space and time. The only thing that such transformations can and will accomplish is that they teach us to draw the line more sharply between what belongs to the purely philosophical, "transcendental," criticism of the concepts of space and time and what belongs merely to the particular applications of these concepts. Here, in fact, the theory of relativity can perform an important indirect service for the general criticism of knowledge,—if we resist the temptation to translate its propositions directly into propositions of the criticism of knowledge.

Kant's doctrine of space and time developed to a large extent on the basis of physical problems, and the conflict carried on in the natural science of the eighteenth century on the existence of absolute time and absolute space affected him keenly from the beginning. Before he approached the problems of space and time as a critical philosopher, he had himself lived through the various and opposite solutions by which contemporary physics sought to master these problems. Here, at first, contrary to the dominant scholastic opinion, he took his stand throughout on the basis of the relativistic view. In his *Neuen Lehrbegriff der Bewegung und der Ruhe* of the year 1758, the thirty-four year old Kant set up the principle of the relativity of all motion with all decisiveness and from it at-

tacked the traditional formulation of the principle of inertia. "Now I begin to see," he says after he has illustrated the difficulties of the concept of "absolute motion" with well-known examples, "that I lack something in the expression of motion and rest. I should never say: a body rests without adding with regard to what thing it rests, and never say that it moves without at the same time naming the objects with regard to which it changes its relation. If I wish to imagine also a mathematical space free from all creatures as a receptacle of bodies, this would still not help me. For by what should I distinguish the parts of the same and the different places, which are occupied by nothing corporeal?" (35, II, 19.) But Kant, in his further development did not at first remain true to the norm, which he here set up so decisively and of which a modern physicist has said that it deserves to be set up in iron letters over each physical lecture hall.[21] He ventured to abandon the concept of inertial force, of *vis inertiae;* he refused to pour his thoughts on the principles of mechanics "into the mill of the Wolffian or of any other famous system of doctrine." But while he opposed in this way the authority of the leading philosophers, he could not permanently withdraw himself from the authority of the great mathematical physicists of his time. In his *Versuch, den Begriff der negativen Grössen in die Weltweisheit einzuführen* of the year 1763, he took his place at the side of Euler to defend with him the validity of the Newtonian concepts of absolute space and absolute time, and six years later, in his essay on the first grounds of the difference of regions in space (1769), he sought to support the proof, that Euler had attempted, of the existence of absolute space from the principles of mechanics, by another, purely geometrical consideration, which "would give practical geometricians a conclusive reason to be able to affirm the reality of their

[21] Streintz (82), p. 42.

absolute space with the "evidence" which is customary to them." (35, II, 394.) But this is indeed only an episode in Kant's evolution; for only a year later the decisive critical turn in the question of space and time had taken place in his Inaugural Dissertation of the year 1770. By it the problem receives an entirely new form; it is removed from the field of physics to that of "transcendental philosophy" and must be considered and solved according to the general principles of the latter.

But the transcendental philosophy does not have to do primarily with the reality of space or of time, whether these are taken in a metaphysical or in a physical sense, but it investigates the objective *significance* of the two concepts in the total structure of our empirical knowledge. It no longer regards space and time as things, but as "sources of knowledge." It sees in them no independent objects, which are somehow present and which we can master by experiment and observation, but "conditions of the possibility of experience," conditions of experiment and observation themselves, which again for their part are not to be viewed as things.

What—like time and space—makes possible the positing of objects can itself never be given to us as a particular object in distinction from others. For the *"forms"* of possible experience, the forms of intuition as well as the pure concepts of the understanding, are not met again as *contents* of real experience. Rather the only possible manner in which we can ascribe any sort of "objectivity" to these forms must consist in that they lead to certain *judgments* to which we must ascribe the values of necessity and universality. The meaning is thus indicated, in which one can henceforth inquire as to the objectivity of space or time. Whoever demands absolute thing-like correlates for them strains after shadows. For their whole "being" consists in the meaning and function they possess for the

complexes of judgments, which we call science, whether geometry or arithmetic, mathematical or empirical physics. What they can accomplish as presuppositions in this connection can be exactly determined by transcendental criticism; what they are as things in themselves is a vain and fundamentally unintelligible question. This basic view comes out clearly even in the Inaugural Dissertation. Even here absolute space and time possessing an *existence* separate from empirical bodies and from empirical events, are rejected as nonentities, as mere conceptual fictions (*inane rationis commentum.*) The two, space and time, signify only a fixed law of the mind, a schema of connection by which what is sensuously perceived is set in certain relations of coexistence and sequence. Thus the two have, in spite of their "transcendental ideality," "empirical reality," but this means always only their validity for all experience, which however must never be confused with their existence as isolated objective contents of this experience itself. "Space is merely the form of external intuition (formal intuition) and not a real object that can be perceived by external intuition. Space, as prior to all things which determine it (fill or limit it), or rather which give an empirical intuition determined by its form, is, under the name of absolute space, nothing but a mere possibility of external phenomena. . . . If we try to separate one from the other, and to place space outside all phenomena, we arrive at a number of empty determinations of external intuition, which, however, can never be possible perceptions; for instance, motion or rest of the world in an infinite empty space, *i. e.*, a determination of the mutual relation of the two, which can never be perceived, and is therefore nothing but the predicate of a mere idea." (34, p. 457; Müller trans., p. 347.)

Accordingly, when Einstein characterizes as a fundamental feature of the theory of relativity that it takes from

space and time "the last remainder of *physical objectivity*," it is clear that the theory only accomplishes the most definite application and carrying through of the standpoint of critical idealism within empirical science itself. Space and time in the critical doctrine are indeed distinguished in their validity as types of order from the contents, which are ordered in them; but these forms possess for Kant a separate existence neither in the subjective nor in the objective sense. The conception, that space and time as subjective forms into which sensations enter "lie ready in the mind" before all experience, not as "physical" but as "psychical" realities, today scarcely needs refutation. This conception indeed seems to be indestructible, although Fichte poured upon it his severe but appropriate scorn; but it disappears of itself for everyone who has made clear to himself even the first conditions of the transcendental formulation of the question in opposition to the psychological. The meaning of the principle of order can in general be comprehended only in and with what is ordered; in particular, it is urged in the case of the measurement of time that the determination of the temporal positions of particular empirical objects and processes cannot be derived from the relations of the phenomena to absolute time, but that conversely the phenomena must determine and make necessary their positions in time for each other. "This unity in the determination of time is dynamical only, that is, time is not looked upon as that in which experience assigns immediately its place to every existence, for this would be impossible; because absolute time is no object of perception by which phenomena could be held together; but the rule of the understanding through which alone the existence of phenomena can receive synthetical unity in time determines the place of each of them in time, therefore *a priori* and as valid for all time." (34, p. 245 and 262; *cf.* 56, p. 332; *cf.* Müller trans., p. 175.)

It is such a "rule of the understanding," in which is expressed the synthetic unity of phenomena and their reciprocal dynamical relation, on which rests all empirical spatial order, all objective relations of spatial "community" in the corporeal world. The *"communio spatii," i. e.,* that *a priori* form of coexistence, which in Kant's language is characterized as "pure intuition" is, as he expressly urges, only empirically knowable for us by the *commercium* of substances in space, *i. e.,* by a whole of physical effects, that can be pointed out in experience. We read, in a passage of the *Critique of Pure Reason,* which appears especially significant and weighty in connection with the development of the modern theory of relativity: "The word *communion* (*Gemeinschaft*), may be used in two senses, meaning either *communio* or *commercium.* We use it here in the latter sense: as a dynamical communion, without which even the local *communio spatii* could never be known empirically. We can easily perceive in our experience, *that continuous influences only can lead our senses in all parts of space from one object to another; that the light which plays between our eyes and celestial bodies produces a mediate communion between us and them,* and proves the coexistence of the latter; that we cannot change any place empirically (perceive such a change) unless matter itself renders the perception of our own place possible to us, and that by means of its reciprocal influence only matter can evince its simultaneous existence, and thus (though mediately only) its coexistence, even to the most distant objects." (34, p. 260; *cf.,* Müller trans., p. 173f.) The spatial order of the corporeal world, in other words, is never given to us directly and sensuously, but is the result of an intellectual construction, which takes its start from certain empirical laws of phenomena and from that point seeks to advance to increasingly general laws, in which finally is grounded

what we call the unity of experience as a spatio-temporal unity.

But is there not found in this last expression the characteristic and decisive opposition between the theory of space and time of critical idealism and the theory of relativity? Is not the essential result of this theory precisely the destruction of the unity of space and time demanded by Kant? If all measurement of time is dependent on the state of motion of the system from which it is made there seem to result only infinitely many and infinitely diverse "place-times," which, however, never combine into the unity of "the" time. We have already seen, however, that this view is erroneous, that the destruction of the substantialistic unity of space and time does not destroy their functional unity but rather truly grounds and confirms it. (*Cf.* above, p. 33ff., p. 54ff.) In fact, this state of affairs is not only granted by the representatives of the theory of relativity among the physicists, but is expressly emphasized by them. "The boldness and the high philosophical significance of Einstein's doctrine consists," we read, *e. g.*, in the work of Laue, "in that it clears away the traditional prejudice of one time valid for all systems. Great as the change is, which it forces upon our whole thought, there is found in it not the slightest epistemological difficulty. For in Kant's manner of expression time is, like space, a pure form of our intuition; a schema in which we must arrange events, so that in opposition to subjective and highly contingent perceptions they may gain objective meaning. This arranging can only take place on the basis of empirical knowledge of natural laws. The place and time of the observed change of a heavenly body can only be established on the basis of optical laws. That two differently moving observers, each one regarding himself at rest, should make this arrangement differently on the basis of the same laws of nature, contains no logical impossibil-

ity. Both arangements have, nevertheless, objective meaning since there may be deduced exactly from each of them by the derivative transformation formulae that arrangement valid for the other moving observer." (40, p. 36f.) This one-to-one correlation and not the oneness of the values gained in the different systems, is what remains of the notion of the "unity of time"; but precisely in it is expressed all the more sharply the fundamental view that this unity is not to be represented in the form of a particular objective content, but exclusively in the form of a system of valid relations. The "dynamic unity of temporal determinations" is retained as a postulate; but it is seen that we cannot satisfy this postulate if we hold to the laws of the Newtonian mechanics, but that we are necessarily driven to a new and more universal and concrete form of physics. The "objective" determination shows itself thus to be essentially more complex than the classical mechanics assumed, which believed it could literally grasp with its hands the objective determination in its privileged systems of reference. That a step is thereby taken beyond Kant is incontestible; for he shaped his "Analogies of Experience" essentially on the three fundamental Newtonian laws: the law of inertia, the law of the proportionality of force and acceleration, and the law of the equality of action and reaction. But in this very advance the doctrine that it is the "rule of the understanding," that forms the pattern of all our temporal and spatial determinations, is verified anew. In the special theory of relativity, the principle of the constancy of the velocity of light serves as such a rule; in the general theory of relativity this principle is replaced by the more inclusive doctrine that all Gaussian coördinate systems are of equal value for the formulation of the universal natural laws. It is obvious that we are not concerned here with the expression of an empirically observed fact, but with a principle which the understanding

uses hypothetically as a norm of investigation in the interpretation of experience, for how could an infinite totality be "observed"? And the meaning and justification of this norm rest precisely on the fact that only by its application could we hope to regain the lost unity of the object, namely, the "synthetic unity of phenomena according to temporal relations." The physicist now depends neither on the constancy of those objects with which the naïve sensuous view of the world rests nor on the constancy of particular spatial and temporal measurements gained from a particular system, but he affirms, as a condition of his science, the existence of "universal constants" and universal laws, which retain the same values for all systems of measurement.

In his *Metaphysischen Anfangsgründen der Naturwissenschaft,* Kant, returning to the problem of absolute space and time, formulates a happy terminological distinction, which is suited to characterize more sharply the relation of critical idealism to the theory of relativity. Absolute space, he urges here too, is *in itself nothing and indeed no object;* it signifies only a *space relative to every other* which I can think outside of any given space. To make it a real thing means to confuse the *logical universality* of any space with which I can compare any empirical space as included in it with the *physical universality* of real extension and to misunderstand the Idea of reason. The true logical universality of the Idea of space thus not only does not include its physical universality, as an all inclusive container of things, but it is precisely of a sort to exclude it. We should, in fact, *conceive* an absolute space, *i. e.,* an ultimate unity of all spatial determinations; but not in order to know the absolute movements of empirical bodies, but to represent in the same "all movements of the material as merely relative to each other, as alternatively reciprocal, but not as absolute motion or rest." "Absolute space is thus neces-

sary not as a concept of a real object, but as an Idea, which should serve as a rule for considering all motions in it as merely relative, and all motion and rest must be reduced to the absolute space, if the phenomena of the same are to be made into a definite concept of experience that unifies phenomena." (35, IV, 383f., 472f.) The logical universality of such an idea does not conflict with the theory of relativity; it starts by regarding all motions in space as merely relative because only in this way can it combine them into a definite concept of experience, that unifies all phenomena. On the basis of the demand for the totality of determinations it negates every attempt to make a definite particular system of reference the norm for all the others. The one valid norm is merely the idea of the unity of nature, of exact determination itself. The mechanical view of the world is overcome from this standpoint. The "unity of nature" is grounded by the general theory of relativity in a new sense, since it includes under a supreme principle of knowledge along with the phenomena of gravitation, which form the real classical field of the older mechanics, the electrodynamic phenomena. That in order to advance to this "logical universality of the Idea," many trusted presentational pictures must be sacrificed need not disturb us; this can affect the "pure intuition" of Kant only in so far as it is misunderstood as a mere picture and not conceived and estimated as a constructive method.

In fact, the point at which the general theory of relativity must implicitly recognize the methodic presupposition, which Kant calls "pure intuition" can be pointed out exactly. It lies, in fact, in the concept of "coincidence" to which the general theory of relativity ultimately reduces the content and the form of all laws of nature. If we characterize events by their space-time coördinates $x_1$, $x_2$, $x_3$, $x_4$, $x'_1$, $x'_2$, $x'_3$, $x'_4$, *etc.*, then, as it emphasizes, everything that physics can teach us of the "essence" of natural

processes consists merely in assertions concerning the coincidences or meetings of such points. We reach the construction of physical time and of physical space merely in in this way; for the whole of the space-time manifold is nothing else than the whole of such coördinations.[22] Here is the point at which the ways of the physicist and of the philosopher definitely part, without their being thereby forced into conflict. What the physicist calls "space" and "time" is for him a concrete measurable manifold, which he gains as the *result* of coördination, according to law, of the particular points; for the philosopher, on the contrary, space and time signify nothing else than the forms and *modi*, and thus the presuppositions, of this coördination itself. They do not result for him from the coördination, but they *are* precisely this coördination and its fundamental directions. It is coördination from the standpoint of coexistence and adjacency or from the standpoint of succession, which he understands by space and time as "forms of intuition." In this sense, both are expressly defined in the Kantian Inaugural Dissertation. *"Tempus non est objectivum aliquid et reale . . . sed subjectiva conditio, per naturam mentis humanae necessaria, quaelibet sensibilia certa lege sibi coordinandi et intuitus purus . . . Spatium est . . . subjectivum et ideale et e natura mentis stabili lege proficiscens veluti schema omnia omnino externe sensa sibi coordinandi."* (35; II, 416, 420.) Whoever recognizes this law and this schema, this possibility of relating point to point and connecting them with each other, has recognized space and time in their "transcendental meaning," for we can abstract here from any psychological by-meaning of the concept of form of intuition. We can thus conceive the "world-points" $x_1$ $x_2$ $x_3$ $x_4$ and the world-lines, which result from them, so abstractly that we understand under the values $x_1$ $x_2$ $x_3$ $x_4$ nothing but certain mathemati-

[22] Einstein (17), p. 13f.; (18), p. 64.

cal parameters; the "meeting" of such world-points involves a comprehensible meaning only if we take as a basis that "possibility of succession," which we call time. A coincidence, which is not to mean identity, a unification, which is still a separation, since the same point is conceived as belonging to different lines: all this finally demands that synthesis of the manifold, for which the term "pure intuition" was formulated. The most general meaning of this term, which indeed was not always grasped by Kant with equal sharpness, since more special meanings and applications were substituted involuntarily in his case, is merely that of the serial form of coexistence and of succession. Nothing is thereby presupposed concerning special relations of measurement in the two, and in so far as these depend in particular on the relations of the physical in space, we must guard against seeking to find an exhaustive determination in the mere "forms of possibility" of the relations of the "real." (*Cf.* below VI.) When, *e. g.*, in the mathematical foundations of the theory of relativity the formula is deduced for the "distance" of the two infinitely close points $x_1$ $x_2$ $x_3$ $x_4$, and $x_1+dx_1$, $x_2+dx_2$, $x_3+dx_3$, $x_4+dx_4$, this cannot indeed be conceived as a rigid Euclidean distance in the ordinary sense, since there is involved in it, by the addition of time as a fourth dimension, not a magnitude of space but rather one of motion; but the fundamental form of coexistence and succession and their reciprocal relation and "union" is unmistakably contained in this expression of the general linear element. Not that the theory, as has been occasionally objected, presupposes space and time as something already given, for it must be declared free of this epistemological circle, but in the sense that it cannot lack the form and function of spatiality and temporality in general.

What seems to render understanding difficult at this point between the physicist and the philosopher is the fact

that a common problem is found here, which both approach from entirely different sides. The process of measurement interests the critic of knowledge only in so far as he seeks to survey in systematic completeness the concepts, which are used in this process, and to define them in the utmost sharpness. But any such definition is unsatisfying and fundamentaly unfruitful to the physicist as long as it is not connected with any definite indication as to how the measurement is to be made in the concrete particular case. "The concept exists for the physicist," says Einstein in one place neatly and characteristically, "only when the possibility is given of finding out in the concrete case whether the concept applies or not." (18, p. 14.) Thus the concept of simultaneity, for example, only receives a definite meaning, when a method is given by which the temporal coincidence of two events is determined by certain measurements, by the application of optical signals; and the difference which is found in the results of this measurement seems to have as a consequence the ambiguity of the concept. The philosopher has to recognize unconditionally this longing of the physicist for concrete determinateness of concepts; but he is ever again brought to the fact that there are ultimate ideal determinations without which the concrete cannot be conceived and made intelligible. To make clear the opposition in formulation of the question which is here fundamental, one can contrast to Einstein's expression one of Leibniz. *"On peut dire,"* we read in Leibniz' *Nouveaux Essais, "qu'il ne faut point s'imaginer deux étendues, l'une abstraite, de l'espace, l'autre concrète, du corps; le concret n'étant tel que par l'abstrait."* (43, V, 115.) As we see, it is the unity of the abstract and the concrete, of the ideal and the empirical in which the demands of the physicist and the philosopher agree; but while the one goes from experience to the idea, the other goes from the idea to experience. The theory of

relativity holds fast to the "pre-established harmony between pure mathematics and physics"; Minkowski, in the well-known concluding words of his lecture, "Space and Time," has expressly taken up again and brought to honor this Leibnizian term. But this harmony is for the physicist the incontestable premise from which he strives to reach the particular consequences and applications, while for the critic of knowledge the "possibility" of this harmony constitutes the real problem. The basis of this possibility he finds ultimately in the fact that any physical assertion, even the simplest determination of magnitude established by experiment and concrete measurement, is connected with universal conditions, which gain separate treatment in pure mathematics, that any physical assertion involves certain logico-mathematical constants. If we desire to bring all of these constants into a short formula, we can point out the concept of number, the concept of space, the concept of time, and the concept of function as the fundamental elements, which enter as presuppositions into every question which physics can raise. None of these concepts can be spared or be reduced to another so that, from the standpoint of the critique of cognition, each represents a specific and characteristic motive of thought; but, on the other hand, each of them possesses an actual empirical use only along with the others and in systematic connection with them. The theory of relativity shows with especial distinctness how, in particular, the thought of function is effective as a necessary motive in each spatio-temporal determination. Thus physics knows its fundamental concepts never as logical "things in themselves," but only in their reciprocal combination; it must, however, be open to epistemology to analyze this product into its particular factors. It thus cannot admit the proposition that the meaning of a concept is identical with its concrete application, but it will con-

versely insist that this meaning must be already established before any application can be made. Accordingly, the thought of space and time in their meaning as connecting forms of order is not first created by measurement but is only more closely defined and given a definite content. We must have grasped the concept of the "event" as something spatio-temporal, we must have understood the meaning expressed in it, before we can ask as to the coincidence of events and seek to establish it by special methods of measurement.

In general, physics sees itself placed by its fundamental problem from the beginning between two realms, which it has to recognize and between which it has to mediate without asking further as to their "origin." On the one side, stands the manifold of data of sensation, on the other a manifold of pure functions of form and order. Physics, as an empirical science, is equally bound to the "material" content, which sense perception offers it, and to these formal principles in which is expressed the universal conditions of the "possibility of experience." It has to "invent" or to derive deductively the one as little as the other, *i. e.*, neither the whole of empirical contents nor the whole of characteristic scientific forms of thought, but its task consists in progressively relating the realm of "forms" to the data of empirical observation and, conversely, the latter to the former. In this way, the sensuous manifold increasingly loses its "contingent" anthropomorphic character and assumes the imprint of thought, the imprint of systematic unity of form. Indeed "form," just because it represents the active and shaping, the genuinely creative element, must not be conceived as rigid, but as living and moving. Thought comprehends more and more that form in its peculiar character cannot be given to it at one stroke, but that the existence of form is only revealed to it in the becoming of form and in the law of this becoming. In

this way, the history of physics represents not a history of the discovery of a simple series of "facts," but the discovery of ever new and more special means of thought. But in all change of these means of thought there is nevertheless revealed, as surely as physics follows the "sure course of a science," the unity of those methodic principles upon which rests the formulation of its question. In the system of these principles, space and time take their fixed place, although they are not to be conceived as fixed things or contents of presentation. The ancient view believed that it possessed and encompassed the spatio-temporal unity of being directly in presentation. To Parmenides and fundamentally the whole ancient world being was given "like the mass of a well-rounded sphere." With the reform of Copernicus, the security of this possession was gone once for all. Modern science knows that there is a definite spatio-temporal order of phenomena for knowledge only in so far as knowledge progressively establishes it, and that the only means of establishing it consists in the scientific concept of law. But the problem of such a general orientation remains for thought and becomes the more urgent the more thought knows it as a problem never to be solved definitively. Precisely because the unity of space and time of empirical knowledge seems to flee eternally before all our empirical measurements, thought comprehends that it must seek it eternally and that it must avail itself of new and ever sharper instruments. It is the merit of the theory of relativity not only to have proved this in a new way but also to have established a principle, *i. e.*, the principle of the co-variancy of the universal laws of nature with regard to all arbitrary substitutions, by which thought can master, out of itself, the relativity which it calls forth.

In the analysis of spatial and temporal measurements, made by the theory of relativity this fundamental relation can be traced in detail. This analysis does not begin by

accepting the concept of the "simultaneity" of two processes as a self-evident and immediately known datum, but by demanding an explanation of it—an explanation, which, as a physical explanation, cannot consist in a general conceptual definition, but only in the indication of the concrete methods of measurement, by which "simultaneity" can be empirically pointed out. The simultaneity of such processes as take place practically in "the same" point of space or in immediate spatial adjacency is at first presupposed; we assume, as Einstein explains, the determinability of "simultaneity" for events, which are immediately adjacent spatially, or, more exactly, for events in immediate spatio-temporal adjacency (coincidence), without defining this concept. (17, § 3.) In fact, recourse here to a mediating physical method of measurement seems neither desirable nor possible; for any such method would always presuppose the possibility of making a temporal coördination between diverse events, thus, *e. g.*, of establishing "the simultaneity" of a definite event with a certain position of the hands of a clock found at the "same" place. The real problem of the theory of relativity begins only when we are no longer concerned with temporally connecting spatially adjacent series of events with each other, but rather series of events spatially remote from each other. If we assume that there is established for the two points of space A and B a certain "place-time," then we possess only an "A-time" and "B-time" but no time *common* to A and B. And it is seen that every attempt to establish such a common time as an empirically measurable time, is bound to a definite empirical presupposition concerning the velocity of light. The assumption of the uniform velocity of light enters implicitly into all our assertions concerning the simultaneity of what is spatially distant. A time common to A and B is gained when one establishes *by definition* that the "time," which light takes in going

from A to B is equal to the "time," which it takes in going from B to A. Let us assume that a ray of light is sent at A-time $t_A$ from a clock found in A to B, and then at B-time, $t_B$, the ray of light is reflected to A and reaches A again at A-time, $t'_A$; then we establish by definition that the two clocks of A and B are to be called "synchronous" if $t_B - t = t'_A - t_B$. Thus for the first time an exact determination is made of what we are to understand by the "time" of an event and by the "simultaneity" of two processes; "the time" of an event is what is told us by a motionless clock found at the place of the event simultaneously with the event, a clock which runs synchronously with a certain motionless clock and indeed synchronously with the latter at all times." (16, p. 28f.)

That the "forms" of space and time as definite forms of the coördination of different contents already enter into the concrete determinations, which are here made for the procedure of the physical measurement of time, scarcely needs special explanation. The two are immediately assumed in the concept of the "place-time"; for the possibility is involved in it of grasping a definitely distinguished "now" in a definitely distinguished "here." This "here" and "now" does not signify indeed the whole of space and time, to say nothing of all the concrete relations within the two to be established by measurement; but it represents the first foundation, the unavoidable basis of the two. The first primitive difference, which is expressed in the mere positing of a "here" and a "now" remains thus, for the theory of relativity, too, an indefinable on which it grounds its complex physical definitions of space and time values. And while for these definitions it appeals to a definite assumption concerning the law of the propagation of light, this, too, involves the presupposition that a certain condition that we call "light" occurs *in succession* at different *places* and according to a definite rule, in which what space and

time mean as mere schemata of coördination, is obviously contained. The epistemological problem seems indeed to be heightened when we reflect on the reciprocal relation of space and time values in the fundamental equations of physics. What is given in these equations is the four-dimensional "world," the continuum of events in general, the temporal determinations in this continuum not being separated from the spatial. The intuitive difference between a spatial distance and a temporal duration, which we believe ourselves to grasp immediately, plays no rôle in this purely mathematical determination. According to the temporal equation of the Lorentz-transformation:

$$t' = \frac{t - \frac{v}{c^2} x}{\sqrt{1 - \frac{v^2}{c^2}}}$$

the time differential $\Delta t'$ between two events with reference to K′ does not disappear when the time differential $\Delta t$ of the same disappears with reference to K; the purely spatial distance of two events with reference to K has as a consequence in general the temporal sequence of the same with reference to K′. This leveling of space and time values is developed even further in the general theory of relativity. Here it is seen to be impossible to construct a reference system out of fixed bodies and clocks of such a sort that place and time are directly indicated by a fixed arrangement of measuring rods and clocks relatively to each other; but each point of the continuous series of events is correlated with four numbers, $x_1$, $x_2$, $x_3$, $x_4$, which possesses no direct physical meaning, but only serve to enumerate the points of the continuum in a definite but arbitrary way. This correlation need not have such properties that a certain group of values $x_1$ $x_2$ $x_3$ must be understood as the spatial coördinates and opposed to the "temporal" coördinate $x_4$. (18, p. 38, 64.) The demand of Minkowski that "space for itself and time for itself be completely degraded

to shadows" and that only "a sort of union of the two shall retain independence" seems thus now to be realized in all strictness. Now at any rate, this demand contains nothing terrible for the critical idealist, who has ceased to conceive space and time as things in themselves or as given empirical objects. For the realm of ideas is for him a "realm of shadows," as Schiller called it, since no pure idea corresponds directly to a concrete real object, but rather the ideas can always only be pointed out in their systematic community, as fundamental moments of concrete objective knowledge. If it thus appears that physical space and time measurements can be assumed only as taking place in common, the difference in the fundamental character of space and time, of order in coexistence and succession is not thereby destroyed. Even if it is true that, as Minkowski urges, no one has *perceived* a place save at a time and a time save at a place, there remains a difference between what is to be *understood* by spatial and by temporal discrimination. The factual interpenetration of space and time in all empirical physical measurements does not prevent the two from being different in principle, not as objects, but as types of objective discrimination. Although two observers in different systems K and K′ can assume the arrangement of the series of events in the orders of space and time to be different, it is still always a *series of events* and thus a continuum both spatial and temporal, which they construct in their measurements. Each observer distinguishes from his standpoint of measurement a continuum, which he calls "space," from another, which he calls "time"; but he can, as the theory of relativity shows, not assume without further consideration that the arrangement of phenomena in these two schemata must be similar from each system of reference. There may thus, according to Minkowski's "world postulate," be given only the four-dimensional word in space and time, and "the pro-

jection into space and time" may be possible "with a certain freedom"; this only affects the different spatio-temporal interpretations of phenomena, while the difference of the form of space from that of time is unaffected.

For the rest, here too the transformation-equation reestablishes objectivity and unity, since it permits us to translate again the results found in one system into those of the other. Also, if one seeks to clarify the proposition of Minkowski that only the inseparable union of space and time possesses independence, by saying that this union itself, according to the results of the general theory of relativity, becomes a shadow and an abstraction, and that only the unity of space, time and things possesses independent reality,[28] then this classification only leads us back again to our first epistemological insight. For that neither "pure space" nor "pure time" nor the reciprocal connection of the two, but only their realization in some empirical material gives what we call "reality," *i. e.,* the physical being of things and of events, belongs to the fundamental doctrines of critical idealism. Kant himself did not weary of referring repeatedly to this indissoluble connection, this reciprocal correlation of the spatio-temporal form and the empirical content in the existence and structure of the *world of experience.* "To give an object," we read, "if this is not meant again as mediate only, but if it means to represent something immediately in intuition, is nothing else but to refer the representation of the object to experience. . . . Even space and time, however, pure these concepts may be of all that is empirical, and however certain it is that they are represented in the mind entirely *a priori,* would lack nevertheless all objective validity, all sense and meaning, if we could not show the necessity of their use with reference to all objects of experience. Nay, their representation is a pure schema, always referring to

[28] See Schlick (79), p. 51; *cf.* p. 22.

that reproductive imagination, which calls up the objects of experience, without which objects would be meaningless." (34, p. 195; *cf.* Müller trans., p. 127f.) The *"ideal"* meaning, that space and time possess "in the mind" thus does not involve any sort of *particular* existence, which they would possess prior to things and independently of them, but it rather expressly denies it—the ideal separation of pure space and pure time from things (more exactly, from empirical phenomena), not only permits but demands precisely their empirical "union." This union the general theory of relativity has verified and proved in a new way, since it recognizes more deeply than all preceding physical theories the dependency belonging to all empirical measurement, to all determination of concrete spatio-temporal relations.[24] The relation of experience and thought that is established in the critical doctrine does not contradict this result in any way, but rather it confirms it and brings it to its sharpest expression. It is indeed at first glance strange and paradoxical that the most diverse epistemological standpoints, that radical empiricism and positivism as well as critical idealism have all appealed to the theory of relativity in support of their fundamental views. But this is satisfactorily explained by the facts that empiricism and idealism meet in certain presuppositions with regard to the doctrine of empirical space and of empirical time, and that the theory of relativity sets up just such a doctrine. Both here grant to experience the decisive rôle, and both teach that every exact measurement presupposes universal empirical *laws*.[25] But the question becomes all the more urgent as to how we reach these laws, on which rests the possibility of all empirical measurement, and what sort of validity, of logical "dignity" we grant to them. Strict positivism has only one

[24] On the "relativization" of the difference of space and time, *cf.* also below, VII.

[25] (8), p. 191ff.; *cf.* Sellien (81), p. 14ff.

answer to this question: for it all knowledge of laws, like all knowledge of objects, is grounded in the simple elements of sensation and can never go beyond their realm. The knowledge of laws possesses accordingly in principle the same purely passive character that belongs to our knowledge of any particular sensuous qualities. Laws are treated like things whose properties one can read off by immediate perception. Mach attempts, quite consistently with his standpoint, to extend this manner of consideration to pure mathematics also and the deduction of its fundamental relations. The way in which we gain the differential quotient of a certain function, as he explains, is not distinguished in principle from the way in which we establish any sort of properties or changes of physical things. As in the one case we subject the thing, so in the other case we subject the function to certain operations and simply observe how it "reacts" to them. The reaction of the function $y=x^m$ to the operation of differentiation out of which the equation $\frac{dy}{dx}=mx^{m-1}$ results "is a distinguishing mark of $x^m$ just as much as the blue-green color in the solution of copper in sulphuric acid." (49, p.75.) Here we find clearly before us the sharp line of distinction between critical idealism and positivism of Mach's type. That the equations governing larger or smaller fields are to be regarded as what is truly permanent and substantial, since they make possible the gaining of a stable picture of the world,[26] that they thus constitute the kernel of physical objectivity: this is the fundamental view in which the two theories combine. The question concerns only the manner of establishing, only the exact grounding, of these equations. Idealism urges against the standpoint of "pure experience" as the standpoint of mere sensation, that all equations are results of measurement; all measurement, however, presupposes certain theoretical principles and in

[26] See Mach (49), p. 429.

the latter certain universal functions of connection, of shaping and coördination. We never measure mere sensations, and we never measure with mere sensations, but in general to gain any sort of relations of measurement we must transcend the "given" of perception and replace it by a conceptual symbol, which possesses no copy in what is immediately sensed. If there is anything that can serve as a typical example of this state of affairs, it is the development of modern physics in the theory of relativity. It is verified again that every physical theory, to gain conceptual expression and understanding of the facts of experience, must free itself from the form in which at first these facts are immediately given to perception.[27] That the theory of relativity is founded on experience and observation is, of course, beyond question. But, on the other hand, its essential achievement consists in the new interpretation that it gives to the observed facts, in the conceptual interpretation by which it is progressively led to subject the most important intellectual instruments of classical mechanics and the older physics to a critical revision. It has been pointed out with justice that it has been precisely the oldest empirical fact of mechanics, the equality of inert and heavy masses, which, in the new interpretation it has received from Einstein, has become the fulcrum of the general theory of relativity. (24a.) The way in which the principle of equivalence and with it the foundations of the new theory of gravitation have been deduced from this fact can serve as a logical example of the meaning of the pure "thought-experiment" in physics. We conceive ourselves in the position of an observer, who, experimenting in a closed box, establishes the fact that all bodies left to themselves move, always with constant acceleration, toward the floor of the box. This fact can be represented con-

[27] *Cf.* Duhem (15, p. 322): *"Les faits d'expérience, pris dans leur brutalité native, ne sauraient servir au raisonnement mathématique; pour alimenter ce raisonnement, ils doivent être transformés et mis sous forme symbolique."*

ceptually by the observer in a double manner: in the first place, by the assumption that he is in a temporarily constant field of gravity in which the box is hung up motionless, or, in the second place, by the assumption that the box moves upward with a constant acceleration whereby the fall of bodies in it would represent a movement of inertia. The two: the inertial movement and the effect of gravitation, are thus in truth a single phenomenon seen and judged from different sides. It follows that the fundamental law that we establish for the movement of bodies must be such that it includes equally the phenomena of inertia and those of gravitation. As is seen, we have here no empirical proposition abstracted from particular observations, but a rule for our construction of physical concepts: a demand that we make, not directly of experience, but rather of our manner of intellectually representing it. "Thought-experiments" of such force and fruitfulness cannot be explained and justified by the purely empiristic theory of physical knowledge. It is not in contradiction with this that Einstein refers gratefully to the decisive stimulus, which he received from Mach (20); for a sharp distinction must be made between what Mach has accomplished as a physicist in his criticism of Newton's fundamental concepts, and the general philosophical consequences he has drawn from this achievement. Mach himself has, as is known, granted wide scope to the pure "thought-experiment" in his own logic of physics; but, more closely considered, he has thereby already left the ground of a purely sensualistic founding of the fundamental concepts of physics.[28] That there is no necessary connection between the theory of relativity and Mach's philosophy may be concluded from the fact, among other things, that it is precisely one of the first advocates of this theory, Max Planck, who among all modern physicists has most sharply criticized and fought

[28] See Mach (50, p. 180ff.); *cf.* (8), p. 316ff. and (39), p. 86f.

against the presuppositions of this philosophy. (69.) Even if one takes the theory of relativity as an achievement and outcome of purely empirical thought, it is thereby a proof and confirmation of the constructive force immanent in this thought by which the system of physical knowledge is distinguished from a mere "rhapsody of perceptions."

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*The entire essay of which this article is a part will be published in the course of the year in *Substance and Function*, by Ernst Cassirer.

# CRITICISMS AND DISCUSSIONS

## PSYCHOLOGY AND LAW

THE late Dr. Mercier's *Crime and Criminals* (London: University of London Press), is worth the serious attention of both social psychologists and of lawyers. Indeed it is becoming more and more desirable that workers in these two fields of human endeavor should co-operate in a much needed co-ordination of the results of modern work in their two departments. This book points the way, though in the opinion of the writer of this note it does little more than call our attention to the fruitful field to which that road leads. It is written, as readers of Dr. Mercier's former works would expect, in a vigorous style, clear and unhesitating. The opening sentence of the preface strikes the keynote of the work: "With the exception of logic, there is no subject on which so much nonsense has been written as this of criminality and the criminal." It is a bold beginning, and raises hopes which are not altogether fulfilled, particularly in the earlier chapters which are somewhat patchy and disjointed.

In the chapter on the "Factors of Crime," we are told that every act is compounded of two elements, instinct and reason, and the author points out that the conduct of animals, e. g., of spiders in making webs, is not purely instinctive but is conditioned by the circumstances giving rise to the conduct; "into every instinctive act there is an intrusion of reasoned action." On these lines is developed a rather crude biological interpretation of human conduct: "in man the reasoned factor encroaches more and more in discovering means to attain his ends, but the ends, the ultimate ends. are always instinctively determined." There are two factors: the internal factor,—"the group of instincts inherent in the actor and the degree and kind of intelligence with which he is endowed"; and the external factor,—"the circumstances in which the actor is, and that act upon him and control and elicit, or modify his action." Then

follows a discussion of the psychology of crime which gives us nothing newer than that human conduct may be ultimately traced to a small number of instinctive desires, hunger, self-preservation, and propagation of one's species; and that "the middle-aged spinster, rising at an uncomfortably early hour on a winter's morning to attend early celebration would be indignant if she were told the truth that her action is prompted by the craving for self-sacrifice which is part of the fundamental instinctive desire for motherhood." Others. beside the spinster, will no doubt be surprised and willing to join issue on this bold generalization which even if true explains nothing, or at any rate gives us no practical guide to present-day human conduct, and gives us no help in assessing the guilt of a criminal or formulating a criminal code.

In discussing the nature of crime, the author seems to miss entirely the point of view of the lawyer; such a statement as "my own definition of crime coincides in the main with that of Austin and Stephen with this difference, that I shall regard it as consisting of acts and omissions that are infractions of law, not as it is, but as I conceive it ought to be," goes a long way to justify a class which Dr. Mercier does not spare in abuse, the conscientious objector to military service; and when in a later chapter it is seriously suggested that every breach of contract should be punished as a crime, as well as any carelessness which puts temptation in the way of the potential criminal. the lawyer feels that Dr. Mercier fails to appreciate the lessons of the eighteenth century. Criminal law must be practical and its machinery practicable; it is easy enough to point out illogicalities in any system of law and more particularly in systems of criminal law. Even if it be granted that the person who unsuccessfully attempts a murder is normally as guilty as if he succeeds in killing his fellow citizen it does not follow that it is expedient to punish the attempt as severely as the completed act; to hold so means to hold further, that the mere compassing or imagining of crime, without any overt act is always necessary in treason, would justify the full punishment meted out to the crime itself. Dr. Johnson made this quite clear when he said that if Garrick felt a murderer whenever he played Richard III, he ought to be hanged every time he played the part. But in truth the infliction of a less punishment for an attempt is logical on Dr. Mercier's own criterion of criminal legislation, viz., the prevention of anti-social acts; an attempt does less harm than the completed act.

Criminal law must ever be a crude approximation to an ideal, a system consciously defective in the interests of the non-criminal class. If the net of criminal law is so tightly drawn that no anti-social act goes unpunished, as Dr. Mercier seems to desire when he would punish for high treason all who waste public money or for theft those who waste their employer's time or use their master's property for private pleasure although not impairing its value, honest men would be open to needless risks, would be restricted in their lawful occupations and ever open to blackmailing prosecutions. "The life of law is not logic but experience," and Dr. Mercier has failed to appreciate the vast amount of human and social experience which is wrapped up in our legal systems; crude appeals to animal instincts and austere applications of ethical principles do not carry us very far in criminal jurisprudence. It is regrettable that the learned author did not begin his study of crime from the point of view which he so ably and clearly sets forth in the seventh chapter of this book. In this chapter on criminals he says: "According to this doctrine of mine, all men are by nature potential criminals since all are actuated by instinctive desires that urge their possessor to seek the gratification of them, and since no man yet attained to the perfection of socialization that we witness in the social insects, in whom gratification of selfish desire harmonizes completely with the common welfare."

This is unquestionably a very valuable point of view and it is to be deeply regretted that death has deprived us of an elaboration by the author of this theory, which is sprung rather suddenly on the reader towards the end of the work and leaves the feeling that it requires closer reasoning in statement and more justification than that produced.

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## SAMUEL BUTLER AND EDUCATION

SINCE his death in 1902, Butler has come more and more to be recognized not merely as the author of amusing and caustic satire, but as a profound and original thinker on Psychology and Evolution. Whether his "discoveries" of Gandenzio Ferrari and Tabachetti, of the authorship of the *Odyssey* (in propounding which heresy his plausibility rivals Mrs. Gallup's), and his elucidation of Shakespeare's *Sonnets,* are, or ever will be, accepted by orthodox authorities, I cannot say; but on the subject of Education—though he omits it from the last of his seventeen most interesting "finds"—Butler left many scattered remarks, which together form a thing that deserves and indeed requires serious consideration. Yet I can recall no reference to Butler in any book on Education except that by Professor Nunn)[1] and even there it is Butler's biological theory of "Invention" rather than any of his views on Education, that is quoted. Such neglect is not surprising, for Butler was always a "literary pariah"; nor were his remarks on Education likely to conciliate dons or schoolmasters. Yet Butler had the pedagogue in his blood. His grandfather during his head-mastership of Shrewsbury (1798-1835) proved himself one of the most revolutionary of nineteenth century schoolmasters;[2] and Butler made ample amends for his portrait as George Pontifex by editing *The Life and Letters of Dr. Samuel Butler* (1896), "insofar as they illustrate the scholastic, religious and social life of England from 1790-1840." Brought up at Shrewsbury under his grandfather's successsor Benjamin Hall Kennedy (the notorious Dr. Skinner of *The Way of All Flesh*), Butler not only acquired the "class spirit" of the English public schoolboy,[3] but enough classics to ensure his doing well at

[1] *Education: Its Date and First Principles,* by Prof. T. P. Nunn: Chap. 11.
[2] See Adamson's *Short History of Education.*
[3] See J. B. Yeats, "Recollections of Samuel Butler," in *Essays, Irish and American.*

Cambridge; for it should never be forgotten, in reading his strictures on academism, that Butler was bracketed twelfth in the first part of the Classical Tripods—thus doing better at the university than many a man whose only distinction in life has been a good degree. He had really tasted of the grapes before he declared them sour.

The word "academicism" includes the bulk of what Butler attacked in educational aims and practice. It signified for him not merely that remoteness from life with which academies of one kind or another are usually charged. but also those faults in their training which have been exposed in recent years by experimental psychology. Thus, to give but two instances, Butler foresaw the downfall of "mental gymnastic" from its bad eminence, and he emphasized the value of "learning by doing," rather than by attending to rules and theories. In the well-known account of the Colleges of Unreason in Crewhon, he satirizes under the name of "Hypothetics," the curriculum and methods of English Public Schools and Universities. The Professors of these Colleges, he writes, "argue thus—that to teach a boy merely the nature of things which exist in the world around him, and about which he will have to be conversant during his whole life, would be giving him but a narrow and shallow conception of the universe, which it is urged might contain all manner of things which are not now to be found therein. To open his eyes to these possibilities and so to prepare him for all sorts of emergencies, is the object of this system of hypothetics. To imagine a set of utterly strange and impossible contingencies and require the youths to give intelligent answers to the questions that arise therefrom, is reckoned the fittest conceivable way of preparing them for the actual conduct of their affairs in after life. Thus they are taught what is called the hypothetical language for many of their best years—a language which was originaly composed at a time when the country was in a very different state of civilization to what it is at present, a state which has long since disappeared and been superseded. Many valuable maxims and noble thoughts which were at one time concealed in it have become current in this modern literature and have been translated over and over again into the language now spoken. Surely then it would seem enough that the study of the original language should be confined to the few whose instincts led them naturally to pursue it. But the Crewhonians think differently; the store they set by this hypothetical language can

hardly be believed; they will even give anyone a maintenance for life if he attains a considerable proficiency in the study of it; nay, they will spend years in learning to translate some of their own good poetry into the hypothetical language—to do so with fluency being reckoned a distinguishing mark of a scholar and gentleman."[4] This attack upon the Classical education had doubtless more pungency in 1872 than it has now. But the satire of the first sentences quoted—that directed against hypothetical studies—is, as already stated, no stray shot, but one instance of Butler's oft-repeated censure of "Academicism." "The more I see of academicism the more I distrust it," he writes in the *Note-books*.[5] "If I had approached painting as I have approached bookwriting and music, that is to say by beginning at once to do what I wanted, or as near as I could to what I could find out of this, and taking pains not by way of solving academic difficulties, in order to provide against practical ones, but by waiting till a difficulty arose in practice and then tackling it. thus making the arising of each difficulty be the occasion for learning what had to be learnt about it—if I had approached painting in this way I should have been all right. As it is I have been all wrong, and it was South Kensington and Heatherley's that set me wrong. I listened to the nonsense about how I ought to study before beginning to paint, and about never painting without nature, and the result was that I learned to study but not to paint. . . . Fortunately for me, there are no academies for teaching people how to write books, or I should have fallen into them as I did into those for painting and, instead of writing, should have spent my time and money in being told that I was learning how to write. If I had one thing to say to students before I died . . . I should say: "Don't learn to do, but learn in doing. Let your falls not be on a prepared ground, but let them be bona fide falls in the rough and tumble of the world; only, of course, let them be on a small scale in the first instance till you feel your feet under you. Act more and rehearse less.'" So he "regards dumb-bells with suspicion as academic,"[6] thus forestalling modern theories of physical training (and even the change that has come upon army "physical jerks"), and their avoidance of the old formal gymnastic. The excellent cooking at Oxford

[4] Crewhon, chapter XXI. (Fifield, London; E. P. Dutton and Co., New York). For permission to use this and other passages from the work of Samuel Butler, I am indebted to the courtesy of Mr. A. C. Fifield).

[5] *The Note-books of Samuel Butler*, p. 104. (Fifield, London: E. P. Dutton & Co., New York).

[6] *Ib.*, p. 219.

and Cambridge, which is better than the curriculum, "is taught by apprenticeship in the kitchens: there is no Chair of Cookery,"[7] whilst it would be as reasonable to "have a professor of wit as of poetry."[8]

In all these passages (and many more might be quoted) we find a consistent and surely a very sound doctrine as to the acquisition of any form of skill, and perhaps any form of knowledge. Our pupils should learn in the same way as an apprentice learns, i. e., by actual performance, by getting on with the job, not thinking overmuch about rules, or about possible but remote contingencies. This difference between apprenticeship and academicism is clearly illustrated in a most suggestive chapter ("Considerations on the Decline of Italian Art") in *Alps and Sanctuaries*. Speaking of the rapidity with which his friend Mr. Festing Jones learned to draw, Butler asks, "How did he learn? On the old principle, if I am not mistaken. The old principle was for a man to be doing something which he was pretty strongly bent on doing, and to get a much younger one to help him. The younger paid nothing for instruction, but the elder took the work, as long as the relation of master and pupil existed between them. I, then, was making illustrations for this book, and got Jones to help me. I let him see what I was doing, and derive an idea of the sort of thing I wanted and then left him alone—beyond giving him the same kind of small criticism that I expected from himself—but I appropriated his work. That is the way to teach, and the result was that in an incredibly short time Jones could draw. The taking the work is a *sine qua non*. If I had not been going to have his work, Jones, in spite of all his quickness, would probably have been rather slower in learning to draw. Being paid in money nothing like so good. This is the system of apprenticeship *versus* the academic system. The academic system consists in letting them do it, with just a trifle of supervision. 'For all a rhetorician's rules,' says my great namesake, 'teach nothing, but to name his tools'; and academic rules generally are much the same as the rhetorician's. Some men can pass through acad-

[7] *Ib.*, p. 222.
[8] *Ib.*, p. 221.

emies unscathed, but they are very few, and in the main the academic influence is a baleful one, whether exerted in a university or a school. While young men at universities are being prepared for their entry into life, their rivals have already entered it. The most university and examination ridden people in the world are the Chinese, and they are the least progressive." [9]

It should be added that Butler, with his usual sanity, remarks that "this proposition, like every other, wants tempering with a slight infusion of its direct opposite"; as again he says. "It is with this as with everything else—there must be a harmonious fusing of two principles which are in flat contradiction to one another." Still he held generally that knowledge must be acquired as and when it is wanted and wanted so badly as to cause discomfort. "There are plenty of things that most boys would give their ears to know, these and these only are the proper thing for them to sharpen their wits upon. If a boy is idle and does not want to learn anything at all, the same principle should guide those who have the care of him—he should never be made to learn anything till it is pretty obvious that he cannot get on without it. This will save trouble to boys and teachers, moreover it will be far more likely to increase a boy's desire to learn. I know in my own case no earthly power could make me learn till I had my head given me; and nothing has been able to stop me from incessant study from that day to this." [10] Now that the theory of "mental gymnastic" is discredited, and psychologists have proved that there is little or no transference of one form of training to another, we may well believe that Butler's advice for the treatment of the lazy boy indicates not merely the line of least resistance, but the policy that will in the end prove most fruitful.

For his views on the larger question of the apprenticeship system there is much to be said. We had in England during the war

[9] *Alps and Sanctuaries of Piedmont and the Canton Ticino*, chap. XII. (Fifield, London.)

[10] *Note-books*, p. 103. The substance of this note is embodied in the marvellous speech of the "Dinner Guest" (*The Way of All Flesh*, chap. XXXI). Cf. the remark of Butler's disciple, Bernard Shaw: "If the child finds that it can no more go to the seaside without a knowledge of the multiplication and pence tables, than it can be an astronomer without mathematics, it will learn more than it always does at present, in spite of all the canings and keepings in." (*Parents and Children*, p. lxxvii.)

an extraordinary example of its success in the training of soldiers, particularly of officers; for no one will deny the efficiency of the average temporary officer: to compare him with the finest type of regular is naturally unfair. This astonishing piece of education was achieved, for the most part, without anything which approached academicism; and when the army did try to be academic, the process was not merely retarded—it ended in laughter. It may be urged that such rough and ready teaching is only emergency measure; yet the proof of the pudding is in the eating: this kind of training produced what was required. The men were keen on learning, and they learnt by actual performance—in many cases their falls being on ground by no means prepared. In the training for more peaceful professions many traces of apprenticeship remain. The lawyer still takes articled clerks, who learn their business in the daily routine of the office. Even in a highly specialized science like medicine, the most useful part of the course is that spent in the hospitals; though one may be permitted the inconsistency of rejoicing that medical apprentices are forbidden to practice. The best engineer has usually been through the shops, and has probably served his time there; whilst captains of industry, I am told, look with little favor on the latest outgrowth of academicism, a university course in commerce. In the training for another profession, that of teaching, the demonstration school is an essential part of the Education Department. But it would clearly be a vast improvement if it were economically possible for students in training to serve a year's apprenticeship in school before entering upon his professional training. As things are, the student is still permitted to attach the theory before he has had a chance of learning its significance in practice. The sterility of many education courses results from the theory being taught to young graduates who can have no conception of the problems involved or of the "hypothetical difficulties" to which his attention is directed.

Butler has thus put his finger on the growing pains in educational theory. The recognition of auto-education is indeed a discovery of the obvious. Its success in Adult Education in England has

given rise to a movement of far reaching importance. One may predict that only on such lines can the continued education of "young persons" hope to prosper. Educationists may do worse than study the works of Sam Butler, where they will find certain fundamental truths stated with a vigor and humor which they do not always receive from professional exponents of Pedagogy.

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# BOOK REVIEWS

A First Course in Statistics. By *D. Caradog Jones*. G. Bell & Sons, London, 1921. Pp. viii, 286. Price, 15s. net.

The author of this book explains in his preface that "the whole is meant not as an exhaustive treatise, but merely as a first course introducing the reader to more serious works." His aim is to a great extent sociological, to teach the non-expert to understand and apply correctly such statistics as occur in ordinary life; at the same time he considers that the book may be of service in the sciences, since the principles are fundamentally the same. The science of statistics is based upon the study of the crowd rather than of the individual; hence we find on page 11 an explanation of the idea of Frequency Distribution, followed immediately by a chapter on Classification and Tabulation. We then have the fundamental ideas of average, median, mode and weighted mean fully explained; and their applications and characteristics are illustrated by well-chosen examples. Chapter VI discusses Dispersion or Variability, and in this the root-mean-square deviation is introduced tentatively, the proof being given later. The next chapter deals with the plotting of frequency distribution curves and the symmetry or skewness (lack of symmetry) of the curves obtained; it is followed by chapters on graphs treated more mathematically. There is next given an important chapter on Correlation, followed by illustrative examples of a sociological nature, such as the correlation between overcrowding and infant mortality in London districts, between unskilled wages and rents, and so on; and this brings Part I of the volume to a close. The whole of this part should be well within the understanding of the general reader; and the clear exposition, if at times somewhat lengthy, has much to recommend it. Part II starts by introducing the reader to Probability and Sampling, with many good illustrative examples. This is followed by over fifty pages on "curve fitting," and a couple of chapters on the normal curve of error and the frequency surface for correlated variables; and the book closes with an appendix containing some mathematical proofs of a more difficult nature.

In all, a very excellent text-book, which should have a place of its own more especially with the general reader; its weak point is the somewhat (to a mathematical reader) cumbrous nature of some of the work in the second part, which must, however, be excused owing to the professed aim of the author—that he is writing an introduction to the subject for the general reader. This general reader of course includes the scientist whose mathematical reading is not of an advanced order.

J. M. Child.

MIND AND WORK; THE PSYCHOLOGICAL FACTORS IN INDUSTRY AND COMMERCE. By *C. S. Myers*, M.A., M.D., Sc.D., F.R.S. London: University of London Press, 1920. Pp. xii, 204. Price, 6s.

A lucid account of the relation to human efficiency of fatigue and of well or ill-arranged rest periods; of monotony and variety of occupation; of compatibility and incompatibility of temperament and vocation; of the speed and noise of machinery; of piece-work, day-work, and overtime; and of irritation and goodwill between employers and employees.

The author intends his book as a plea for the establishment of a National Institute of Industrial Psychology in which the conditions for the wisest and most economical application of the human element in the work of production and exchange could be studied impartially and scientifically. The case is convincingly put. Notable examples of what has already been done by scientific methods to increase the efficiency and well-being of those engaged in industry are given, with several photographs by way of concrete illustration. There is little doubt that an industrial system organized in accordance with such principles as those outlined in this book would be much more stable than the present one. Dr. Myers' volume deserves, therefore, the close and earnest attention of our industrial leaders and social reformers.

FRANK WATTS.

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KARL MARX ON VALUE. By *J. W. Scott*. London: A. & C. Black, Ltd. Pp. vii, 54. Price, 3s. 6d. net.

This little book, by the Professor of Philosophy at Cardiff University College, is a brief summary of Marx's Theory of Value and of the chief arguments that have been brought against it. It is written in very simple language, which sometimes approaches what Mr. Caliban would call "the prattling style." But it makes no claim to originality either of idea or of manner of presentation, and it is, of course, very far from exhaustive of the subject.

G. C. FIELD.

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THE PHILOSOPHICAL THEORY OF THE STATE. By *Bernard Bosanquet*. Third Edition. London: Macmillan & Co., Ltd. Pp. lxii, 320. Price, 15s. net.

A third edition of Mr. Bosanquet's famous work is very welcome. The additions made are not great but of considerable interest. A few footnotes are added, and there are seventeen new pages in the introduction on "How the Theory stands in 1919." Mr. Bosanquet claims that there is nothing in recent events or recent movements which would necessitate the abandonment or modification of any of his views. Certainly a re-reading of the present work can only confirm previous impression of its depth and significance. It is a work of permanent importance today as much as on the day that it first appeared. But for all that, one feels a certain doubt whether Mr. Bosanquet has succeeded in taking quite sufficiently seriously certain recent movements of thought, particularly in the direction of a criticism of the claims of the state as against other institutions. It is not always perfectly evident that he realizes the possibility of real differences of principle on this point.

G. C. FIELD.

INSTINCT AND THE UNCONSCIOUS: A CONTRIBUTION TO A BIOLOGICAL THEORY OF THE PSYCHO-NEUROSES. By *W. H. R. Rivers*, M.D., D.Sc., F.R.S. London: Cambridge University Press, 1920. Pp. viii, 252. Price, 16s. net.

Mr. H. G. Wells has recently given us in his *Outline of History* the story of the evolution of man as it can be pieced together from a study of prehistoric remains and of recorded events. Such an account can never be more than a second-hand description of what life has been. In addition we need to win a sympathetic understanding of the process by which the gradual development of human intelligence has been achieved. Dr. Rivers has in this work done us a real service, therefore, in providing a new method for approaching the study of mental development. His thesis is that the story of our mental evolution is to be traced in the organization of the human nervous system and that it is a story of progress from crude undiscriminating sensitivity coupled with excess of feeling and blind ungraduated reaction to greater and still greater delicacy of sense discrimination and an adequately proportioned measure of feeling and response. According to the English school of neurologists, the nervous system, in so far as function is concerned, is arranged in "levels," one above another, forming a hierarchy in which each level controls those below and is controlled by those above. When disease or injury brings about a loss of such control, we may therefore observe the behavior which is characteristic of an earlier stage of development. Every abnormality of behavior is consequently a clue to the method of our evolution. It is the special aim of the author to show that the more primitive reactions (accompanied by "hit or miss" effects) which are incompatible with those activities characteristic of a higher level of mental development usually become *suppressed* quite automatically, or to use the Freudian terminology, are thrust down into the *Unconscious;* and in Dr. Rivers' words it is his purport to consider "the general biological function of the process by which experience passes into the unconscious." This leads naturally to a study of the psycho-neuroses which are symptomatic of that particular form of suppression which the Freudians call *repression.*

The book itself is a masterly piece of work which no student of human nature can afford to ignore. Chapter IV, which contains an account of the experimental neurological work of Dr. Henry Head and his collaborators, upon which Dr. Rivers has based his thesis, is a model of lucidity and concise expression. Especially valuable are Dr. Rivers' attempts to define his terms which lead in nearly every chapter to a thorough examination of psychological first principles. There will be disagreement with many of Dr. Rivers' definitions and views, but none can fail to recognize that they represent clear thinking and precise knowledge. Many modern books depend for their success upon the brilliant marshaling of new illustrations to prove old points of importance; but here we have old illustrations aptly used to prove recognized new points of possibly even greater importance.

The book is essentially an original piece of thinking, destined to excite that useful kind of controversy in which new advance in science usually begins.

FRANK WATTS.

THE TREND OF THE RACE. A study of Present Tendencies in the Biological Development of Civilized Mankind. By *Samuel J. Holmes,* Ph.D. Harcourt, Brace and Company, New York, 1921. Pp. 384.

This book is the outgrowth of a course of lectures on Eugenics which Dr. Holmes has been giving for several years in the University of California. Its aim is to present an account of the various forces which are at present modifying the inherited qualities of civilized mankind. The point of view and method are those of a careful biologist broadened and enriched by his interests along sociological lines. The style is popular enough to make the book very readable for the laymen but technical enough to guard against suggestions of misleading inferences. The spirit is manifestly that of a cautious scientist who while eager to release for public use the positive results of investigations in his field still maintains the reserve of one who is aware of the limitations of those results. In its contents and its organization the book will lend itself to use as a valuable reference for students in other fields.

The book tends to fall into two parts. The first seven chapters comprise an introductory orientation and a discussion of the inheritance of the human traits which are of especial significance in relation to the progressive and retrogressive development of mankind. The following eight chapters treat of the selective agencies that determine what types of human inheritance tend to prevail over others and the relation of these selective agencies to various factors in our social environment. A final chapter provides a general summary outcome of the discussion. As the work does not lend itself to epitomizing our review must limit itself to a statement of the topics treated with only a hint or two of the author's particular conclusions.

The first chapter is introductory. It begins by emphasizing the distinction between the hereditary transmission of acquired characters and the influence of the social environment on hereditary. That distinction made, the problem is stated as follows: What are the forces, both biologically hereditary and environmentally selective which are now modifying the inherited qualities of civilized peoples? Chapter II continues the orientation with an exposition of our present knowledge concerning the hereditary mechanism and several other cognate problems. Here Dr. Holmes sets down in a srtiking way his own position with respect to the relative importance of hereditary and environment: "Experience is often fallacious in ascribing great effects to trifling circumstances. Many a person has amused himself with throwing bits of stick into a tiny brook and watching their progress; how they are arrested, first by one chance obstacle, then by another; and again, how their onward course is facilitated by a combination of circumstances. He might ascribe much importance to each of these events and think how largely the destiny of the stick has been governed by a series of trifling accidents. Nevertheless all the sticks succeed in passing down the current and in the long run they travel at nearly the same rate. So it is with life itself in respect to the several accidents which seem to have had a great effect upon our careers. The one element that varies in different individuals but is constant in each of them is the natural tendency; it corresponds to the current in the stream, and inevitably asserts itself." What follows is a pointed discussion along more or less familiar lines of the inheritance of mental defects and disease, the heritableness of crime and delinquency,

the inheritance of mental capacity, and the various phases of the problem of birth rate decline. The second part of the book, which deals with the selective agencies which influence race development, clears up the present status of the principle of natural selection and examines such subjects as the selective influence of war, sexual selection, assortative mating, the differential marriage rate, consanguineous marriages and miscegenation, the rôle of disease and alcohol in relation to hereditary defects, the alleged influence of order of birth and age of parents upon offspring, the racial influence of industrial development, and the selective function of religion. The following extracts from the concluding chapter will suggest the tendency of the author's own position with respect to some of these problems:

"We can only judge of the present tendency of our biological development by a study of the forces which are now producing modifications in the inherited qualities of mankind. In our study of these forces it has been found that some of them are working in the direction of racial improvement while others are quite evidently having an opposed influence. What the resultant will be can be determined only by some estimate of their relative potency. . . . The one agency which appears to be most clearly working towards racial improvement is natural selection. At any rate there is a large amount of evidence that it is favoring the maintenance of physical vigor and keeness of mind. To a certain extent it retains what might be considered its primitive function of denying the privilege of parenthood to the poorer or uglier individuals, of the species, but the more capable and independent spirits, especially among the women, are coming to be denied this privilege also. The influence of group selection as manifested in war and otherwise, may also retain some of its original racial benefits, but, under our present regime, its dysgenic effects not improbably outweigh whatever it may contribute to racial improvement. The general influence of reproductive selection or differential fecundity is quite evidently pernicious. tI tends to extinguish the posterity of the most capable and to fill the world with the subnormal and inefficient, thereby constituting the most serious menace of all the forces which are influencing human heredity. Religious selectior. while formerly eliminating through persecution many of the better minds and while still continuing the racial evil of a celibate clergy in the Catholic church, now exercises the effects mainly upon the birth rate of different stocks. Its influence in maintaining the high birth rate of the Jews, who are certainly endowed with an unusual degree of intelligence and energy is rapidly waning and the differential fecundity it now helps to maintain is mainly in favor of elements, which for the most part, have not demonstrated a superior inheritance. The manifold racial effects of industrial development are in many respects bad. Industry may intensify the action of natural selection in eliminating persons whose physique and intelligence are below the general level, but, on the other hand, its influence on differential fecundity may more than counteract its tendency to racial improvement. Its effects in encouraging celibacy in increasing numbers of capable and self-reliant women who qualify themselves for an economically independent career promises to be a serious racial danger. Education itself, the basis of so much advancement, has proven, up to the present, a dysgenic agency. Its devotees commonly fail to reproduce themselves, and since education is becoming extended to more and more of those who are capable of acquiring it the racial damage thus

caused is correspondingly increased. The effect of our modern life upon the trend of germinal variability . . . is a subject about which we know little. Alcoholism while helping to dispose of a number of undesirables, is open to grave suspicion as a cause of defective inheritance. . . . Those forces which have been called to action as a result of the development of our culture are in large part racially destructive. We cannot say that they are entirely so because there are counter tendencies which sometimes arise. All those agencies which bring about the present well-marked correlation between sterility and success in life tend to rob the race of its best inheritance. It is chiefly the primitive evolutionary factors which operate among the lower animals that are making for racial improvement in man. Civilization brings in its train so many factors that undermne its own biological foundation, that from the racial standpoint at least, we may well ask with E. Carpenter, "Is Civilization a Disease'?"

As a whole the reading of the book creates the somewhat paradoxical impression that, on the one hand, the problem is hopelessly complex, and on the other that biology now stands on firmer ground with its new method and technique.

EDWARD Z. ROWELL.

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# THE MONIST

## THE PHILOSOPHY OF POSSIBILITY

NO INSISTENCE has been more characteristic of modern philosophy than that which lays it down that philosophy has to do with experience and its actualities. Possibilities have been too often treated as airy nothings, mere ideas or mental terms, and nothing more. That philosophy is the science of the possible is a suggestion far from likely to be taken up even if it were found feasible or desirable, so insistent are the demands of the actual world for explanation. If we should find the actual to be the source of all possibility, that would certainly not render the suggestion, with its isolation of the possible, a more warrantable enterprise. But, in any case, the philosophy of possibility seems to me worthy of more attention than it has received. In spite of pure empiricism, possibility has not failed to catch some attention, all through the history of philosophy from the Greeks onward, however fugitive and sporadic that attention may have been. And so far from being a mere figment of the imagination, possibility exists both as idea and as fact; a possibility contains some actual idea; it is indeed as real, objectively. as actuality itself. But yet the possible implies in every case, that the idea has fallen short of reality. For every possibility an *ens essentiae* may be claimed, though not, of course, an *ens existentiae*. Every possibility has a real foundation in some nature or being, proximate or remote. That there is some reality in possibilities must be thus early kept in

mind. There are two forms of existence, the actual and the possible. What exists as actuality is, from the subject's point of view, content of a presentation, or reducible to such. Actual experience has a large margin of possible experience, and this extension of the actual cannot be left out of sight. There is not one of the States of America which does not exist for me actually, and not as mere possibility, and yet any one of these is only possible experience for me until I set foot upon it, and it becomes for me actual in presentative experience. What exists as possibility, on the other hand, is, from the subject's point of view, content of a conception. Such conceived content exists only as possibility, not actuality. Possibility has been declared to be just the ideal we have of anything. Possible existence may be of a kind, as we have just seen, that it can become real content of what is as yet only possible experience. But that is not the sense of possibility with which we are at this point concerned. We are concerned with it only in the sense in which the existent is present to consciousness purely as possible. The purely possible, of course, precludes existence. Possible being is not yet existent, but is taken as capable of coming into being, or existing. The idea of possibility exists antecedently to all created being. All knowing is a knowing of what is at least possible existence. All really possible objects are conceivable: all real possibilities are rational, I mean as objects of thought: the impossible is self-contradictory or irrational. The possible, in the logical sense, is what is free from contradiction; but all possibility is possibility of something, however indeterminate. The philosophy of possibility cannot evade the question of the origin of possibilities. Can we trace possibility simply to the human mind? Do possibilities not exist before the human mind comes into being? Will the possibilities not exist after the human mind has ceased to exist? Can we even ascribe the possibilities to

the universe? If the universe were done away, would possibilities not remain in undiminished form? For, are the possible universes not infinite? And, is not possibility necessary and eternal? These are among the questions that may be asked. The philosophy of possibility can hardly be satisfied to accept possibilities as accounting for themselves. We are compelled to think of the ideas of possibility as existing in some mind or spirit, and ultimately, in a Sovereign Mind or Spirit, wherein they gain eternal basis and fixity. But, even if such an ultimate origin be deemed unnecessary, it still holds that necessary ideas of possibility, like other necessary truths, contain "the determining plan and the regulative principle of existent things themselves." But if the ideas or principles exist before contingent things in this manner, then must they be grounded in some necessarily existing substance. Rosmini, who was severely critical of Kant's treatment of the categories, did possibility the honor to regard it as the only one, out of Kant's twelve forms, which really is an original and essential form of the human intellect. Höffding, less correctly, in my judgment, would educe all the categories to the two concepts, quantity and cause. Rosmini's position would perhaps be a primary consideration in making philosophy a science of the possible, were such a philosophy feasible. Our knowledge, so far at least as it consists of thought, should then only be concerned with possibilities. Possibility would be the fundamental, all-embracing category. We cannot, however, carry out this notion of confining all that is thought only to the possible, for we must know the real, and seek objective being or existence. Otherwise there would be a false limitation of thought. Besides, being allows itself to be brought under concepts, and thereby shows itself to be logically determined. But this is not all that Rosmini did. Aristotle had confused the purely logical issue, so far as it was concerned, by plac-

ing possibility and necessity in things, in conformity with his insistences on potentiality and actuality. But we shall consider Aristotle later. The transition from the possible to the actual was, for him, effected through motion. Kant did not improve matters when he made possibility and necessity mere subjective conditions of the thinking subject. What Rosmini did was to ground both possibility and necessity in the nature of being. Being so meant has no material associations, is only indeterminate form. But being, of course, is object, not subject. What I have already said of possibilities, from the subject's point of view, holds of them as known in their groundedness in being. But that is not to say that a merely possible essence has no ontological or objective basis independently of our conception of it. Possibility and necessity I have coupled together, since, possibility being analyzed, what is really possible is found to be necessarily so. And it may be remarked that it is with necessary truths that science, in the strict sense, has to do—with essences rather than existences. Mere scientific facts are sterile until they become fecundated by ideal or necessary truths.

When we consider the concept of mere possibility, we find that, in its positive aspect of capability of existence, it has a certain ontological basis and a certain objective reality, that is, as an objective concept. For what is meant by the really possible? Is it not some ideally constituted being or entity, conceived as capable of coming into being or existence in the world? But that means that it is already real as an objective concept. And this means that possible essence is not any fanciful and arbitrary creation which one chooses to conceive. Possible existence lends itself to no such freaks of the mind. Possible being, as a concept, needs some sort of objective reality to justify it. Possible existence is not to be thought of save as capable of being produced in the order of things existing; for

that alone is meant by its real possibility. Thus an acorn may, certain ideal conditions being made actual, be an oak, but it cannot possibly be an elm. In the same way the child may possibly be the man, but not the kangaroo. Possible essence does not require to be precontinued in the entity which is the real basis of possibility, in any formal and actual manner: it is enough that it virtually or ideally so pre-exists. Possible things are such that they may yet exist *de facto,* and do presently exist virtually in their causes. What has been said in this paragraph appertains to the region of metaphysical possibility, rather than that of logical possibility, although the mental process of conceiving must, of course, be of a logical or consistent character. But indeed this is not all; for, metaphysical as may be the entity which is the real basis of possibility, the possible essence must obviously be logically pre-contained in that existence which is the only basis of its reality.

The Scholastic philosophers have been by no means alone in realizing the importance of the philosophy of possibility, but they have been paramount in the attention they have given to it, and have made us their doctors for all time. One of the distinctions whose importance they have realized is that between intrinsic possibility and extrinsic possibility. In intrinsic possibility, the conception of a being or object, in its capacity for existence, is such as to involve no inner contradiction, no inner repugnance of being or character. Such intrinsic possibility is, in its inherent character, logical; yet must it find foundation in some reality.

A very broad example of intrinsic impossibility may be found in Lotze's criticism of Kant's scheme of Categories when, in the first volume of his "Metaphysics," he says that "that kind of theoretical security for an unconditional completeness, which Kant was in quest of, is something intrinsically impossible." But simpler examples, such as a round

square, lie ready to hand. The absolute character of inner or intrinsic possibility is noteworthy, compared with the relative character of outer or extrinsic possibility. But the two forms together constitute the full concept of possibility. And the inner possibility is the presupposition of the outer possibility. Such inner possibility is in character metaphysical. What is intrinsically impossible is also extrinsically impossible, but what is, in a given case, extrinsically impossible, is not therefore intrinsically impossible. Intrinsic possibility, in briefest terms, then, means mere non-absurdity.

Extrinsic possibility denotes the capacity for existence of a being or object due to the fact that something else has power to actualize it, as intrinsically possible. More briefly, extrinsic possibility means merely, being causable. Within the sphere of created things are many intrinsically possible things which are yet impossible, extrinsically. Extrinsic possibility may be physical, or it may be moral. Ultimate possibilities belong to the order of the necessary and immutable. Aristotle held, in his "Metaphysics," that the actual is anterior to the possible, alike in respect of being and of knowledge.

Aristotle contended that the passage from possibility to actuality takes place in certain fixed and unchanging ways, whereby the true nature of the real is made manifest. The potential is not to be confounded with mere possibility, as if anything whatsoever were to be reckoned possible. The possibility of the actual is for him the only possibility. What cannot be actualized is impossible. Determinate possibility, not possibility of the abstract and unlimited sort, is Aristotle's insistence. That is to say, the possible means possibility of realization in certain and definite ways. The impossible means incompatibility with the actual. In these positions the Scholastic philosophers have largely followed him. They have held that actual being, not possible, is the

first of all being. Their position has been that we cannot know possibility or things possible, without the concept of actuality or of things actual. Thus the concept of the possible presupposes the concept of the actual. The possible, is already, in part, the real. It has had to be admitted, of course, that in the case of secondary created existences or things, the idea of their possibility preceded their actual being. But, it is said, the contention as to the possible is not made in the sense of comparing it with the real in the same object. What is meant is, that a possible thing does not, in becoming real, give itself reality, its reality not being attained save through some other being, actual or existing.

Hegel puts the matter rightly when he says that "possibility should come second"—after actuality; for only "in abstract thought" does the possibility conception come first.

From the Scholastic philosophers I pass to that admittedly profound but neglected thinker, C. H. Weisse, who went to the root of the philosophy of possibility by raising —but not for the first time—the question of the possibility of God. The concept of God was for Weisse no presuppositionless affair. The original possibility of God includes for him every other possibility, and is the sole content of thought-necessity. This thought of the original possibility of God is the basal thought of his system, in which the concept of the possible may be said to count for more than the concept of being. But he held that no abstract necessity of reason can give us more than empty forms of possibility. His position, then, is that there is only one truth which is originally necessary to thought, namely, that only God is possible, and that in His possibility is contained the possibility of all things. In his view, the becoming real of this original possibility or thought-necessity of God means a real thinking absolute Subject. Now, interesting as these positions of Weisse are, they tempt one to some

critical reflections. The idea of any being implies, of course, the possibility of that being. But can we apply that to the case of God? Is not such mere possibility excluded by the fact that the true idea of God is that of the necessary being or existence? If He is necessarily the primal and perfect actuality, raised above all conditions of potentiality, then He is not possible in the sense that was spoken of. Kant, it should be noted, had early taken up the ground that God's existence, as a necessary Being, was antecedent to the possibility of His existence—the possibility, in his view, depending on His existence. What Kant really meant was, that possibility logically presupposes actual existence as its base or foundation, but his mode of putting the matter was not very happy. But Kant put the matter much better when, in his Critique of Judgment, he spoke of "the irrepressible tendency of reason to suppose some unconditionally necessary existence, or original ground, in which the distinction of possible and actual no longer holds good." The old rule or saying, *"to be possible* comes before *to be"* ("prius est posse esse quam esse"), may, it seems to me, do very well for things finite, but it can have no applicability to One who is *Ens a se*—Being in and of itself. As such, God is not possibility at all, but the prime metaphysical necessity. Leibniz differs from Weisse when, in dealing with the Anselmic proof, he maintained that if such a Being as God be possible, He exists. For he held that those who would deny this proposition would deny the possibility of Being in and of itself. But if that were so, he says, then all things through another would be impossible, and nothing could exist. That would imply that the necessary Being called God does exist, since we find possible beings in actual existence, but it does not yield an *a priori* argument. But the question of the possible had occupied the mind of Descartes, long before Leibniz, and it is interesting in the present connection to recall

the statement of Descartes,—"But we must make a distinction between *possible* and *necessary* existence, and observe that possible existence is included in the notion or idea of all things of which we conceive clearly or distinctly, but that necessary existence is included in the idea of God alone."

It would be a serious mistake to suppose that Descartes is here making a new distinction. The possible, the necessary, and the impossible, were suggestively dealt with, very much earlier, by that powerful logician Wyclif, whom I have found more interesting in his "Logica" in this respect, than his great opponent, Occam, in his logical "Summa." Wyclif gives the first place to God's existence as absolutely necessary, but speaks of a secondarily necessary as self-necessary, geometrical theorems, for example. "Necessary" means, for Wyclif, "impossible not to be": "Impossible" means "necessary not to be." The meaning of impossibility, for him, answer to those of necessity. Descartes, then, holds that we cannot conceive of God save as existing, and, in his fifth "Meditation," he maintains that, as the idea of a triangle involves its having three angles equal to two right angles, so the idea of God carries with it His existence. It is possible to conceive a mountain or a valley without either of them existing, he says, but it is not possible to conceive any other Being than God to whose essence belongs existence. These are aspects of Descartes' teaching whose tenableness I am not now concerned to discuss; what I am concerned with is, that in the fore-shadowed idea of necessary Being there is already much that would give pause to Weisse's line of argument as to the possibility of God. The only possibility of God that would be left for consideration would be, whether the notion itself of God as the sole necessary Being was one that was intrinsically repugnant, self-contradictory, or impossible, in the character of its Being. If the possibilities

of all things derive from God, as the sole necessary Being, then clearly He Himself cannot be subject to the law of possibility. Aquinas said, "Deus est actualitas totius possibilitatis." But I do not dwell on these aspects, though it did not seem possible wholly to avoid their discussion, because it is the law of general possibility, with which this paper is mainly concerned.

The philosophy of possibility has to contend against many injustices. Pure empiricists will have none of it as a theory, from Hume onwards, although Hume did not deny the possibility of knowledge. The actual is their limit of possibility, its only measure. Bain and Mill are examples. The universe is for such empiricism just what it is, a closed system, impervious to influence from without. Said Lewes, "nothing really exists till it exists, and nothing exists possibly, for possibility is only the uncertainty of our ignorance." Mill talked of "possibilities of sensation, although abstracting all substance and causality in such a manner as to leave said "possibilities" deprived of the conditions of possibility. But the philosophy of possibility is not furthered by certain empiricists telling us that the universe might just as well have been one in which 2 and 2 would have made 5, or the square would have had the form of the ellipse. It is concerned only with possibilities that are real and rational. But what do we get from an idealist like Bradley? When dealing with the principles of logic, he says that "reality in itself is neither necessary, nor possible, nor impossible." For these predicates, he holds, exist only in our reflection. And if our knowledge and reflection were great enough to take in all the facts, "nothing would ever appear possible. The real would seem necessary, the unreal would seem impossible." But, as we are not such impossible beings as his supposition demands, the more relevant task is to discuss possibility as it exists for us. We are only such beings that

we have to accept the law of uniformity of nature, although no final logical reason is possible to us for the huge assumption that nature is uniform. Scientific inquiry resolves itself, it would seem, into the study of possibilities. A scientific hypothesis is but one conception among alternative possibilities. Kepler is said to have made nineteen false hypotheses regarding the form of the planetary orbits, and the theory in which he finally rested, that these orbits are ellipses, was but a possibility or an hypothesis until verified by facts. Uranus first became a possibility to Herschel, and Neptune to Leverrier through gravitation laws, and then they became facts or discoveries. The possible is futurist in character, although there is a sense, of course, in which the Actual may be said to be, before all things possible. But that is a sense with which we are not here concerned, and besides, the question of possibility cannot be raised as to present or actual fact, but should be kept as a question of the future. Possible relations the scientific inquirer seeks to establish, as when, for example, Faraday attempted to discover a possible relation between gravity and electricity. Another example of possibility that had to be considered was, the possibility of expressing the conditions of motion by means of differential equations, while another instance has been the discussion, with reference to modern electrical theories, of the possibility of ultimate mechanical explanations. In the biological sphere, beyond the chromidial unit lay, as an unforeseen possibility, the cell, awaiting discovery. But why enter on examples when the whole progress of science has been strewn with theories of possibilities which have in time been replaced by other and more perfect theories? The field of physical possibility seems of boundless scope, and this is the very inspiration of science. But there are other fields of possibility, such as psychic possibilities, moral possibilities, logical possibilities, metaphysical possibilities,

epistemological possibilities, and so forth. They are not less real after their kind than the physical possibilities. Of the possibilities in every sphere, one may say that they must at least be possible to thought. Anything, the concept of which is self-contradictory, is itself impossible. Kant does not treat the categories, among them that of possibility, with the clearness and exactitude that might be wished, accepting them, as he does, in a merely empiric way. He acknowledges a pure use of the categories to be possible, that is, not self-contradictory, but says that such a use has no kind of objective validity. The categories contain mere possibilities and depend on experience for their validity and confirmation. And experience depends on the categories for its possibility. Thought is, however, no guarantee of reality. Kant failed to appreciate the dynamic aspect of the world, and viewed each category too much as separate and complete in itself. There was lack, therefore, of developmental view of the whole. In an abstract sense, what is thinkable, is possible. This would appear to be the use of the possibility category most accordant with Kant's system. But it is present only in the sense that anything is possible which is conform to the formal condition of experience. His first postulate of empirical thought is, in his own words, that "that which harmonizes with the formal conditions of experience is possible." Kant recognizes possibility of a real or empirical character, which is not without some kind of empirical basis to rest upon. It has been objected that this form of possibility is vague and indeterminate, but that can hardly have much weight with us after all that has been advanced in the foregoing pages. A potency contained in its cause, e. g., is surely a fairly determinate possibility. I think there is something in Cohen's position, that the possible is not the real only in concept; that possibility is a synthetic determination of relation; and that it cannot be drawn off from

reality within the context of experience.[1] How different it may be noted, is the impossible, which remains mere thought—thought vanquished by the real; for in the impossible, thought and reality are mutually opposed. It is just the value of the categories that they render synthetic *à priori* judgments possible, and so advance our knowledge of reality. In such judgments, the categories are the *à priori* elements, conferring on them necessity and universality, and so rendering them scientific. Otherwise, objects might be given to us in experience, but they would not be known.

I have been mainly concerned to show the reality of the possible. If we took it as the merely conceivable, out of all relation to everything else, we should have a merely abstract and empty possibility. But as real possibility, possibility or capacity concretely conceived, it is a highly important category. For this is dynamic possibility, force that is pressing towards expression. For possibility is but a moment in the movement towards existence. Existence is *complementum possibilitatis,* so, at least, Leibniz and Wolff styled it. Wolff was, of course, right enough in presupposing that not everything thought possible is also real. And perhaps his position, that the existence of a thing is a completion of its possibility may, in the view of some, be allowed to stand, in the case of what we have called real possibilities. But it will not stand for possibility in general—not for those abstract possibilities already discussed. For he was mistaken in supposing that everything, which happened to be essentially free of contradiction, has the capability of existing. We cannot assume that, in the case of something abstractly possible, but not actual, its non-existence merely means lack of existence. Exception may be taken to Wolff's position that existence is completion of possibility, even in the case of

[1] H. Cohen, "Kant's Theorie der Erfahrung," p. 234.

real possibilities, for if possibility involves that a thing is already fully determined, it may be contended that it is not susceptible of further determination. What I have already said of Aristotle's positions should be remembered here, but attention must be fixed on what the concept of real possibility involves, such possibility being always determinate. Real possibility belongs to something that realizes itself. But there are differences in the possible here. There is possibility as we see it where some effectuating cause is at work. And there is possibility where, on a teleological view, our own self-conscious purpose is a determinative influence at work for the realization of an end. There may be several possible ways of reaching this end. An egg in virtue of its evolutive property produces a bird, but the egg remains for itself an egg. It is thought which sees in it the possibility which is eventually realized. The possibility of development shows potency thus becoming real. There is this peculiarity, then, in possibility, that the real or existent conditions are complemented by the thought or ideal conditions. This supplementary functioning by thought is very important in the study of possibility. For one may be very skeptical indeed as to possibility being given us through empirical perception; it is thought or reflection upon the experience which finds for us the possible, whether present or future. We cannot be content with the given in perception; the Spirit by means of thought frees itself from the impression. We only reach the concept of real possibility as we think of a continuity, which underlies the changing experience. Logical concepts may stretch out beyond experience, but it is otherwise when we deal with real possibility. For this is just the potency of the real, and possibility may here be limited by dependence on circumstances. The potentiality is merely "an antedated, presupposed, and hypothetical actuality." I have shown real possibility to have a certain

dependence on something empirically given, an actual basis in which part of the possibility-conditions is already realized, and yet it is the very triumph of the category of possibility that it frees us from the dominion of the empirically given. If I think of the world itself as one of unrealized possibilities, set in a universe of many other worlds, also of unrealized possibilities, I am away beyond all experience, I am emancipated from the thraldom of the empirically given. Thus does the category of possibility, with its illimitable horizon, raise me above empirical reality. But it is the insistence of Kant that the use of the categories does not extend beyond the limit of the objects of experience. But the possible is still the thinkable, and the thinkable blends with the intuitional in the formulation of possibilities. Reason plays important part in this realm of posited possibilities, whose realization it demands. Science has continually to reckon with possibilities which it has not yet found in sensible reality. In the more abstract domain of mathematics, there is a whole world of possibilities, with chains of necessary connection. There are necessary possibilities, it must even be said, connected with reason's ideal of reality, and when the idea of the absolute, a possible and necessary idea, has been thought, the limit of the category of possibility has been reached. Away from the sphere of the necessitated, in the region of personal spirit, there is the greatly neglected notion of spontaneity, which yields real possibilities. The possibility of communion with others, in matters of knowledge, belongs to the same personal sphere.

It will be evident that a great deal has been said to prepare the way for the statement that there are degrees of possibility. The possible does not always carry one and the same meaning, a fact which furthers this result. The whole philosophy of probability, with which we are not here concerned, rests upon the power to differentiate be-

tween different degrees of possibilities. Probability is not only the guide of life; it is the guide of science also. From fantastic and merely conceivable possibilities, which constitute the lowest degree of possibility, we pass up to high degrees of real possibility. That is to say, we pass from the barely possible up to the highly probable. And we have already noticed such differences in kind as logical possibilities and ontological (or objective) possibilities. When it is a question of propositional truths, the possibilities are matters of logical grounds and consequences, not of objective conditions and results. And anything is possible, logically, that is conceivable, that is, so far as it is not self-contradictory. But this, of course, carries no reality coresponding to it. Possibility is another matter when connected with empirical reality. There the possible, whose causes and conditions exist or will come into existence, must be consonant with, or conform to, the laws of Nature. But when, in respect of degrees of possibility, actuality is pronounced to be the maximum of possibility, I doubt whether we should so speak, for in actuality, as it appears to me, possibility is no longer present, but has been already sublated, or, if you prefer, sublimated. We saw that hypotheses were alternative possibilities, not all of which are, in like degree, possibly true. The vast scope for degrees of objective possibility, in the outer world, needs no emphasis. Things may be possible, too, without being known to be so. There is not only our subjective recognition of outer possibilities, but there are the subjective possibilities connected with our own purposes and endeavors, in which also there are degrees of possibility as to their realization.

The great value of the category of possibility, particularly for science, is, I think, its lesson of the need for the open mind in respect of the future. But, of course, this applies also to the philosophical questions, and theories,

which arise out of scientific problems or determinations. Many a scientific hypothesis, and many a philosophical theory, have had to succumb to the tests of ever-advancing thought. The same possibility awaits many of those current today. Evolution by means of natural selection; the turning of thought from mechanical to vitalistic tendencies; the postulation of energy as a quantity constant in amount; the theory of heat; the theory of relativity; the theory, have had to succumb to the tests of ever-advancing tist theory; the theory of value; these, and many more, have possibilities that lie hidden in the future, favorable or otherwise. In the sciences of Nature, possibility amounting to probability is all that we have. But a short time ago, it did not seem possible that any chemical element could change; but radium burst the bonds of seeming impossibility. On the philosophical side, there is the question of open or closed systems. A closed system is one which contains no real possibilities. Every event is to it either actual or necessary. The actual, too, is supposed necessary, and there is nothing possible in such a system but the necessary. New potentialities, of course, there may be, but all is predetermined and fatally certain. The system being closed, there is no purpose: it is non-teleological. There are systems, however, like that of Leibniz, for example, with its pre-established harmony, which, though closed, are not non-teleological, although there is still too much fixedness for the freedom of the moral life. The Hegelian system seeks to mediate between the idea of a finished universe and the freedom of the moral life, but it must, strictly regarded, be taken as, in reality, a closed system. The Hegelian "Logic" expressly says that there must be no talk of possibility or of the possible. We must bring out, says Hegel, the necessity hidden behind every semblance of contingency. In such a system the possibilities are merely apparent, not real and genuine.

Against all this, one must stand for an open system—a growing or unfinished universe, with room for the free, creative possibilities of the good will. There must be no limit set to the possible increase of ethical and other higher values.

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# DEWEY'S THEORY OF VALUE

## I

IN THE BELIEF that John Dewey's theory of value is not only the hub of his philosophy, but that it also constitutes the chief contribution of Pragmatism to current philosophy, I wish, in this paper, to show what Dewey's theory of value is, to indicate its relations and significance, and to adduce corroborations of the theory from certain unexpected quarters.

No one can hope to understand Dewey's theory of value who does not grasp rather fully the meaning that he attaches to the term "experience." Such an intellectualistic penumbra hangs over the word that some readers never understand that Dewey means other than what they would mean if they used the word in a similar connection. That he does mean something different, and something more, he has emphasized most vigorously in the introduction to his *Essays in Experimental Logic*. It seems that Professor Dewey himself did not for a long while get fully clear the significance of the distinction; and he gives Mr. S. Klyce credit for pointing out to him this and "other indispensable considerations." Upon this point Dewey explains: "Our words divide into *terms* and into names which are not (strictly speaking) terms at all, but which serve to remind us of the vast and vague continuum, select portions of which only are designated by words as terms."[1]

[1] *Essays in Experimental Logic*, p. 8n.

Now the word "experience" and such other words are not terms at all; they are what Mr. Clyce calls "infinity and zero" words. As opposed to such a word as "desk" (which is a term referring to a definite object, on which consciousness is or may be focussed), the word "experience" refers to the fringe or penumbra of the situation in which "desk" or any other term is the focus. As Dewey says: "The word 'experience' is . . . a notation of an inexpressible as that which decides the ultimate status of all which is expressed; inexpressible not because it is so remote and transcendent, but because it is so immediately engrossing and matter of course."[2] Such a word, then, connotes what is before and after and around that which at any given time is denoted. "I shall only point out," says Dewey, "that when the word 'experience' is employed in the text, it means just such an immense and operative world of diverse and interacting elements."[3]

Experience, as this will indicate, is a much broader term than knowledge. Instead, therefore, of putting the question as some philosophers have, i. e., whether there are different ways of *knowing*, we must cease begging the question and ask whether there are not different ways of *experiencing*, of which knowing is only one. We certainly must be content to put the matter in this way, if we are to understand Dewey; for Dewey makes this point in wholly unambiguous language. "Knowing," says he, "is one mode of experiencing, and the primary philosophic demand . . . is to find out *what* sort of an experience knowing is—or, concretely how things are experienced when they are experience *as* known things. . . . To assume that, because from the *standpoint of the knowledge experience* things are what they are known to be, therefore, metaphysically, absolutely, without qualification, everything in

[2] *Essays in Experimental Logic*, p. 10n.
[3] *Essays in Experimental Logic*, p. 7.

its reality (as distinct from its "appearance," or phenomenal occurrence) is what a knower would find it to be, is from the immediatist's standpoint, if not the root of all philosophic evil, at least one of its main roots."[4]

While Dewey has carried the analysis of experience somewhat into detail in the foregoing citation, we may rest, for the purpose of this paper, in his usual dichotomy. If we analyze the various aspects of our experience, we find on the one side what Dewey refers to variously as "appreciation," "realization," "direct, non-cognitive doing-suffering," and on the other side what he refers to as "reflection," "thinking," "cognition," "judgment." Experience, of its own intrinsic nature, falls thus into a dualism—qualitative at least—and the nature and relation of these two parts constitute both ethics and logic. Perhaps nowhere better than in *Democracy and Education* has Dewey plainly set over against each other these two aspects of experience.

"To value means primarily to prize, to esteem; but secondarily it means to apprize, to estimate. It means, that is, the act of cherishing something, holding it dear, and also the act of passing judgment upon the nature and amount of its value as compared with something else. To value in the latter sense is to valuate or evaluate. The distinction coincides with that sometimes made between intrinsic and instrumental values. Intrinsic values are not objects of judgment, they cannot (as intrinsic) be compared, or regarded as greater and less, better or worse. They are invaluable and if a thing is invaluable, it is neither more nor less so than any other invaluable. But occasions present themselves when it is necessary to choose, when we must let one thing go in order to take another. This establishes an order of preference, a greater and less, better and worse. Things judged or passed upon have to be estimated in relation to some third thing, some further

[4] *Influence of Darwin on Philosophy*, p. 229.

end. With respect to that they are means, or instrumental values."[5]

From this quotation as a basis, Dewey's theory of value may, I think, be fairly summarized in the four following theses:

## II

1. *Experience, which furnishes the context of all values, is largely non-cognitive.*[6] It is well to emphasize this non-cognitive basis of cognition itself; for upon this plane of experiencing lie most of the contents of our living. Here are included our loves and our hates, our eating and our sleeping, our friendships and our animosities, our illness and our health; here too are the fine arts;[7] here the dumb gladness that welcomes the dawn, the quiet contemplation of the sun's trailing glory at eventide, and the silent watching of the passing night. This is the primal and ever the larger aspect of human life. It is the good-in-itself, from which reflection rises and for whose sake reflection exists as an instrument.

2. *Experience becomes cognitive only when incompatibilities demand more than mere appreciation for their successful resolution.*[8] Dewey is primarily interested in intrinsic values, in the appreciative life described above. Indeed, no living being, thinks he, ever becomes interested in "extrinsic" value until he must in order to save and extend some of the "intrinsic" content of his appreciative life. Then judgment comes into play as an instrument that is justified by resolving the difficulty back into a situa-

[5] *Democracy and Education*, p. 279.

[6] "Experience is primarily an active-passive affair; it is not primarily cognitive." *Essays in Experimental Logic.*

[7] "For Dewey's feeling estimate of the place of art in life, see *Reconstruction in Philosophy*, p. 212. Cf. *Human Nature and Conduct*, p. 159ff.

[8] "Difficulties occasion thinking only when thinking is the imperative or urgent way out. . . ." *Reconstruction in Philosophy*, p. 139. *Cf. Democracy and Education*, p. 280.

tion that permits intrinsic goods again to become possible. If judgment begins building upon itself a hierarchy that forbids the energy of life to dip again into its stream, it meets Nemesis upon its upward way, who robs it of its vaunted glory and leaves it quite inane. "In fact, 'good' is an empty term unless it includes satisfactions experienced."[9]

3. *Such adjustment*[10] *arises from, exists for the sake of, and dips again into, non-cognitive experience.* Thinking becomes thus an instrument, but an indispensable instrument, for the continuous preservation of the values for which it exists.

4. *Moral judgments do not discover value outside experience, but reconstruct and create values within experience.* Let it be carefully borne in mind what has been said about the genesis of judgments of all kinds. Value judgments are not unique, as Dewey thinks, save in that they deal with a content that in direct experience was valuable. Value judgments arise out of a situation made embarrassing by a conflict between two equally valuable parts of experience or out of some other equally unsatisfactory turn of experience; and the judgments are but citations of what seems necessary to make experience once more satisfactory. Judgments are—if one wishes to put it so—discoveries of what, under the circumstances, one ought to do, if he is successfully to resolve the unsatisfactory situation. The judging process is not only the discovering of what is to be done, but it is also the first step of the action itself. It is not a discovery of something already existent (though it has such as its data), because the situation to which the judgment looks as an end that is to resolve the maladjustment, has never yet existed and would not come to exist but for the process of action which is initiated by the judg-

[9] *Democracy and Education*, p. 412.

[10] "It comes after something and out of something, and for the sake of something." *Essays in Experimental Logic*, p. 75. See also *Ibid.*, p. 36.

ment itself."[11] The restored equilibrium is the creation of the active life through the judgment as its tool. "At whatever risk of shock," thinks Dewey, "this doctrine should be exposed in all its nakedness. To judge value is to engage in instituting a determinate value where none is given."[12] The contention that judgments of value are practical includes two points: "one, that the judgment of value is never complete in itself, but always in behalf of determining what is to be done; the other, that judgments of value (as distinct from the direct experience of something as good) imply that value is not anything previously given, but is something to be given by future action, itself conditioned upon (varying with) the judgment."[13] For an ill person to decide that it is well for him to see a doctor is but for him to take the first step in the creation of the good of his then situation, i. e., the seeing of the doctor, a good that but for his judgment would never come to exist at all.

It is well for Dewey to insist upon the practical nature of judgments of value: for if he can establish that fact, he has not only added something to our understanding of such judgments, but he has also in a way as effective as it is indirect warded off from his general theory criticisms from both idealists and realists. From both schools alike have come charges of subjectivism against pragmatism. Those who incline toward subjective idealism have wished to claim Dewey; those who have espoused the cause of objective idealism have joined with the realists in saying that Dewey is a subjectivist. Only in so far as Dewey's general theory of knowledge is involved in his theory of value am I interested here in discussing his views on epistemology. Dewey has elsewhere attempted to show that those who, with Bertrand Russell, state the existence of

[11] *Essays in Experimental Logic*, pp. 358-359.
[12] *Ibid.*, p. 368.
[13] *Ibid.*, p. 361.

the world as a logical problem are but unconsciously chasing their own shadows; for the very words in which they are forced to state the problem show that they have already answered the question in the affirmative before the question is put.[14] But as regards the epistemological gulf in the study of value, if it were true that there is such a thing as the objective world and such a thing as the subjective world, and the integrity of the subjective were determined by its "correspondence" with the objective, then it were a most serious charge against Dewey to say that he is a subjectivist. For in the field of value, it means that he proclaims what could be hardly less serious than moral solipsism. To both the idealist and the realist who feel that they must have the objectivity of moral judgments before their judgments can command respect, Dewey seems, if not himself corrupt, at least a corrupter of youthful America.[15]

How, then, does Dewey escape the seriously meant charge of denying the objectivity of value? He escapes by pleading a change of venue, by denying the jurisdiction of the court. "I can but think," says he, "that much of the recent discussion of the objectivity of value and of value-judgments rests upon a false psychological theory. It rests upon giving certain terms meanings that flow from an introspective psychology which accepts a realm of purely private states of consciousness. . . . To refer value to choice or desire, for example, is in that case to say that value is subjectively conditioned. Quite otherwise, if we have steered clear from such a psychology."[16]

Once granted that there is another way of approaching the problem than the subjective-objective route; i. e., once granted that not all experience involves the knowing relation, one will have little difficulty in seeing that Dewey's claim that value-judgments are practical, is an effective

[14] *Logic*, p. 281ff.
[15] W. H. Sheldon, *The Journal of Philosophy*, 18:309-20.
[16] *Essays in Exp. Logic*, p. 364. See also *Democracy and Education*, p. 195.

refutation to either the claim that they are subjective or the claim that they are objective. Even though one himself does not accept all that Dewey puts into his conclusions, he must then admit, I think, that Dewey is at least consistent with himself. This robbing of the subjective-objective problem of its meaning is by no means the least thing for which students of value have to thank Dewey, as I see it. So long as we did not see that value-judgments arise out of a specific non-cognitive situation in need of adjustment, that it is the first step in an action that seeks to resolve the difficulty, and that it dips then to a non-reflective level where intrinsic goods are again possible—so long did we wander from the meaning of concrete experiences and lose ourselves in the attempt to discover the good, *überhaupt*.

This has been the besetting sin of philosophers beginning with and including Plato.[17] But whether they have been on the search for *"the* end, the *summum bonum*, the final goal" or for inflexible standards with which to sound conduct for "eternal values," they have uniformly returned empty-handed. The sufficient reason why they have failed, says Dewey, is that "in the abstract or at large, there is no such thing as degrees or order of value." "It is reasonable to believe," says he in another place, "that what holds moral knowledge back is above all, the conception that there are standards of good given to knowledge apart from the work of reflection in construction of methods of action."[18] The reason for this faith on Dewey's part he elsewhere indicates in these words: "Physical knowledge did not as matter of fact advance till the dogma of models or forms as standards of knowledge had been ousted. Yet we hold tenaciously to a like doctrine in morals for fear of moral chaos."[19] Just as the abolition of this point of view

[17] *Influence of Darwin on Philosophy*, p. 50.
[18] *Logic*, p. 382.
[19] *Ibid.*, p. 381.

in the natural sciences has led to such phenomenal widening of the boundaries of our knowledge, even "in like fashion," thinks Dewey, "we may anticipate that the abolition of *the* final goal and *the* single motive power and *the* separate and infallible faculty in morals, will quicken inquiry into the diversity of specific goods of experience, fix attention upon their conditions, and bring to light values now dim and obscure."[20] Renouncing, then, once for all "the diversion of intelligence from discrimination of plural and concrete goods . . . which has done more than brute love of power to establish inequality and injustice among men,"[21] and leaving henceforth to "poetry and to art, the task (so inartistically performed by philosophy since Plato) of gathering together and rounding out, into one abiding picture, the separate and special goods of life,"[22] we shall "converge all the instrumentalities of the social arts, of law, education, economics, and political science upon the construction of intelligent methods of improving the common lot."[23]

## III

It is this ringing call for man to live in his own world and this justification for his so doing, that constitutes the essence of Dewey's philosophy. While others have cried "lo, here; lo, there!" Dewey has continually insisted that the kingdom of good is within human experience. It is difficult to see why this course should need to be emphasized. For other-wordliness is supposed to have passed with many other anemic beauties of the mediaeval world. And this is all that Dewey is really saying in his theory of value: values are immanent in human experience. Take them for what they are: if they are many, so much the better; if

[20] *Darwin*, p. 70.
[21] *Ibid.*, p. 75.
[22] *Ibid.*, p. 71.
[23] *Ibid.*, p. 69.

they are found within classes that aforetime were vulgar, judge the classes by the values, rather than the values by the classes. When Dewey insists that values are *here,* he also indicates the very important social doctrine that they are *everywhere* here. No class has a monopoly upon them; if so, the chief task of man is to see that monopoly ended. To make values common to all men, to deepen them, and to guarantee them—this is the threefold problem common to philosophy, to science, to government. This is the Problem of Man. The first step in the solution of the problem is the whole-hearted recognition that values are immanent in human experience, rather than secluded in some transcendental or conceptual realm accessible to common men only through priestly or philosophic or governmental intermediaries. It is this recognition that philosophers have often failed consciously to make. I say "consciously," for since all the values there are, *are* really in human experience, it would not be marvelous if philosophers were found, when off guard, to speak more wisely than when in the cold ecstasy of philosophic sophistication. With this hypothesis let us ramble a bit in the out-of-way preserves of the two modern system-builders.

Of all moderns who have found human experience most lacking, whether taken as a whole or by parts, F. H. Bradley perhaps comes first. His *Appearance and Reality* succeeds so fatally well as practically to mean for man, as Schiller facetiously suggested, the *disappearance* of reality. Begin where he will in the evaluation of human experience, Bradley can find nothing that is not shot through and through with inadequacies, contradictions, infinitude. "Surely," he would say, "nothing here can be thought to be finally valuable." It is only in the Absolute that value can be found (though he even quibbles at predicating good of the Absolute).[24] Here, it would seem, is a complete

[24] *Appearance and Reality,* p. 411.

antithesis to Dewey's theory of value; for Dewey finds all values in human experience, whereas Bradley finds none possible in it. If human experience has any values at all, they can appear as valuable only in the Absolute, never in themselves. Here we see a traditionally recurring effort to belittle man's world carried through to its logical completion. Man's world remains good only for its bads.

But the Absolute, in relation to which alone can anything else possess value, gets its value from its reconciling nature. The Absolute seems the complete embodiment of the seventh beatitude: in it all the contradictions of a bedeviled world are set right. In it all questions cease from troubling and all problems are at rest. But how is this consummation (so devoutly to be wished!) attained? It is attained by the abolition of thought (relationally infected thought!); for the Absolute is not only trans-temporal, trans-spatial, but trans-rational as well.[25] If solution of contradictions is the great end that the Absolute attains, then absence of reflection is the great means through which the great end is reached.

At this stage, the question will inevitably arise: If the solution of contradictions is the end to be sought, and the means to its solution is the absence of thought, then why go to such length to attain what is already present at the outset? For unless the term "thought" be used in such a double sense as really to destroy its meaning, thought certainly does have its *genesis* in just the sort of situation that Bradley thinks it attains only at the end. The appreciative realm in which we daily live, move, and have our being is, as such, non-reflective; and being non-rational, it has neither contradictions nor problems. These do arise, but their birth means its death. We do not value and evaluate the same object at the same time. Whether one recognizes it or not, it is the deep concern with which

[25] *Appearance and Reality*, p. 172.

he views the interruption of his a-logical experiencing by a hostile environment, a concern growing out of the unspeakably, the un-rationally profound significance this mode of experiencing has for his life,—it is this concern, I say, that leads to such persistent efforts, as Bradley's, to think out the contradictions and restore the values. In his description of the Absolute, Bradley has described with vivid accuracy a genuine part, the most meaningful part, of *human* experience. In a paradoxical epigram, half-playfully placed in the preface to his *Appearance and Reality* (an epigram apparently evolved in a moment of detached musing), Bradley has done more than in the entire book that follows accurately to analyze human experience and precisely to indicate the relations between its parts. Says he: *"To love unsatisfied the world is a mystery, a mystery which love* satisfied seems to comprehend."[26] (Italics mine.) Not even the incorrigibly intellectualistic sentence[27] with which he closes the excerpt can obscure the genuine significance that Bradley himself finds in immediate experience, such as is represented by "love." Indeed, Bradley's epigram indicates in a sentence Dewey's own theory of value. Bradley's naïve insight at the beginning states, it seems to me, quite as intelligibly as does his much belabored dialectic that follows for six hundred pages, the nature of thought and reality and their relations to each other. Since Bradley graciously leaves to the reader "how seriously"[28] he shall take that epigram, I for one have long preferred to take it more seriously than I do the remainder of *Appearance and Reality*.

Another corroboration of Dewey's analysis and emphasis that comes from quite as unexpected a source and comes, even as in Bradley's case, from a moment of relaxation

[26] *Appearance and Reality*, p. xv.

[27] "The latter is wrong only because it cannot be content without thinking itself right."

[28] *Appearance and Reality*, p. xv.

rather than from a serious intellectual endeavor, is found in a recent note of Bernard Bosanquet's.[29] Bosanquet, who through labored volumes loses human experience and values in a conceptual Absolute at their close, Bosanquet, whose "inmost aspiration"[30] was, and is, to be able to say to the "critics of Absolutism": "Mark now, how a plain tale shall put you down,"—even Bosanquet has here told so plain a tale as to justify the feeling that the Absolute may be brought down as effectively by its friend as by its enemies. Bosanquet, surveying the *Studio,* feels an obvious impatience with those who (following his own footsteps) would push values out of human experience. "In a world of supreme values, wholly beyond doubt," he found himself saying, "What is the use of talking? Why do we not look?" There before him, the great apostle of the Absolute, were some pictures, "any one of them fit to bring heaven into our time and place." In that moment of insight, he confesses, "that the *inexhaustibleness* of values, of *human experience,* is *altogether beyond the need of reasoning,*" and the thought left him, as he further confesses, "a little indifferent to the *precise remoter* inferences which we may draw from it, and a little impatient of any discussion which implies that we are not *constantly* in presence of *supreme realities* and *immeasurable values.* (All italics mine.) This "Undesigned Coincidence," as Bosanquet entitles his brief note, furnishes further corroboration of the theory of value enunciated in the earlier pages of this paper. Undesigned these corroborations are, but perhaps of greater weight because they *are* designed; for they indicate an *unbiased* recognition of just such a state of reality as Dewey's theory of value has so logically elaborated.

[29] *The Philosophical Review,* 30:216.

[30] *The Principle of Individuality and Value,* p. vi.

## IV

If one must have in every philosophy a *metaphysics*, I wonder if we might not call Dewey's theory of value, his metaphysics? This is suggested to me by the discovery that Dewey is treating in his theory of immediate values what those who have monopolized the term "metaphysics" treat under that head. The Absolute is the metaphysical object, *par excellence;* and Dewey also has his doctrine of the absolute or of absolutes. Every act of thinking finds its specific absolute in the restored immediate experience to which the act is instrumental. Immediate experience *is* absolute in the sense that it is the *end* of the problematic situation, and as such is consequently "invaluable."[31] Immediate experience enjoys the same surcease from the exigencies of thinking as does Bradley's Absolute. It seems to me that all the really distinctive and valuable attributes of the historical Absolute is preserved in Dewey's absolute(s), for the doctrine of the Absolute (absolutists to the contrary notwithstanding) has ever been an effort to guarantee emotional satisfaction through the procedure of rationalization. Dewey guarantees it, not by rationalizing, but by recognizing it for what it is and setting about with scientific foresight to *make* it permanent. In short, Dewey has succeeded in showing how within human experience itself, thought, instead of breeding vast contradictions through its relational nature, is, in the ascent of man, the instrument evolved for the resolution of the difficulties that arise on a lower level. Thought, precisely because it is relational, is the one means through which the absolute(s), which we so much prize, may retain their absoluteness; i. e., may remain unproblematical.

[31] See *Reconstruction in Philosophy*, p. 175.

## V

I wish now to return briefly to Dewey's own theory in order to consider what seems to me the most significant objection made to it, an objection raised curiously enough not more by critics than by Dewey himself. Dewey's theory of value, as we have seen, calls for a dichotomy within experience; i. e., appreciation and reflection. But do these actually exist, separated from each other, as the discussion seems to imply? However appreciative it may be, is not all experience judgmental, *implicitly* at least? It is too easy to beg the question with the use of the term "implicit." That all experience, however purely appreciative, may (and does) become judgmental on occasion, Dewey both admits and affirms. Moreover, he explains what is "the occasion." But if the critics should affirm that all experience is *actually* judgmental, they would be doing no more than Dewey himself seems more than once to admit.[32] With this admission one might still maintain that these two aspects—judgmental and appreciative—predominate in different situations, as Dewey himself affirms. But once admit that all experiences are all the time actually infected with more or less of the judgmental aspect, it seems to me that the distinction loses much of its significance. It may still be justified as an explanatory device, but it can no longer be sharply descriptive of actual experiences. Believing that the division is not only explanatory but accurately descriptive of human experience as well, I do not feel that Dewey need make the admission (if the passages quoted below and other similar ones constitute an admission.)[33]

[32] "No experience having a meaning is possible without some element of thought. But we may contrast two types of experience, according to the proportion of reflection found in them." (*Democracy and Education*, p. 169.)

[33] "That something of the cognitive . . . enters in as a catalyzer . . . in even the most aesthetic experiences, seems to be altogether probable." (*Logic*, p. 394.)

The fact that one is always (during waking hours) thinking, leads too easily to the conclusion that all our experiences are more or less thought experiences. True, of course, that all our experiences are thought experiences while they are being thought about. But the question really is: Are all our experiences all the time being thought about? It seems clear to me that they are not, even to the degree that Dewey thinks probable. Thinking is never of things in general, never *überhaupt*. To the contrary, thinking is always quite utterly specific, as Dewey has repeatedly emphasized. Ignoring the apparent facts of sleep (in which one may afterwards distinguish satisfactory or unsatisfactory rest, though he remember not so much as a dream during the sleeping period), I should say that, since thought is genuinely specific, one is always *experiencing* more than he is thinking about; and this "more" (though it may at any time through developing disharmonies become the object of judgment) is for the time being experienced *only* appreciatively.[24] To use Dewey's illustration, I may *value* my meal while *evaluating* the argument of my friend; or I may *evaluate* grit in my bread while *valuing* the presence of my friend. It seems to me that the two may not only be thought of as separate, but that they are actually quite separate; i. e., that Dewey's oft-repeated dichotomy does constitute a social and ethical metaphysics.

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[24] It is, I think, because he does not clearly recognize this that Mr. Delton Thomas Howard in *Dewey's Logical Theory* (Cornell Studies in Philosophy, No. 11), is betrayed first into admitting that "it is doubtless true that men think only occasionally and with some reluctance" (p. 124), later into the declaration that "there is nothing in evidence to show that thinking is a special kind of activity, which operates now and then" (p. 127), and finally into the distinction that "the moment of *real*, earnest thinking is at the high tide of life, when all the powers are awake and operating" (p. 132).

# HISTORY AND PHILOSOPHY

HAS HISTORY any data or method to contribute to philosophy comparable in importance to the data and method of the physical sciences? I propose to maintain that it has; in spite of the undoubted importance of the physical sciences to philosophy. For the purpose of the argument, philosophy or metaphysics may be taken to mean *the study of the nature and number of what is real;* and this may be held to include the philosophy of Realists and modern Idealists, of Mr. Russell, Mr. Bradley, M. Bergson and Signore Croce. By a mistake of historical interpretation it is frequently supposed that, as physical science advances, so metaphysics recedes; and it is often said that what is now the subject-matter of astronomy or biology or psychology was once subject matter for philosophy. Probably, however, it would be truer to say that when any section of reality or any class of reals comes to be studied in segregation from other sections, when the relation of its elements or its peculiar type of reals comes to be thought of in a special way, then what before was studied simply as real is studied in science as one kind of real. The same "reals" or the same section of reality still continues to be subject-matter for philosophy, as it was before, because it is real: stars, for example, do not cease to be data for philosophy because the laws of gravitation have been discovered. The sphere of philosophy has not shrunk: indeed, in some cases, the new science may in-

crease the data or improve the method of philosophy. New types of reals may come into sight, by the closer inspection in a new science of a separate section of reals. New ways of arriving at truth, especially in some restricted field, may be found in the detailed work of the physical scientist. Aristotle may have been wrong as a metaphysician in his classification of the heavenly bodies as a unique type of reality: but the modern metaphysician as well as Aristotle must have some place in his universe of discourse for the stars. In the same way recent advance in psychology has not deprived the metaphysician of the right or the duty of placing "mind" among the reals of which he must take account, or of showing that there is no ultimate real referred to in the traditional use of the word "mind." Thus the data of philosophy or metaphysics are actually increased by the advance of special sciences; and a new method, which is found to be applicable in a restricted field, may be useful also in the wider field of the study of the real as such. The place of philosophy is not less, but more, important as new data and new methods come to be known.

From these preliminary considerations it will appear in what sense philosophy may be conceived to gain from physical science. But it is essential, further, to distinguish more clearly what is here meant by history. For the purpose of the argument, history may be taken as the name for all those classes of study which divide the field of *knowledge of sections* of realities with the physical sciences. Thus it includes the study of events in the record of human society or individual human life, the study of literature and the non-philological study of language, the study of music and of art generally,—all that field of knowledge which is sometimes called "the humanities," as contrasted with "science." Clearly this distinction is arbitrary. There are (i) some subjects of study which are on the border line, as for example psychology or geography in

its social aspects; and (ii) in "history" there are elements of science, as appears from the use of statistics. It would be difficult, with the use of the terms here suggested, to say whether the theory of evolution is history or science. Nevertheless, the distinction is adequate for its use here; for there is a sufficient exactness in the distinction to allow of showing what the study of art and human affairs may have to add to the data of metaphysics. The kind of realities studied in history are sufficiently distinct from those studied in physical science if we say that they are "mental" realities, but this does not involve that there is no science of mind. The methods of history, as distinct from science, are sufficiently obvious, particularly in regard to that mental activity usually called appreciation. One further preliminary statement is needed. The form of mathematics which is logic, or the logic which is the basis of all thinking, is not to be identified with the physical sciences. No one denies that mathematics in this, the Platonic, sense is fundamental. It contributes data and method to philosophy; but its contribution should not accrue to the credit of science *as opposed* to history; for history, no less than science, must be based upon, and must use, the laws of number, identity, difference and the rest. It is true that, in what is commonly meant by history, logic does not seem to be, even in the vaguest sense, mathematical; but that is in part a defect of historical practice, not a necessary characteristic of history in its truest sense. In any case, no knowledge at all is possible except in dependence upon the processes, and in reference to the realities referred to, in mathematics in this widest sense of the word. We are not, therefore, comparing the data and method of mathematics in this sense with the data of history.

History adds to the data of philosophy (*a*) mnemic causation, in the language of Mr. Russell's *Analysis of Mind*. This is characteristic, not only of individual expe-

rience in sensation, but also of the racial or group experience which is too much neglected by skilled psychologists. Matter, indeed, in the study of physical sciences seems to preserve its past experiences. Metal struck once is in a new state to receive a second blow; but clearly mind includes, or is affected by, or represents, past experience in some way differently from this. The manner in which mind so becomes the past in the present is set out in history. This, of course, does not assume that there is "a" mind, or Mind in the traditional sense; for it is equally true of history, if mind is a perspective or a "section" of material things. That kind of reality, which is referred to when we speak, not of "the real pen," but of "the pen in thought or mind" (what used to be called "the idea of the pen"), is studied in history as part of a process in time. And of that kind of reality one characteristic is mnemic causation, which is of very great importance to philosophy. Indeed anyone, who, with a very full knowledge of chemistry or physiology, set out to reach a philosophy *without* a knowledge of mnemic causation, would be likely to misrepresent "the nature and number of what is real"; nor would infinite progress with his physical science supply the omission, although it might suggest that there was an omission. Causation in the world of chemistry or physics would not indicate the peculiarity of causation in the world of mind or thought. There is, of course, no peculiarly superior status to be given to mnemic causation; but its difference from other types of causation is important for philosophy. The characteristics of this type of causation as presented in history will be described later.

Secondly, (*b*) history adds to the data of philosophy uniqueness or identity of events and of "persons." In history an event or a person is not simply nor chiefly an example of a law or a specimen of a class, as they tend to be regarded in science. Of course, history does include some

study of "laws" in the sequence of events, and many speak as though we could "learn" from history by reference to similarities between our own situation and some other situation in the past. This implies the scientific element in history, but it does not include that other element in history which is referred to in the saying that "history *never* repeats itself." The uniqueness of each moment, or point-instant (in Professor Alexander's language) in the historic series is a fundamental fact. But philosophies much dominated by scientific conceptions seem to imply that we could explain the unique moment or individual by reference to a law or an all-absorbing force or reality within which the uniqueness disappears, or of which the unique may be conceived to consist. Bergson, for example, in his *élan* seems to explain away the uniqueness of the moments in a process; but it is bad philosophy to treat as "explained" what has been omitted, nor is it possible to suppose that uniqueness or individuality is not "objective," but only the creation of a spatializing mind. It is "given" just as obviously and irrefutably as any process; and this *datum* is presented in the study of history. Science also, if it is in the widest sense mathematical, presents the unique and the individual or particular; for in any section of space the points are each unique and particular; and in sciences implying evolution there are point-instants in the process described as development. But in history, especially of human or mind process, the instant in the process of time is in a new aspect seen to be unique and particular. What is this new aspect? It can best be understood by reference to the misleading implications of the common idea of progress. The majority think that one generation exists or works for the next, that the future is in some sense the justification or the explanation of the present and the past; but this is clearly a mistake. Whatever the results of my action upon the fortunes of future generations, I exist, so

to say, in my own right and not for the sake of what is to come. Each event, each instant in historical process is what it is, independently of its relation to other members of the series. The excellence of a state of mind is not entirely to be tested by reference to its results on the future, and this is the kind of excellence to which George Meredith refers when he speaks of a kind of reality:

> "Whose fleetingness is bigger in the ghost
> Than time with all his host."

The importance of time or of "motus," as the scholastics called it, or of *durée,* is well recognized by M. Bergson; but he seems to have misinterpreted the character of time in omitting to give an important and permanent place in his interpretation to the uniqueness of the event or the point-instant. As it has been seen, the phraseology of Professor Alexander in his *Time, Space and Deity* has been used here, and a further conception from the same source may emphasize the importance of history in this matter. He speaks of time as the "mind" of space or of a similarity between time-space and mind-body; but this "mind" aspect of time is more clearly to be seen in history than in science.

Thirdly (*c*) history gives the characteristics of mnemic causality and mind or thought in general. For example, under the general term, history, we have included the study of arts. Arts are peculiar to humanity. The processes and products of the arts are parts of the real world and are important to philosophy. To neglect them in a systematic metaphysics is to omit facts, and they cannot be rendered in the terms of science. A product of art, a painting, a melody or a novel or poem, is not, as Aristotle thought, an imitation, and even the wildest reinterpretation of his terms will not prove him to be on the correct lines for the placing of art-products in the world of realities. That element in the painting which is not the amount of the paint

and canvas nor the spatial relation of the parts, that element which makes us say of it that it is beautiful or not beautiful, is not secondary; nor is it reflection of anything else. It is "in" the painting. It is a unique creation. To know how it came there, who was the painter, and other facts "about" it, is not to know *it;* but *it* is the subject for knowledge for one section of history as the study of "the humanities." Now we have no reason to suppose that any other reality but "man" can produce or appreciate art. The perceptions by animals of sounds or of likenesses in pictorial art is no proof that they perceive or appreciate that particular "it" to which a person who knows what a good painting is refers when he says it is beautiful.

In this section of the data provided by history the "group" characteristics of thought or conation or mental activity in general should be included. All art is social. Indeed all mental activity is social. This may seem to be very obvious; but psychologists, logicians, and even philosophers tend to forget it. For example, in logic it is often said that language is an instrument of thought; but clearly its main characteristic is to be an instrument, not of thought, but of *communication,* and "of thought" only because communication is essential to thought. Indeed, all the so-called laws of thought and rules for deduction and induction are spoken of as if "a" mind were active in a non-mental world; but we know of no such mind. All we know is minds in the plural, always in relation to one another. Conation and, still more, "feeling" are social. They are mental activities only as of *many minds;* and of this fact philosophy must take account. But its importance as a fact in the real world is nowhere more clearly to be seen than in history. Finally, truth, goodness and beauty, all of which are in some sense realities, are social; and cannot be understood except in reference to more than one mind, as Professor Alexander has shown in *Space, Time*

*and Deity.* But the operation of these realities and their connection with non-mental realities are to be seen in history.

This fact, the social character of mental activity, is not only important as a *datum,* but also because it indicates a method in philosophy. The traditional logic as well as the traditional psychology is "atomistic" or individualistic. The judgment is treated as fundamentally an act of "a" mind in a non-mental *milieu;* but clearly no mind exists or acts in that environment or atmosphere. Even the proposition, which Mr. W. E. Johnston, in his new *Logic,* has shown to be an "objective" fact distinct from the judgment, can hardly be understood without reference to the plurality of minds. For example, it is probable that the chief distinction between truth and falsehood, or reality and illusion, arises from the fact that the "real," as an object of thinking or mental activity, is a perspective for *more than one mind.* Illusion is obviously what is *not* for more than one, except in cases where one dominates or expunges the perceptiveness of the "others" in the experience. The interaction of minds is clearly of the first importance to epistemology and philosophical method; and this interaction is shown in history.

Against the arguments used above an objection may be raised as follows: It may be said that great advances in philosophy have followed the discoveries of physical science, but no noticeable effect upon philosophy can be traced to advances in historical research or humanistic studies. To this objection there are at least two replies: First, the actual practice of historians must not be identified with the characteristics of history, and secondly, the experience of the recent past is not typical of the whole of experience, for in the Renaissance and in the eighteenth century history was useful to philosophy. As a preliminary, it must be repeated that advances in philosophy due

to "mathematics" in its widest sense are, of course, incomparably greater than those due to other sources; but, as was said above, what is due to mathematics in this sense must not accrue to the credit of physical science as contrasted with history. As for the practice of historians, it is true that historians are commonly untrained in philosophy or in ethical theory. They accept the social ideas and standards of their grandmothers. They pass or imply ethical judgments without even noticing that they are applying an uncriticized traditional criterion. Nevertheless, history is in essence a sphere for the comparison of ethical standards and for the examination of the character of mind in society. In the Renaissance the humanists, and in the eighteenth century Leibniz and Hume were as much historians as "scientists," in the narrow sense of that word. The data supplied by Hegel, although misinterpreted by him, may fairly be regarded as historical. Finally, one of the defects of contemporary philosophy, especially in regard to the nature of mental activity, is due to the too great dependence on the physical sciences and the too little attention to the data and methods derived from history. Clearly there must be an advance in history itself before it can contribute much to philosophy. The use of records is at present crude, the ethical judgments of historians are commonly primitive, and the criticism of art is "childish"; but in a more advanced stage of "the humanities" philosophy would gain much in data and method. Even as it now stands history has much to offer.

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# THE RELATION OF SPACE AND GEOMETRY TO EXPERIENCE

## VII. Conflicting Measurements

IN OUR last lecture we defined vectors—i. e., directed distances—in terms of such notions as we had already taken as primitive in the definition of parallelism, and of no other notions. We then defined in terms of these notions and that of some particular spatial region alone, which we suppose to be spherical, in all future applications of this definition, the set of all vectors-in-the-sphere: that is, the class of all directed distances inside our sphere. We defined, again introducing no new notions whatever into our definition, the magnitude of each vector inside a given sphere in terms of the diameter of the sphere as a unit, or as we put it, the *index* of a vector-in-the-sphere. This definition of the index of a vector was so framed as to give perfectly unambiguous indices of the vectors in any region whatever, no matter how it might be shaped, in terms of that region, without involving, for example, that this region should have anything at all analogous with a diameter, but it was also so framed that, in an ordinary Euclidean space, if the "sphere of measurement" is really spherical in shape, the system of measurement of vectors-in-itself defined by it will agree completely with the system of measurement that is characteristic of ordinary Euclidean space.

Our purpose in this lecture is to obtain from the theory of measurement developed in the last lecture a general system of measurement by which we can determine the distance between any two points in space. In our last lecture, we left this problem still unsolved in two distinct ways. In the first place, the theory of measurement which we developed in the last lecture only told us how to measure the distance between two points both of which lie within our sphere of measurement. This leaves open the problem how we are to measure distances between two points of which one or both lie outside our standard sphere. In the second place, we have not discussed in any manner whatever the problem, how we are to determine what our standard sphere of measurement is to be, and how it is to be discriminated from other sets of generalized points. These two questions and the further problems to which they give rise form the subject-matter of this and a large part of the following lecture.

As we have just said, our first task is to find a method of extending the system of measurement which we have developed for all distances inside a given sphere to all distances in space, so that we may be able to compare any distance in space with the diameter of our standard sphere. You will remember that we defined a certain vector throughout space as the extension of a certain vector-in-a-sphere R, when and only when in all the cases where two points that are separated by R they are separated by the vector throughout space in question. It is clear that it is only natural to assign to a given vector throughout space that is the extension of R the index of R as an expression of the magnitude of the vector throughout space. Of course, we have said nothing which makes it a consequence of the definition of the extension of a vector-in-a-region that a vector in space cannot be the extension of vectors-in-a-given-sphere having distinct indices,

though this cannot happen if our fundamental notions live up to their names. When this happens, we shall say that the vector in space in question possesses *both* of these indices, at least for the present. Since by this attribution of an index to the extension of a vector in our standard sphere we have found a way of measuring distances between points in any part of space, it might seem that we have obtained a satisfactory definition of a distance, which will apply to any distances whatever. A little reflection, however, will convince one that this is not the case. There are certain vectors which are not the extensions of any vectors in our standard sphere, in general. If our standard sphere is one inch in radius, it is obvious that no distance of two inches can be the extension of any vector inside our standard sphere, for there can be no two points both inside our standard sphere separated by a vector two inches long. The question is therefore before us, how are we to measure in terms of the diameter of our standard sphere distances larger than this diameter? This problem is a particular case of the more general one as to how we measure any magnitude with a scale smaller than the magnitude itself. We have that problem on hand, for example, when we wish to measure a yard and a half of cloth with a footrule. We solve this problem, as a matter of practice, as follows: we first apply the footrule to one end of the piece of cloth, and make a step of one foot. We then start from the end of the piece already measured and make another step of one foot. We find that after making in this manner four distinct successive steps, each a foot in length, provided that all these steps have been taken in a straight line pointing directly towards the further end of the piece of cloth, if we now take a step of only half a foot in length, we shall precisely reach the end of the strip of cloth. That is, if we take four successive steps of the whole length of the footrule, and so dispose

them that we are led by them as near as possible to the end of the strip of cloth, we shall have to take a further step of the length of half a foot, measured by the rule, to reach the end of the strip of cloth. It is not essential, however, that our first four steps should each be a foot in length: we might first have taken a step seven inches in length, measured by the footrule, then one eight inches in length, then one eleven inches in length, then one ten inches in length, then one six inches in length, then one five inches in length, and finally, another step seven inches in length. That is, if we cover the space from one end of the strip of cloth to the other in a finite number of steps which are measurable by our footrule and which are so disposed that the sum of the lengths of these measurable steps is as small as possible, we call this sum of all the lengths of these steps the total length of the strip of cloth.

Let us now return to the problem of the measurement of distances which are too large to fit into our sphere of measurement. Suppose that the distance between the point A and the point B is of this kind. Then it will be of course impossible to go from A to B by a single step which belongs to the extension of some vector situated inside our standard sphere, but it may be possible to make the transition from A to B by the intervention of a finite sequence of successive steps each of which is an instance of the extension of some vector in our standard sphere. Let these steps be instances of the vectors which form the extensions of the vectors-in-our-sphere S, S′, S″, ....., $S^{(n)}$, and let the index of $S^{(k)}$ be $i^{(k)}$, where $S^{(k)}$ stands for the $k$th step in the chain connecting A and B. Let the sum $i+i'+i''+\ldots.+i^{(n)}$ be called I. I represents, then the total length of the chain of linear segments representing the steps by which the transition from A to B is made. The length I evidently depends on the particular chain which we select to connect A and B, and on what particular

vector-in-our-sphere we regard as furnishing each member of our chain as its extension, when a member of our chain represents the extension of two distinct vectors-in-our-sphere. In ordinary geometry, there is a certain minimum length which chains connecting A and B actually possess—that is, I has a certain minimum value, and there are actually chains of a finite number of vectors which are extensions of vectors in our standard sphere which have this value of I as the sum of the indices of their members. Such chains stretch in a straight line from A to B: they exist, for every distance between two points which is in magnitude less than the diameter of our standard sphere may easily be shown to be an instance of the extension of some vector-in-our-sphere, and Archimedes' axiom holds in ordinary geometry—that is, since when any two distances $l$ and $m$ be given, there is some integer $k$ such that the distance $kl$ is greater than $m$, so that we may get anywhere by a finite number of steps as small as we please. If, however, the collection of generalized points which we select as spherical should turn out, after all, not to be spherical, or if our initial relation of apparent intersection among convex solids should belie its name, we have no proof at hand that there exists a chain connecting A and B whose length is actually the shortest that such a chain can have: there may be chains, for example, in which I may be made to assume any value you please greater than two, while there may be no chain for which I assumes precisely the value "two." It would seem highly unnatural to say that in this situation, which, as we have seen, can never occur in ordinary geometry, A is at no distance from B. We wish, therefore, to obtain a definition of the distance between A and B which will be the least possible value of I when such a value exists, and which will be, to put it crudely, sufficiently like that number to be called naturally the distance from A to B when there is no single value of

I which is smaller than all its other values. The quantity which we thus define as the distance between A and B must further be such as always to exist when A and B are ordinary generalized points. Furthermore, it must not be such as to make our system of measurement trivial too often, by causing too many distances equal to zero, or in some similar manner.

In our last lecture, we took notice of the fact that if we are given any set S of positive real numbers, and if all the members of S are less than some given positive number *n*, there is some single positive real number which we may call *x*, which is at least as great as any member of S, but which is also such that if *e* be any positive real number, however small it may be, there is some member of S greater than *n*—*e*. x is called the upper limit or maximum of S. Whether S be made up of numbers all of which are smaller than some fixed real positive number or not, it may be shown in a similar manner that S also determines a single positive real number *v*—which may be zero—which is smaller than any member of S, but which is such that if *e* be a positive real number as small as you please, *y*+*e* is greater than some member of S. This number *y*, which is uniquely determined by the class S is called the *lower limit or minimum of* S. The existence of the lower limit or minimum of any class of positive or zero real numbers may be proved as follows: let S be the class in question, and let S′ be the class of all positive zero real numbers smaller than any member of S. S or S′ may or may not contain any terms. Let us suppose that S is not the null-class, the class with no members, and that it actually contains some terms. In this case, S′ may or may not contain some members. If S′ contains no member, it is obvious that *e*, however small it may be, is greater than some member of S: that is, o+*e* is greater than some member of S. This is true because, by hypothesis, there is no

positive or zero number smaller than every member of S, as such a number would belong to S′, which we suppose without members. Since S is made up entirely of positive or zero real numbers, it can contain no member less than zero. Consequently o is the lower limit or minimum of S. If S′ contains members, let $y$ be the upper limit or maximum of S′. Then $y$ is at least as great as any member of S′, and it follows, since we can easily show that S′ contains members which approach as closely as we choose to the members of S, that there are members of S smaller than $y+e$, however small a positive real number $e$ may be. On the other hand, there can be no member of S smaller than $y$. For suppose that $z$ is such a member of S. Then, in accordance with the way in which we have determined S′, since it is made up of all positive or zero real numbers less than every member of S, there can be no member of S′ greater than or equal to $z$: that is, there can be no member of S′ greater than or equal to $y-e$, where $e$ is the positive number $y-z$. Consequently $y$ fails to satisfy the definition of the upper limit or maximum of S′. But $y$ is by definition the upper limit or maximum of S′, so that our supposition that there is a member of S smaller than $y$ engenders a contradiction. We see as a result of this that $y$ satisfies both of the conditions which go to make up the complete definition of the minimum or lower limit of S, so that there exists a minimum or lower limit of S in this case, as well as in that where S′ has no members. If we now consider the remaining alternative concerning the natures of S and S′—that is, if S contains no members whatever, as is the case when it is made up of all odd multiples of ten or of all integers that are commensurable with $\pi$—we shall, to simplify matters, make an *ad hoc* definition of the lower limit or minimum of S, and shall say that this lower limit or minimum is zero. This latter definition is, it is true, somewhat artificial. If we make this convention,

it follows from what we have said that any conceivable collection of numbers has at least one minimum or lower limit: the uniqueness of this limit, to which we have already referred, may be demonstrated by a very simple proof, quite analogous to that which we gave last time for the uniqueness of the *maximum or upper limit* of a class of positive numbers.

We have just seen how it is always possible to assign to a given set of positive or zero real numbers one and only one positive or zero real number which is smaller than or at least as small as any member of the set, but to which the members of the set approach as near as we please, and we have called this number the minimum or lower limit of the set. Let us see whether we can define the distance between two points as the minimum or lower limit of the set of values of I for different values of I for the various paths connecting the points in question. In the first place, we must show that, provided that there is a smallest value of I, this must be the minimum or lower limit of all the possible values of I for the two points, since this condition is necessary if we are to regard the distance between two points as the length of the shortest path between them, as we do in ordinary space. This is true because the smallest of a set of numbers is at least as small as any member of the set, while there is no degree of approximation with which you cannot make it represent some member of the set, since it is itself a member of the set; consequently the smallest of a set of numbers, provided there is such a number, is the minimum or lower limit of the set. It results from this that the length of the shortest path from A to B —i. e., the least value of I for the two points in question— which we should naturally call the true distance between A and B, if it exists, is precisely that lower limit or minimum of the possible values of I which we have just agreed to call the distance between A and B. However, if there

is no shortest distance between A and B along any path made up of the vectors that we have already measured—that is, if there is no least value of I—there must always, by what we have just seen, be a minimum or lower limit of the values of I, which will have many of the properties that are characteristic of a minimum value of I, and which will satisfy the definition which we have just formulated of the distance between A and B.

Given our sphere of comparison, then, we have thus been able to define in terms only of those fundamental notions which we have already explicitly formulated the distance between any two of those generalized points which correspond to ordinary geometrical points. Upon a slight investigation, we should find that we can prove, independently of any assumptions concerning the formal properties of the objects exemplifying our fundamental notions, a few simple geometrical theorems concerning distances. We can prove, for instance, that if A, B, and C are any three points, the sum of the distance from A to B and that from B to C is not less than that from A to C. We can also prove that the distance from A to B equals that from B to A. We cannot, however, prove from our definitions alone, that the distances that we have so far defined have *all* the formal properties of distances in ordinary geometry. In ordinary geometry, for example, if the mutual distances of four distinct points are given, and the distances of a fifth point from three of these are known, the distance of this fifth point from the remaining vertex of the tetrahedron formed by the four original points is determined to have one of only two possible values, while our definitions do not secure to any distance any precise quantitative relation to other distances, in general.

We have given the definition of distance just developed with the original intention of using it only for those distances that do not fit into our sphere of reference—the dia-

meter of which, by the way, is still the unit of our sphere of measurement. If, however, we examine into the definition, we see that it may also naturally be applied to distances which fit into our sphere of measurement—for any two generalized points which are separated by a vector lying in our sphere of measurement, and consequently by the extension of this vector, are thereby connected by a chain consisting of that single step, and consequently have a distance in the sense in which a distance is defined in terms of a chain of vectors. It is further obvious that in a system of generalized points which behave in a decent geometrical manner, a chain connecting A and B and consisting of a single vector is at least as short as any other chain connecting A and B, so that the magnitude of the vector from A to B is the same as the distance from A to B, in the sense of the minimum length of a chain from A to B. We can consequently throw away our first definition of the distance from A to B entirely, and be sure that if we define all distances in space by chains, our definition will be natural—at least as natural, that is, as our direct definition of distances by means of vectors. We have thus attained a definition of all the distances in space which involves besides the notions that we have taken as primitive only that of a certain standard sphere of measurement.

The question now arises, what sphere is the actual unit sphere of our measurements? If it turns out that we can select no set of generalized points as the unique unit sphere, how are we to determine the various spheres which shall serve as the bases of our measurements, and if these spheres give us different values as expressions of the magnitude of a given distance, how can we reduce the various results which we obtain to a single internally coherent system of measurement? It is easy to indicate two directions in which we may approach this problem, corresponding respectively to the two definitions which we gave of parallel

lines in a preceding lecture. It will be remembered that in the process of obtaining one of these definitions of parallel lines we defined a certain collection of sets of generalized points called a-spheres entirely in terms of our experience of the intersection of convex solids. We saw that if our experience of the intersection of convex solids records two convex solids as intersecting when and only when they both are situated in a certain region of space and approach one another closer than a certain distance which is the same everywhere in space, our a-spheres, *qua* sets of generalized points, will be spheres of a certain uniform size. This result will seem more familiar to you if I put it in another form. Many of you have seen the various processes of the differential and integral treated in the textbooks of the more old-fashioned sort as if they dealt with operations of division or summation among certain entities called infinitesimals. These infinitesimals were regarded as magnitudes, all very small, but equal, in general. By the English mathematicians of the eighteenth century, who, like their philosophical colleagues, were of a more empiricist attitude to their subject than those of the Continent, these infinitesimals with which the calculus seems to deal were regarded as if they were the smallest objects accessible to our direct sensory experience. Since it was an essential property of the infinitesimals, in their mathematical use, that they should be equal, in general, those mathematicians who held this view were driven to the supposition that all just-noticeable sensible objects—all *minima sensibilia*—are equal. Now, it will be remembered that our a-spheres bear a very close analogy to the *minima sensibilia* of the philosophers, in that they mark the lower boundary of the sensibilia we are considering—namely, convex solids —with respect to their magnitude. Our assumption that all a-spheres are equal is consequently one of those things which was involved as an unquestioned presupposition in

the empiricist treatment of the calculus, although it involves the admission of far less than is necessary for the support of that view. The fact that it is such a natural hypothesis, and that it is an element in a view of the greatest historical importance, makes the consideration of a theory of measurement based on this hypothesis a thing of the utmost interest, whether it be strictly true or not that all *minima sensibilia* are what we should ordinarily regard as of equal magnitude. Let us first notice, however, that it is extremely improbable that the delicacy of our sensory discrimination, and as a corollary the size of our *minima sensibilia,* is in any ordinary sense the same throughout all those parts of space which are more directly accessible to our sense-experience.

On the assumption that we could naturally call all a-spheres equal, and that our space has all the normal geometrical properties, it is a matter of indifference which a-sphere we use as a unit of measurement, and all the diameters of all a-spheres are equal. It will not, however, be in general a matter of indifference which a-sphere we take as our standard sphere, if this assumption is not fulfilled. In such a case, it may be possible to get several numbers, all of which express with equal right the distance between two points, A and B, according to the a-sphere which we choose as our unit of measurement. If we are to be able to regard some single number as the only true distance AB, as measured in terms of the diameter common to all a-spheres as a unit, we must find some way of obtaining from all these different measures of this distance some single quantity, which is uniquely determined by all these various measures, and which coincides with their common value if they agree. There is, of course, a certain degree of arbitrariness in our selection of this distance, but one may easily show that if we regard the true distance AB as the minimum or lower limit of all the values which we can get for

the distance AB by using various a-spheres as standard spheres, we shall obtain an entirely unequivocal definition of the distance AB. In the first place, this distance will always exist, be finite, and uniquely determined, since, as we have seen, all these things are true of the minimum or lower limit of any set of positive or zero numbers. In the second place, if all the distances of A from B, measured by a-spheres, agree, they will all coincide with their minimum or lower limit, as one may see on inspection. We have thus obtained a definition of the distance between any two points in space which involves no notion other than that of our experience of the intersection of convex solids, which secures that any two points in space shall be at one and only one distance from one another, and which will completely agree with our usual notions of the distance between two points, provided that our experience of the intersection of convex solids has such properties as we should naturally expect it to have, and provided, moreover, that we can naturally call all *minima sensibilia* equal.

In the system of measurement thus obtained, some of the theorems of ordinary geometry will still hold good, irrespective of any assumptions about the nature of our fundamental experience; for example: the distance AB will equal the distance BA, the sum of the distances AB and BC will be at least as large as the distance AC, and so on indefinitely. For the most part, however, the geometrical properties of our distances will be dependent on the nature of our experience of the intersection of convex solids, and will not, in general, agree with those characteristics of ordinary Euclidean geometry, if our fundamental experience has not such properties as we should naturally expect it to have. For example, the theorem that, if any tetrahedron is given, the distance of a point from one of its corners must assume one of a certain pair of values, once the distance of the point from each of the other three

corners is known, which is true in ordinary geometry, cannot be deduced from our definition of distances alone.

However, the object of this course of lectures is to show how we can eliminate from geometry all presuppositions concerning the particular formal properties of our original notions, and yet define our points, lines, distances, etc., in terms of these in such a manner that all the ordinary formal geometrical properties of these latter entities should follow from their definitions alone. To do this, we must be able to obtain a definition of distance which will depend on our experience of the intersection of convex solids alone, and which will of itself be a sufficient guarantee that our distances satisfy the usual geometrical laws, and it may further be shown that, once we have a perfectly satisfactory definition of all the distances in space, we have it in our power to give perfectly satisfactory definitions of all geometrical entities. As we have just seen, we have not yet succeeded in completely performing this task; the problem, however, is an extremely interesting one, and one of the utmost philosophical importance. Let us turn our attention to it for a little while.

Our last definition of distance has practically indicated to us how we should make a survey of the universe, for it correlates with every pair of ordinary generalized points the distance between them. However, it will be a bad survey, in general. By this I mean distances which it causes to separate points will not check up, will not "gee" with one another. You all know that a surveyor always makes more observations than are merely sufficient to indicate the position of each point he is mapping a single time only: he observes the same point five or six times over from various observation-stations, and records all his observations. In an ideally perfect survey, all these observations would come out in complete accord with one another, but as a matter of fact, they seldom or never come out in com-

plete accord with one another in any actual survey. The several determinations of each triangulation-station indicate on the map, not one point, but a number of points, situated in more or less close proximity to one another. It is a part of the task of the surveyor to fix a single point, which may be said to be the best representative of these several points, in terms of the collection made up of all of them. This he does by means of a certain mathematical theory known as the theory of least squares. This theory does not concern us here, except in so far as it fulfills the function of reducing a set of unharmonious measurements to a harmonious system; we shall have nothing to say concerning its technical details. Its function is to make every single point which forms a station in a certain survey correspond to one point on the map representing the results of this survey, and to one only. Then the surveyor can go and take the distances between these uniquely determined points on his map, and say that they represent—when the appropriate alterations of scale are made—the distances between the points on the earth's surface corresponding to the points on the map, and we may be *a priori* sure that the proper geometrical relations will hold among the distances so determined.

In order to make the discussion of surveying which we have just given more simple to grasp, we have been guilty of a slight inaccuracy of statement. We said that the several determinations of each triangulation station indicate on the map, not one point, but a number of points. As a matter of fact, however, the several determinations of each triangulation-station indicate no specific point whatever on the map until we have already determined a certain correspondence between points on the map and points on the surface of the earth. This presupposes that we are able to locate certain points on the map before others and to discriminate between such as we shall locate first and

those which we locate later. We do not possess, however, any criterion for such a location. It is consequently necessary for us to possess some means of deriving at one blow from our confused measurements of the distances between our stations quantities which we can regard as the expression of the magnitudes of these distances, among which the proper geometrical relations will of necessity hold, if we are to be able to map our triangulation-stations in such a manner that to each triangulation station there will correspond one and only one point on the map. This task can also be accomplished with the aid of the method of least squares, in the case of any survey involving only *a finite number of points.* From any survey, then, however bad it may be, provided that it involves only a finite number of points, a set of finite quantities may be obtained, one of which is correlated with each pair of the points surveyed, and among these quantities the formulae which correlate distances in an ordinary Euclidean space will hold.

However, the bad survey of the universe which was made by our definition of distances in terms of a-spheres involves the measurement, not of a finite number of distances, but of an infinite number of distances, provided that the space that we have defined in terms of our experience of the intersection of convex solids is comparable in richness with the space of ordinary geometry. Now, the method of least squares is unable, as far as I know, to bring order into surveys that are bad at an infinite number of points. We do not at present possess a method of deriving a well-behaved Euclidean set of distances from the hodge-podge set of distances which we obtain, in general, from our definiton of distances in terms of a-spheres. I see no reason, however, for supposing that the problem of obtaining a method which should perform the same function for infinitely irregular systems that the theory of least squares fills for finitely irregular systems should be essen-

tially insoluble. The solution of this problem is of absolutely vital importance for the philosophy of space, for however we may define distance, nothing is more certain than that the distances of points in space, as we first learn and observe them, only approximately satisfy the laws of geometry, yet we are absolutely sure that there are actual distances which our observed distances represent and which satisfy these laws precisely. If we were enabled by a theory analogous to that of least squares, to derive from our chaotic observed distances, distances whose geometrical properties and relations should be secured *a priori*, we could immediately explain this phenomenon, on which so much importance has been laid by philosophers.

In this lecture, we have extended the system of measurement determined by a sphere from its inferior to the whole of space. On the basis of that definition of parallelism which starts with a-spheres, we saw that a system of measurement could be developed, but that this would have certain defects. We saw that this system of measurement does not secure *a priori* the geometrical properties of space, but that it would only require the formation of a branch of mathematics which should deal with problems essentially similar in character to those dealt with in an already existing branch to give us a space whose geometrical properties would be *a priori*. In our next lecture we shall take up the problem of defining distances on the basis of our alternative definition of parallel lines, and we shall finally discuss the philosophic problems to which both these systems of measurement give rise.

## VIII. Geometry an a Priori Science

IN OUR last lecture, we developed a theory of measurement upon the basis of that definition of parallel lines which involved the notion of a-spheres. In this lecture, we shall first develop an alternative theory of measurement, in which we follow out that definiton of parallelism which involved, not only the relation of apparent intersection among convex solids, but also four selected convex solids, which we agreed to call "unit spheres." If the distances of any point in space from the centers of these four spheres are known, and our four spheres, to put it in ordinary geometrical terms, are four spheres whose centers are not all coplanar, the position in space of the point in question is completely determined, and if the positions of two points are determined in this manner, their distance from one another is completely determined. As we intend in what follows to secure the proper geometrical interrelations of our distances, and to cause the theorems of geometry to be satisfied, by means of defining the position of any point in space in terms of its distances from the four centers of our standard spheres, the first notion which we must define is that of the distance of a point from the center of a sphere. Now, it is obvious that, whether a point $x$ is inside a sphere S or not, the furthest point in S from $x$ lies on the line through $x$ passing through the center of S, and is that one of the points on the surface of S and also on the line in question which is further from $x$. If $y$ be the point in S furthest from $x$, it therefore follows that the distance from $x$ to $y$ is the distance from $x$ to the center of S, plus the radius of S. One might consequently think that we

might define the distance of $x$ from the center of S as the distance of $x$ from the most remote point of S, minus the radius of the sphere. We wish, however, to have our definition applicable even where S is not precisely a sphere, and we consequently do not wish to make the assumption that there is any single point of S most remote from $x$ an essential condition of the usefulness of our definition. Now, if there is any single point in S remote from $x$, the distance of this point from $x$ will be the maximum or upper limit of the distances of points in S from $x$. However, it will be much more usual, in general, for there to exist a maximum or upper limit of the distances of points in S from $x$ than for there to exist some single point at precisely that distance from $x$ and belonging to S. We shall consequently regard the distance of a point from the center of S as the maximum or upper limit of the distances of points in S from the point in question, minus the radius of S. Now, if we make S itself our standard sphere of measurement, in the sense explained in our last two lectures, since the diameter of S will be our unit, the radius of S will be one-half. Consequently, it will be natural to regard the distance of a point from the center of S as the maximum or upper limit of the distances of points in S from the point in question, minus one-half. This definition, as may be readily seen, involves no other notions than such as we have already taken explicitly as primitive, together with such notions as "one-half," or "limit," which, as the modern logicians have shown, are purely logical notions.

Given the four equal, non-coplanar spheres that served as the foundation of our second theory of parallelism, we are now able to define the distances of any point we please from the center of each of them, in terms only of such notions as we have already taken explicitly as primitive. It is interesting to note that our definition involves absolutely no reference to such entities as the centers of these

spheres. Be that as it may, we can treat our definitions as if they yielded us the distances of a point in space from four given points in space—the centers of our four spheres. We intend to use these four distances as the coördinates of our points, and to determine the position of any point in space by means of these four distances. We finally desire to determine the distances of any two points in space from one another in terms of the distances of each of the points from each of the four centers of our equal spheres. Our actual coördinates will not, however, be the four distances of a point from our four centers, since we wish our coördinates to be absolutely independent of one another, as this will facilitate such a definition of the distances in space as will secure automatically their satisfaction of the laws that bind distances to one another in ordinary geometry. Now, in ordinary Euclidean space, if we know the distance of a point from each of the vertices of a triangle, we know that its distance from any fourth point in space can assume one of only two possible values. We shall consequently determine a point, not by its distance from each of our four centers, but by its distance from three of our centers, and by whether its distance from the fourth center has its greatest possible value, the other three distances being given, or not. We shall, for example, write the coördinates of a point whose distance from the center of the standard sphere A is *a*, whose distance from the center of the standard sphere B is *b*, whose distance from the center of the standard sphere C is *c*, and whose distance from the center of the standard sphere D has its maximum value, *a*, *b*, and *c* being fixed, in the form $(a, b, c, +)$, while if the distance of our point from the center of D has not its maximum value, and everything else remained unchanged, we should represent our point by $(a, b, c, -)$. In these definitions, we have introduced no notion that we have not already taken explicitly as primitive.

The three coördinates and a sign by which we determine a point are completely independent of one another if our space already obeys the laws of ordinary geometry. If, however, we know the coördinates of two points, we do not yet know, by these data alone, the distance by which the two points are separated. This knowledge depends further upon a knowledge of the shape and size of the tetrahedron formed by the four centers of our spheres of reference—that is, on a knowledge of the remoteness of the four centers from one another. To discover this, we must already possess a definition of the distance between the centers of two unit spheres. A definition which is in every way analogous to the definition which we have already given of the distance of a point from the center of a sphere reads as follows: the distance between the centers of two spheres, S and T, is the maximum or upper limit of the distances between points in S and points in T, whether measured with reference to S or to T as the standard sphere, minus the radius of S plus the radius of T, which is one, since the diameters of S and of T are regarded as possessing a common value, and this common value is taken as our unit of distance.

We now possess sufficient data to transform the system of coördinates which we have already obtained into an ordinary Cartesian system of coördinates. If we form certain functions of the coördinates already defined for a point $p$ in a perfectly determinate manner—which, it is true, are a little too intricate for us to exhibit here—and call these X, Y, and Z, we may be sure that, if our points, lines, distances, etc., have such properties as we should naturally associate with the names we give them; and if our four "spheres" are actually four equal spheres whose centers do not all lie in the same plane, our coördinates X, Y, and Z, will be an ordinary set of rectangular Cartesian coördinates whose axes are determined in a certain definite man-

ner by our four spheres of reference, and whose unit of distance is the common diameter of all four standard spheres. We may also be sure that even if our space is not so completely subject to the ordinary laws of geometry, many of our points will, in general, still determine coördinates, and any point that determines the coördinates X, Y, and Z will determine them uniquely. We shall call the values of X, Y, and Z that are determined by a certain point $p$ the *fundamental coördinates* of $p$.

It is by no means a necessary consequence of the definition of the fundamental coördinates of a generalized point that to a given point there must always correspond some set of coördinates. Let us call those generalized points that have fundamental coördinates *proper* generalized points. By the definition of a proper generalized point, it must have one set of fundamental coördinates and one only. It does not follow from this, however, or from anything else we have yet said, that no two distinct proper generalized points have the same set of fundamental coördinates, whatever the formal properties of our fundamental notions may be. We desire, however, to obtain ultimately a definition of a spatial point which will secure by itself that no two points hold the same position in space—that is, that no two points have the same fundamental coördinates. How shall we do this? We shall do it by introducing a new definition of a point in place of our definition of a generalized point, just as we formerly introduced the definition of a generalized point in place of our very first definition of a point. Our final definition reads as follows: a *revised point,* we shall say, is a class other than the null class, or class without any members—of all the proper generalized points that have a given set of fundamental coördinates. Obviously, we assign this set of coördinates to the revised point in question. It is evident, then, that no two distinct revised points can have

the same set of coördinates, for otherwise they would coincide. A revised point, then, must determine a set of coördinates different from that of any other revised point; it further follows from the mode of formation of a revised point that every revised point has a set of coördinates, and that no revised point can have more than one set of coördinates. The appropriateness of calling revised points points will become obvious if you reflect that in an ordinary, well-behaved space a revised point will contain only one member —a certain generalized point—and that the revised point will consequently represent the same point, in our everyday sense of the term (whatever that sense may be) as the single generalized point which is its member. We have thus obtained a set of entities which can naturally be called points, between which and certain sets of coördinates or triads of numbers there subsists a certain correlation which is one-one, and extends over all of our revised points. In ordinary geometry, the correlation between points and their Cartesian coördinates is not only one-one, but also connects all the points in space with all possible sets of coördinates. Consequently, if our revised points are to have all the formal properties of the points in ordinary geometry, we must show, over and above what has been already shown, that every set of Cartesian coördinates—every triad of real numbers—determines some point. Now, it does not follow from the definition of a revised point that we have just given that every set of Cartesian coördinates determines a point. The manifest and obvious way to remove this imperfection in our system of definition is to find some entities that will fill the gaps in our system of points which are left by the absence of a revised point corresponding to a given set of coördinates. We can do this in the following manner: if to a given set of coördinates there corresponds a revised point, we shall say that the revised point is the point, in our final sense of the word,

that corresponds to it, but if there is no revised point corresponding to a given set of coördinates, we shall call the set of coördinates itself the point that fills the position indicated by our set of coördinates. If we thus interpolate sets of coördinates as points into our system, we shall find that our complete set of points will be in one-one correspondence with our complete set of coördinate triads. We shall thus have obtained a space which agrees perfectly in this respect with the space of ordinary geometry.

Now, it is familiar to all those of you that have had an elementary mathematical training that all the theorems of geometry my be reduced to purely algebraic theorems, entirely independent of space and dependent only on the properties of number and quantity, when once a system of Cartesian coördinates has been set up. It has been shown by the modern mathematical logicians that all theorems that deal with number and quantity alone are theorems of pure logic, and are independent of any concrete experience whatever. Since these things are the case, then once a system of Cartesian coördinates has been defined in terms of our fundamental notions, we have done all that is necessary to prove the *a priori* character of geometry, for we can now so define lines, planes, circles, angles, etc., in terms of our set of coördinates—and ultimately in terms of those notions that we have explicitly taken as primitive —that all the theorems of geometry shall result from these definitions and the laws of algebra alone. Consequently, if the space of our every-day life is constituted of the entities that I have just called revised points and of the number-triads of the space that I have just defined, the theorems of geometry are *a priori* true, even though space is actually a function of experience.

This brings me to the end of the technically logical and mathematical portion of the present course. Let us consider what we have accomplished, and let us see in how

far we have fulfilled the purpose which this course was designed to fulfill, and have proved or rendered more probable the thesis which we set out to prove. The thesis of this course of lectures is that, whereas space is a function of experience, the geometrical properties of space are *a priori* certain, and may be proved without involving any reference to the concrete nature of the experience of which space is a function. We claimed that the geometrical properties of space were due to the method of schematization by which space is obtained as a function of experience. To show that such a situation is possible, one thing that must be done is to exhibit a system in which geometrical properties are the results of a method of schematization applied to an arbitrary subject-matter, and of this method of schematization alone. We can fairly claim to have accomplished this task by exhibiting the spatial system resulting from our second definition of parallelism, for in this we have introduced no notion pertaining to a certain spatial system—no notion, indeed, other than that of a certain relation, which we *call* the relation of experienced intersection among convex solids, and four arbitrary terms or sets of terms that enter into this relation, which we call four equal spheres whose centers do not all lie in a single plane—and notwithstanding the arbitrariness of the formal properties of our fundamental notions, we have so framed our definitions that certain of our entities must of necessity possess the formal properties characteristic of geometrical objects. We can further claim that we have pointed out that it would not require the solution of any problems generally different in character from those solved in already existing branches of mathematics (such as the theory of least squares), to derive a system of entities obeying all the formal properties of geometrical entities from certain relations or facts essentially similar to those more or less directly accessible to our experience, in such a manner

that no presuppositions concerning the formal properties of these relations and facts are involved. That is, we have made it seem very probable that, even if we did already possess a geometry, and had no knowledge whatever of points, lines, planes, etc., we could build up from those facts ascertainable by means of a more or less direct experience, by a method involving only such notions as belong to logic and are not dependent on any concrete experience, a system of entities which one could naturally call lines, points, planes, etc., in such a manner that one could be *a priori* sure that the formal properties of these lines, points, planes, etc., would be those laid down by the laws of ordinary geometry. All this, though suggestive of the actual relations that subsist between geometry and experience, is not in any way conclusive evidence as to the nature of these relations. We have given no reason to justify us in supposing that the entities that we have called points, lines, planes, etc., are the same entities that we call by those names in our every-day life. Indeed, the feeling of artificiality that haunts us at every step of the ground we have covered seems to forbid this view, and I think that one will be perfectly justified if he categorically denies that the particular entities that we have called by various geometrical names are those that are called by those names in our every-day life. But the fact that there is nothing inherently impossible in the formatioin of a space whose geometrical properties shall be *a priori* certain, yet which will be a function of experience alone, and the absence of any other existing view of the relation between space and experience that will explain the association of certain geometrical entities with certain empirically known physical entities, and in addition thereto the non-experimental nature of geometry, entitles us to say that we have rendered the view that geometry deals with some schematization, some systematization, some arrangement of experi-

ence highly probable. We have given good reasons for supposing that the apriority of geometry is genuine, but that it is an apriority of method, not of subject-matter, and we have illustrated how an apriority of method which is not apriority of subject-matter is possible in the field of geometry.

We have the question still before us: if space, as we have said, is a schematization of experience, what sort of a schematization is it actually, and how is this schematization related to that which we have exhibited in this course of lectures? The first thing to notice is that it is very highly probable that the schematization by which the space of our every-day life is formed is probably not a fixed, immutable schematism, for which there is one analysis that is always right, while all the other accounts of its structure are always wrong. For instance, the schematization by which the space of a carpenter is reached is necessarily different from that by which a physicist attains his space: one might almost say that with the physicist, his straight lines are his light-rays, for a light-ray in a vacuum is the criterion by which he tests the straightness of anything else, and consequently it is at least reasonable to suppose that the complicated synthesis and organization of experience through which he must go in order to obtain a light-ray—since light-rays, as such, are not given in experience—must play a part in the construction of the lines of his geometry; while this obviously cannot be the case with the carpenter, whose criteria of straightness are simply chalk-lines and T-squares. It seems likely, moreover, that even with one and the same person, the schematism he uses may vary with the problem which he is attacking: on one occasion, the physicist may find it more convenient to regard a point as if it were built up by some process of organization similar to that by which he organizes his experiences under the form of a light-ray; on another

occasion, his method of schematization may be more analogous to that by which he obtains a gravitational line of force. It may be incorrect to speak of "space" as something unique: all that we ought to mention may be "spaces."

On the other hand, I feel fairly sure that certain tasks will have to be performed by all the divers methods of schematization that may lead us from experience to space. For example, the problem to which we devoted our third and fourth lectures—the problem, namely, of proceeding beyond that portion of space which is more or less directly accessible to our experience, and of obtaining definitions of points and of lines which will yield us all the points and lines in space—is one that is inseparable from any method of schematization by which we obtain space from experience, and I believe that the method of making this extension of space that we there developed is essentially similar to that which we use in our actual processes of obtaining space. The problem of an infinite theory of least squares, of turning a bad survey of the universe into a good one, is one that we must meet in almost any process of deriving space from experience. The problem of distinguishing parallel lines from intersecting lines is another problem that is not confined to the system developed in this course. It will thus be true that though there are many different systems of schematization which, on certain occasions, yield us space as a function of experience, and while the method which we have described in the preceding chapters may be different from all of these, there will be certain problems that run through all these methods and the one which we have given, so that an exposition of the method which we have given cannot but throw light on the methods of schematization by which space is actually obtained.

It may seem to you that these tasks which I have just mentioned as essential steps in the attainment of space as a function of experience are, as our introspection shows

us, not performed consciously, while they are too intricate to be performed unconsciously. I think, however, that both of these statements are open to question. In the first place, whenever a man thinks clearly enough to give a definite criterion for the straightness of a line, or the intersection of two lines, or whatever other geometrical property you please of physical, sensible objects, he is really simply rendering explicit some stage in the synthesis of experience which constitutes his space, and in so far as he has succeeded in forming his criteria of straightness, intersection, etc., into a system which must, by an internal necessity, be coherent, he has succeeded in making the complete method of schematization of his space determinate. We have already referred in this course of lectures to the fact that the surveyor, when he obtains a coherent map by a definite method from a mass of disharmonious data, is doing something of essentially the same nature as what we have been trying to do in this course of lectures, for he possesses a method by which he can deduce a map in which the laws of geometry hold from the most hopelessly incoherent and lawless set of observations or experiences. In the second place, it is a well-known fact that the complexity of a mental process is in itself no absolute bar to its unconsciousness. A man may have the most complicated set of criteria by which he determines the straightness of a given line or the flatness of a surface, but he may never have introspectively considered the nature of this process. It may turn out that many or all of the geometrical properties of his space result from these criteria alone, and that these criteria or definitions of straightness, flatness, etc., are so formulated that they constitute a perfectly coherent system, without his ever explicitly knowing that he uses these criteria or definitions at all. It does not seem to me at all unlikely that the mathematician or physicist, who unconsciously performs such intricate processes

of reasoning as differentiation or integration, should perform unconsciously some comparatively simple synthesis whereby space, with all its geometrical properties, is obtained as a function of experience. As to the non-mathematician, who is unable to follow a complicated train of reasoning even consciously, and who can formulate only vague and unclear definitions, there is no reason to suppose that in the space which he obtains as a result of his own process of synthesis from experience the laws of geometry hold in any but a rough and vague and rough way. In short, it is perfectly possible for the methods of schematization by which space may be obtained from experience to be unconscious, at least in the sense that we never see it as a systematic whole.

In closing this course, I wish to make a few remarks about the manner in which the theory of space we have developed has answered that problem which we found the Kantian view of space unable to meet. I refer to the problem of the correspondence of certain physical objects with certain spatial entities. How can we use one of the geometrical lines that we might define in terms of our revised points, for instance, as a criterion of the straightness of such a convex solid as a mark on the blackboard with a piece of chalk may be? The answer to this question is very simple. We have already seen how we can regard a convex solid as a set of points, in our first sense, and we have also seen how we may make a set of points in our first sense determine uniquely a set of generalized points. We are thus able to regard a convex solid as if it were a certain set of generalized points. It is easy to carry this process a little further, and to regard a convex solid as a certain set of revised points determined uniquely by it, which may be regarded as the set of all the revised points inside the convex solid in question. We can then give a simple mathematical definition for the accur-

acy with which the convex solid in question represents a straight line: we can say, for example, that the linearity of a set of points is the ratio of the longest linear segment connecting two points of the region or set to the longest perpendicular segment that contains two points of the set. On Kant's view, it is essentially impossible that anything of the sort should be done, since his lines, forming portions of space, can be given to us only *a priori,* so that we cannot recognize them in an empirical situation, and associate them in a definite manner with empirical objects. We have thus completely established our case against Kant; our case against Mach and the whole Empiricist school of philosophers of mathematics has already been made out.

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# THE FAILURE OF CRITICAL REALISM

*EX PEDE HERCULEM.* A realism in which perception, as such, is incapable of ever apprehending material existents *qua* existents is—obviously—completely debarred from direct awareness of the physical Universe. It may, by means either of explicit arguments or of instinctive and irresistible belief, posit the reality of such a Universe; but only as a World with which the knower can never come into absolutely primal contact and direct relation. Thus the crucial test of realist epistemology is its treatment of the process and content of perception. All the later stages in the development of knowledge—thought, universals, mind—though of more vital importance to philosophy, are nevertheless very largely determined by the conclusions of the prior inquiry into perception. If then the content apprehended in and through perception is never under any circumstances ontologically identical with the material world, realism degenerates into Noümenalism.

This appears to me to be the logical outcome, on its own confession, of the noetic system developed in *Essays in Critical Realism.*[1] It concludes with a "lack of absolute certainty"—a lack not merely occasional but eternal; "what we contemplate is, in the case of perception, *appar-*

[1] Macmillan, 1920. For the sake of brevity I have treated the volume as one whole, without explicit reference to the various authors, except in the few cases in which purely individual opinions are advanced. It may be noted here that perception occupies a "fundamental position in any theory of knowledge," (p. 97).

*ently* the very physical object itself."[2] But—*semper, ubique et ab omnibus—only* apparently; for "this outer existent is not literally grasped; only its what, essence, character is grasped; our knowledge is obviously fallible";[3] and so far as perception, the root of knowledge, carries us, not only sometimes, but always, fallible. Perception is *la connaissance—la verité—manquée.*

This result springs from a distinction which, in spite of some ambiguity, is presented by Professor Drake as fundamental—the distinction, *i. e.,* between "characteristics of objects" and "objects themselves." "Characteristics appear to us; objects themselves," on the other hand, "do not get within our consciousness; the physical existent itself does not get within experience; knowledge is a beholding of (its) what, its nature."[4] But since "objects themselves" are indubitably real—for otherwise ontological realism vanishes—and since further our instinctive belief in their reality is well founded—for if not, then epistemological realism vanishes also—"objects" and their "characteristics" must constitute, in virtue of their dual relation to consciousness, experience or knowledge, two separate categories. The existence of "objects themselves is private, incommunicable";[5] what is known is their characteristics, essence, nature.

This is the case if we take the passage as it stands; but if we compare it with page four, there is some self-contradiction. For (p. 24), "we directly perceive . . . the character of objects"; on the other hand (p. 4), "what we perceive . . . is the outer object itself." Taking these two statements together, then, "objects" and "character of

[2] The word "object" bears a special meaning; *cf.* p. 213. I have therefore used "thing" or "existent" in referring to physical or material objects in the ordinary sense.

[3] Pp. 32, 20.

[4] Pp. 24, 29.

[5] P. 24. *Cf.* p. 240: "The physical thing and the psychic state . . . are unquestionably two and mutually independent." I may refer to a previous discussion of Dr. Strong's theory in *The Journal of Philosophy,* Vol. XVI, p. 428.

objects" are, for perception, identical, and the distinction I have just referred to disappears completely. If, indeed, we confine ourselves, as the volume before us explicitly does, to the epistemological standpoint, it is difficult to conceive how any such distinction between objects and characteristics can ever arise. Unless the content of our knowledge of the world, however extensive and abstract it may in the end become, is based ultimately on the content perceived, it can arise in some way or other only from the activity of the mind; and when this possibility is excluded, noümenalism can be avoided only by identifying the perceived content ontologically, to some extent at least, with physical things themselves, which then become (again to some extent) one with their "characteristics."

"To some extent," that is; for this principle precludes neither the logical grading of the essentiality of the thing's characteristics, nor the ascription to it, for certain limited purposes, of attributes which are scarcely essential at all; as, *e. g.*, in poetry or metaphor or scientific hypothesis. But just as the issue of paper currency must repose on a gold reserve, so all such mental or knowledge processes must be based ultimately on an ontological core of reality directly perceived—a core or nucleus which is surprisingly small, but which is none the less indispensable; as, once more, a very restricted gold reserve would serve to maintain, under an ideal economic system, a disproportionately large credit system. If, on the other hand, perceived content is confined to characteristics which are always distinct from real things themselves, then the "gold reserve" of knowledge becomes wholly mythical, or it is at best locked up in an invulnerable safe the combination which is not only unknown but unknowable.

It is to this result that the epistemology of Critical Realism leads; for if we consider the main trend of the volume, the ambiguity referred to may be disregarded. There is

a universal, fundamental, distinction between "actual characteristics of objects" and "objects themselves"; it is no mere question of degree, of more or less, of extent and inclusiveness. The crucial passage in this respect occurs on page twenty-four: "So far as perception gives us accurate knowledge, it does so by causing the actual characteristics of objects to appear to us. The objects themselves . . . do not get within our consciousness." It becomes necessary therefore to examine the ground for asserting the real existence of objects, as distinct from their apparent, or perceived, or known, characteristics.

The assertion, we find, rests on an instinctive feeling or belief. "We instinctively feel these appearances to be the characters of real objects. . . . We may consider our instinctive and actually unescapable belief justified."[7] This standpoint undoubtedly expresses a most important principle; but its true significance demands careful analysis. For there is a sense in which all our beliefs and philosophical conclusions are unescapable, and (from that point of view) instinctive. Whatever we really believe, we believe because, in the end, we can not help so doing; there is always some ground, logical or otherwise, which compels us to modify or abandon hitherto accepted conclusions; no one seriously adopts a new position except on grounds which—whatever their precise nature may be—he feels to be irresistible. But it is the peculiar task of philosophy to criticize such bases for belief; and having once undertaken this examination philosophy can not, *as philosophy,* rest finally upon any basis which is merely instinctive and nothing more. This does not mean that an instinctive belief is never satisfactory and must never be entertained; if that were the case life would at once lose many of its

[6] I do not see how perception can be said, in any literal sense, to cause anything essential; the phraseology appears rather unfortunate.

[7] P. 6. *Cf.* p. 195, "our instinctive assertion of" the physical world, which we "affirm through the pressure and suggestion of experience." See also note 11 below.

most precious elements; it simply implies that it is never satisfactory *philosophically,* and can never be accepted as a final *philosophic* deliverance. There is a whole universe of difference between an instinctive belief accepted purely as instinctive, and an instinctive belief for which some non-instinctive basis can be exhibited; the second may be philosophical, but the first can never be so.

Of this principle the varieties of belief in the real existence of physical things is an outstanding example. Everyone begins with a purely instinctive belief in their reality;[8] but in the case of the "average man" or "naïve realist" this has become more or less rationalized, in so far as he bases his belief upon his "senses," which he means perception. And every form of Realism merely carries farther —it does not reject or subvert, as does, *e. g.,* subjectivism or solipsism—the critical analysis thus initiated in ordinary experience;[9] no other course is indeed open to it, unless it rejects perception and its content *in toto.* For (to revert to our illustration) while the naïve realist far overestimates the amount of his gold reserve, so that much of what he accepts must be rejected as spurious or as impure, none the less must some minimum of precious metal be preserved intact as the ultimate basis of the whole system of knowledge. All Realisms, in other words, must rest finally—exactly as does naïve Realism—upon perception and its content[10]—upon a content more deeply criticized and more rigidly tested; and this analysis must be carried through to a final verdict. Either this content is—as naïve Realism regards it—itself ultimate physical reality, although the conditions governing both its reality and our knowledge of it are more truly defined; or it is not ultimate physical reality. If then this negative standpoint is adopted, two further alternatives arise. Either ultimate physical reality is

[8] *Cf.* James, *Principles of Psychology,* Vol. II, p. 287.

[9] *Cf.* "Critical realism . . . is a criticism of naïve realism." *Op. cit.,* p. 189; and p. 196.

never ontologically identical with the content of perception, and then—since there is no other mode of knowing it directly—this reality is noümenal; or we fall back once more—at the end as at the beginning—on an instinctive, but non-philosophical, belief in the known existence of physical reality. This in itself is never presented as the content of perception; in no other way is it possible for it to appear to us; and still we believe in its known existence instinctively—simply and solely, *i. e.*, because we can not help doing so.

This is the dilemma which faces, it seems to me, Critical Realism. If it maintains its universal distinction between physical things themselves beyond our consciousness, and their perceived or apparent characteristics, then (as I have argued throughout) it becomes a noümenalism. But if on the other hand it founds its affirmation of the known existence of physical reality on instinctive belief, then it forfeits all title to be regarded as a philosophic system, despite whatever other merits it may possess.[11] Or, if it still claims to be such, it can be at its best only a philosophy of the content of perception—this content being, confessedly, always distinct from ultimate physical reality itself.

It is true that the critical realist, despite this distinction, claims that perception is directly of things (or objects) and not merely of content. "He sees no reason why (the object of perception) should not be called the direct object" (p. 103). But in thus insisting on the directness of perception, it is only in precisely the same sense that memory, thought and conception are direct; in this respect these diverse processes or activities are all alike; "the principle is not different in perception." Such a standpoint, however, seems to rest upon a fundamental misinterpreta-

[10] Adopting the usual distinction between "perception" as a process and "percept" as content.

tion of the relation between these purely ideational processes and perception itself. For while these non-perceptual functions are undeniably direct, still their directness is in its nature far removed from the immediacy of perception, inasmuch as the former necessarily succeed and are based on the latter, and always, implicitly or explicitly, refer back to it for their own substantiation. They can not sustain a conflict with veridical perception, however remote and abstract may be the ideal content wherewith they supplement it. *If* perception is direct, then memory and thought—so far as they remain concerned with the same objects and are free from error—will be direct also. But their immediacy is the result or consequent of the prior immediacy of perception; it is wholly illogical therefore to adduce it as evidence for its own basis; the argument is obviously circular and assumes the very point at issue.

Ideation then always refers back to perception.[12] Its directness is really of a secondary order as compared with the primary immediacy of perception. What then of perception itself? Does it repose upon any cognitive function or activity more fundamental than itself? Naïve realism replies in the negative; for it perception, even when corrected and supplemented by memory, conception, and thoughts, find its ultimate guarantee in further perception and in that alone. The standpoint of the natural sciences, in their relation to physical reality, is the same. In the end, and so far as it is at all possible, science also appeals to observation and experiment—that is to perception characterized by the utmost delicacy and exactness. This principle is in no way contradicted by the use of scientific concepts. For these, when they transcend the abstractness of

[11] Historically, this standpoint closely resembles that of Stoic philosophy, which regarded some "phantasies"—the Greek equivalent of our sense-data—as creating an irresistible belief, not only in their own existence, but in that of their external objective causes..

[12] *Cf.* "Realism takes its start from perception. . . . I have dwelt upon perception because of its fundamental position in any theory of knowledge." *Op. cit.*, pp. 89, 97.

pure mathematics and logic, are derived, and indeed can only be derived, from the prior content of sense-perception. In this respect there has recently occurred a marked change in the attitude of scientific investigators. Molecules, atoms and electrons—perhaps even the aether—are no longer universally regarded as unreal abstractions or as mere "conceptual formulae of calculation."[13] Both in themselves and in their spatial arrangements they are now placed, by the physicists most closely concerned with the investigation of their properties, on the same footing and in the same existential category as the common objects of everyday experience. This holds true even of the recent developments in the physical theory of relativity, despite its extreme abstraction and abstruseness. "The conception simultaneous . . . does not exist for the physicist until he has the possibility of discovering whether or not it is fulfilled as an actual case . . . (until) he can decide by experiment";[14] that is again, from the standpoint of epistemology, by perception. This is the case then so far as naïve realism and physical science are concerned; while if we turn in the opposite direction and trace perception, as so many psychologists do, to some origin in mere sensational content the result is pure subjectivism—the very antithesis of all realism.

For naïve realism, further, this finality of perception is absolute—or very nearly so; and in this, of course, lies its patent defectiveness. *All* that it contains or reveals or yields to knowledge is, with but slight exception and qualification, ontologically and existentially identical with physical reality; even the modern physicist does not hesitate to assume that with sufficiently heightened powers of vision, either natural or artificial, what we should actually see would be the real molecule or atom itself. *Everything* cir-

[13] *Cf.* Dr. A. N. Whitehead's recent volume, *The Concept of Nature*, Chaps. 1, 2; also *Nature*, Nov. 6, 1919, p. 230.
[14] Einstein, *Theory of Relativity*, p. 22.

culates, we may say, in gold currency, except for a few spurious coins and the minimum of paper. But in this respect naïve realism is obviously far too sanguine; investigation quickly detects a large admixture of mere tokens and bank notes as it were; part of least of what perception yields is far from being the identical physical thing itself; naïve realism therefore must be supplanted by some more refined type of epistemological theory.

This leads us to what is, in my opinion, the radical defect of Critical Realism. Just as naïve realism is too absolute in one direction, so Critical Realism goes to too great an extreme in the other. It carries its critical process—in principle perfectly legitimate—altogether too far; so far indeed that it defeats its own aim and undermines its own basis. It saws away the branch which supports it and so falls headlong into noümenalism.

For *every* element which was identified by the naïve realist with physical reality now becomes sharply distinguished from that reality and is regarded as "content"—content which means or indicates or even reproduces physical reality,[15] but which can never be, under any circumstances whatever, existentially identical with it. Of the perceived characteristics regarded originally as existentially inherent in the physical thing not a single one remains—they are all transferred to the category of datum or content, and are therefore wholly distinct from the thing itself. Every element, regarded as a physical quality of the material thing, is thus deprived of its existential physical status and becomes a datum. Every coin, in other words, tendered for examination by the naïve realist is degraded into a mere token. Certainly by its means his activities can go on quite as well as before; but neither singly nor in their totality are these tokens ever identical

[15] "The content which we apprehend must have the property of reproducing something about the object, of conveying in its own medium the form of the object." P. 218.

with the gold currency which he believed himself to be handling. He is, however, not a cent the poorer; his wealth remains there intact, to be just as readily used and computed, if he will but recognize that this can be done only by means of scrip instead of gold. He is given, in short, securities which are absolutely sound, but which are also absolutely irredeemable.

The insistence on the non-physical character of all perceptual content is indeed so emphatic as almost to amount to sheer subjectivism. It is, in the first place, "impossible to identify either the datum or the images . . . with the object. . . . The quality-group found in perception is not physical."[16] And it is extremely interesting to trace the manner in which, as the development of Critical Realism proceeds, this merely negative description of the quality-group as non-physical imperceptibly changes into its definite and positive designation as subjective; so that the philosopher, in his dread of the Scylla of naïve realism, is inevitably overwhelmed in the Charybdis of subjectivism, from which he attempts to escape on a crazy raft of representationism.

The prominence thus attained by these subjective and representative elements in the entire system is sufficiently evident in the following passages; and if it be objected that these are isolated and severed from their context, the reply is that they are cumulative, and that nothing in the context qualifies their definiteness. We find then that "the content with which we automatically clothe these (physical) realities is subjective . . . Knowledge is the insight into the nature of the object that is made possible by the contents which reflect it in consciousness."[17] Here is the dawn of representativeness—the object is reflected in consciousness; after which, *facilis descensus Averno*. For "what

[16] P. 96.

[17] Pp. 197, 200. *Cf.* p. 212, "the content is mental," and p. 240, "psychic state or sensation."

appear to us as physical things are in themselves of psychic nature," and "experience indicates an actual, causally-based agreement between the physical existent perceived and the content of perception. . . . The organism has perfected the agreement between the subjective datum and the object of perception";[18] and thus the earlier metaphor of "reflection" has given place to a more definite and intelligible "agreement." Finally, and almost literally,

> Last scene of all,
> That ends this strange eventful history,
> Is second childishness and mere oblivion,
> Sans teeth, sans eyes, sans taste, sans everything.

For "the knower is confined to the (subjective) datum, and can never literally inspect the existent which he affirms and claims to know."[19] How, in that case, he can ever cognize the "agreement" asserted must remain a mystery; "the situation is unique," continued Professor Sellars, whose position here is stated most explicitly: *"Internally,* or *in the percipient* himself, we have the content of perception."[20]

Thus it becomes perfectly clear that all the elements within his perceptual field which are regarded by the naïve realist as existential constituents of the physical thing, or evens as in their totality constituting that thing, become transferred, without any exception whatever, to the category of non-physical content—content which is, further, subjective, internal, and in one way or another representative or reproductive of the thing itself. This is, I think, an accurate summary of the argument thus far; and taking it as a whole, we are irresistibly reminded of the conclusion of the classic encounter between Alice and the Cheshire Cat. It will be recalled that the Cheshire Cat slowly faded away, till finally nothing remained within Alice's perceptual field

[18] Pp. 202, 203, 240. *Cf.* p. 218, "the content must have the property of reproducing something about the object."

[19] P. 203.

[20] P. 196: italics mine.

except its grin; and in precisely the same way does the physical thing lose all its perceived characters (as physical) so that they are all converted into subjective, internal, content; there is not even a grin left. "I have seen" said Alice, who was of course a naïve realist, "a cat without a grin, but never a grin without a cat." We are not told what become of the vanished animal; but if a critical realist had been present, he would have assured Alice not only that the cat still existed, but also—what is here the fundamental point—that she still continued to perceived it exacly as before! For the conversion into content in no degree interferes with perception; it is rather the means by which it is effected. "I perceive" maintains Professor Sellars, "concrete *things* . . . co-real with the percipient, and independent of him in exactly the same way and to the same degree that they are independent of one another. . . . Thinghood and perception go together";[21] and as we have seen already, Professor Pratt insists that perception remains direct.

The terminology here is of fundamental importance. It is not that we know things,[22] or believe in their existence, or assume them. It is true that, in a phrase reminiscent of Hegel, "we are compelled to think the object";[23] but this is only in the sense that things are perceived by the critical realist to the same degree and with the same actuality of details—though not in the same manner—as by the naïve realist. It is the essence of Critical Realism that, whatever be the true theory of perception, there is absolutely no difference whatever between what the critical realist actually perceives and what the naïve realist perceives—of course excluding, in both instances alike, ordinary error and illusion. The *result* in each case is precisely the same, for each perceives real physical things.[24]

[21] Pp. 196, 197.
[22] On p. 205 we find "I know an object." I consider this later.
[23] P. 198.
[24] Or objects; the distinction is immaterial here.

That is the essence of the claim to be perceptual realism; the only difference being that one understands how it occurs, while the other altogether misinterprets the true character of the processes involved.

It seems to me, however, that the two principles which in conjunction form the basis of Critical Realism—that is the subjectivity or internality of all perceptual content, and the physical nature (in the sense of naïve realism) of the things or objects perceived—can not logically be held at the same time; the maintenance of either necessarily involves the abandonment of the other. If all the physical qualities of the naïve realist are truly mere character complexes which constitute subjective content, then *physical things* can not be perceived, although they may possibly be known, or postulated, or believed in—this would depend on the meaning given to these terms. On the other hand, if physical things are perceived—again in the sense of naïve realism—then some concessions must be made to its standpoint as a serious theory of knowledge. Its "qualities" must be, to some degree at least, not merely representative or reproductive or reflective of the thing, but ontologically identical with it.

For if, as Critical Realism contends, *all* the content involved is, and always has been, subjective, then, although there certainly might have been realists of some type or another, there could never have been any naïve realists in the world at all; and since critical realism is confessedly a development from the primary naïve form, there could have been no critical realists either. In other words, those problems which actually confront epistemology would never have arisen. For those problems originate in the contrast which is set up by naïve realism between objective and subjective, externality and internality, matter and mind, or whatever other terms express this dichotomy. It rests upon certain deliverances of perceptive experience;

it can indeed have no other possible basis, since, as we have seen repeatedly, perception is accepted as foundational of knowledge. But if now the whole content of perception is subjective, it would have been impossible for this contrast, in its actual form, ever to have been in any way established, simply because there would have been nothing whatever in the universe of perception to suggest it to the experient—it would have had no perceptive basis actually, and it could not conceivably have acquired any other. It cannot be argued that it arises from the spatiality of the universe; for all the character traits of perceived space, like those of every other perceived object, are (by the critico-realist hypothesis) themselves subjective. Universal subjectivity of the basal elements of experience, in short, would render impossible (actually) and inconceivable (theoretically) that contrast between objective and subjective which has, as a matter of fact, actually occurred in the evolution of knowledge; for naïve realism, despite all its defects, is after all one marked stage in that evolution.[25]

This conclusion appears to me to vitiate the entire critico-realist standpoint; and further considerations may be advanced in its support. It may be argued that the mere practical demands of individual life and of mutual intercourse necessitate that category of physical objectivity which the realist—either naïvely or critically—employs; but I do not think this need necessarily be the case at all. It can make no difference, for low types of conscious experience, whether the "world" is all subjective or all objective; life would proceed equally well in either case, since neither category can have any significance for the experi-

[25] On account of its fundamental importance in epistemology I may be permitted to refer to a prior consideration of the subject, from a different point of view, in *Mind*, vol. xxvii, p. 304. *Cf.* "We mean independent objects and we interpret these objects in terms of ideas." (P. 194.) My contention may be put in the form that if these "independent objects" are physical, no notion of them could ever have arisen.

ent; while theoretically to regard all the content at that stage as subjective is a gratuitous assumption. In the opposite direction we find that when mind reaches its highest levels the mathematician, the abstract thinker, the poet and artist, perhaps even the true mystic, live in and efficiently react to a realm which wholly lacks physical objectivity. All that is theoretically necessary then for such efficient reaction is the selection—consciously or otherwise —of certain elements out of the whole which serve as types, quite irrespective of *physical* objectivity and subjectivity, and which other elements then represent or reflect. The symbols of the mathematician may be cited as one instance of this general procedure.

This naturally leads to the representative function which, as earlier quotations have shown, is assigned by critical realism to subjective perceptual content. Professor Drake accepts—but only provisionally—the well-known distinction between primary and secondary qualities; and this raises a dilemma already familiar in one form or another. The desk (pp. 23, 24), "really is oblong, but not in itself black, except in the sense that it has characteristics which cause the character-trait black to appear to us." If then the characteristics which cause the trait black to appear are themselves black, as (it would seem) other allied characteristics are oblong, where is the necessity for the trait at all? It is wholly gratuitous to lay down *a priori limits* to what lies within the capacity of the mind to perceive; and it seems sheer superfluity to require a black content whereby to perceive a black object whose blackness is similar to that of the content. But on the other hand, if the object's characteristic is not black, in the same way as the content is black, then what is it? Obviously, we can not tell; our ignorance is complete and final, for we can neither perceive nor conceive its quality or nature;

which means that the thing is, to that degree, a noümenon and perception a pure misnomer.

But, as a matter of fact, critical realism does assert *a priori* limits to the capacity of perception. "What," asks Professor Sellars, "is the fundamental postulate of knowledge? It is the cognitive value of the idea."[26] But this principle, when thus advanced as the basis of a theory of perception, plainly commits realism in advance to complete dependence upon subjective ideas; it postulates an epistemology resting at bottom on ideas—in other words, it begs the main issue from the very outset. The sole fundamental postulate of epistemology is, not the cognitive value of ideas, but the cognitive value of knowledge itself. And this is not the mere tautology that, at first sight, it may appear to be; it means that knowledge, whatever be the precise character of its contributory processes, is always knowledge of reality. Some ideas most certainly have cognitive value; it may indeed prove to be true that nothing has cognitive value except ideas. But such a principle can be adopted only at the conclusion of the inquiry, as a logically established result; it can not be advanced at the outset, as a fundamental postulate. To do so is merely to base realism firstly on that radically defective sensationalistic tradition which vitiates so much current psychology, and which it should be the first task of realism to criticise and confute; and secondly on a misinterpretation—for which, however, some historic justification may be pleaded —of the true meaning of "idea."

In conclusion then it seems to me that realism must make substantial concessions to the naïve realist and—though not of course uncritically—adopt more generally his practical standpoint. At *some* levels of experience the content of perceived physical reality must be identical and consubstantial with the content of experience, and this

[26] P. 198.

without the mediation of any subjective or internal content of any type whatever in any kind of representative or reflective or reproductive capacity. It may easily prove to be the case that the volume of the content in question is, relatively to the whole, very small; but there is an infinity of difference between very little and none at all. We know that in the living organism a minute proportion of certain ions or of vitamines makes all the difference between life and death; and similarly some minimum of physical reality, perceived directly, not merely in the critico-realist sense, but independently of the critical realist's subjective and representative perceptual content, plays an analogous part in realistic epistemology.

A final remark on terminology may not be out of place here. Modes of consciousness vary correlatively with the content; and it seems to me to lead to some confusion if we speak of "knowing" a physical object or thing. Epistemology is, in its own way, a science; and quite apart from any theory of perception, I would suggest that scientific precision requires us to say that we "perceive" physical things; that is, the sole type of consciousness relevant to them is perception, with its subsidiary aspects of sensation and apperception. It will be admitted that we can neither imagine nor conceive existing physical objects, because these functions lack proper sensational content; and this is equally true of knowledge as such. What we "know" is a truth or principle, fact or judgment. Perception is certainly implicit judgment, and knowledge consists in rendering explicit what is there implicit. But still there is an essential difference between implicit and explicit; and to ignore this—to say that we "know" or "think" the object—appears to involve a serious confusion of perfectly distinct categories.

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# EINSTEIN'S THEORY OF RELATIVITY CONSIDERED FROM THE EPISTEMOLOGICAL STANDPOINT*

## VI. Euclidean and Non-Euclidean Geometry

IN THE preceding considerations, however, we have taken up only incidentally an achievement of the general theory of relativity, which, like scarcely a second, seems to involve a "revolution of thought." In the working out of the theory, it is seen that the previous Euclidean measurements are not sufficient; the development of the theory can only take place by our going from the Euclidean continuum, which was still taken as a basis by the special theory of relativity, to a non-Euclidean four-dimensional space-time continuum and seeking to express all relations of phenomena in it. Thus a question seems answered physically which had concerned the epistemology of the last decades most vitally and which had been answered most diversely within it. Physics now proves not only the possibility, but the reality of non-Euclidean geometry; it shows that we can only understand and represent theoretically the relations, which hold in "real" space, by reproducing them in the language of a four-dimensional non-Euclidean manifold.

The solution of this problem from the side of physics was, on the one hand, for a long time hoped for as keenly,

* Translated by W. C. and M. C. Swabey.

as, on the other hand, its possibility was vigorously denied. Even the first founders and representatives of the doctrine of non-Euclidean geometry sought to adduce experiment and concrete measurement in confirmation of their view. If we can establish, they inferred, by exact terrestrial or astronomical measurements, that in triangles with sides of very great length the sum of the angles differs from two right angles, then empirical proof would be gained that in "our" empirical space the propositions not of Euclidean geometry, but of one of the others were valid. Thus, *e. g.*, Lobatschefsky, as is known, used a triangle $E_1$ $E_2$ S, whose base $E_1$ $E_2$ was formed by the diameter of the orbit of the earth and whose apex S was formed by Sirius and believed that he could, in this way, prove empirically a possible constant curvature of our space. (48.) The fallacy in method of any such attempt must be obvious, however, to any sharper epistemological analysis of the problem and it has been pointed out from the side of the mathematicians with special emphasis by H. Poincaré. No measurement, as Poincaré objects with justice, is concerned with space itself but always only with the empirically given and physical objects in space. No experiment therefore can teach us anything about the *ideal* structures, about the straight line and the circle, that pure geometry takes as a basis; what it gives us is always only knowledge of the relations of material things and processes. The propositions of geometry are therefore neither to be confirmed nor refuted by experience. No experiment will ever come into conflict with the postulates of Euclid; but, on the other hand, no experiment will ever contradict the postulates of Lobatschefsky. For granted, that some experiment could show us a variation in the sums of the angles of certain very great triangles, then the conceptual representation of this fact would never need to consist in, and methodologi-

cally could not consist in, changing the axioms of geometry, but rather in changing certain hypotheses concerning physical things. What we would have *experienced*, in fact, would not be another structure of space, but a new law of optics, which would teach us that the propagation of light does not take place in strictly rectilinear fashion. "However, we turn and twist," Poincaré therefore concludes, "it is impossible to attach a rational meaning to empiricism in geometry." (72, p. 92ff.) If this decision holds and if it can be proved, on the other hand, that among all possible self-consistent geometries the Euclidean possesses a certain advantage of "simplicity" since it defines the minimum of those conditions under which experience is possible in general, there would then be established for it an exceptional position from the standpoint of the critique of knowledge. It would be seen that the different geometries, which are equivalent to each other from a purely formal standpoint, as regards their logical conceivability, are yet distinguished in their fruitfulness in the founding of empirical science. "The geometries are distinguished from each other in principle," one can conclude, "only by reference to their epistemological relation to the concept of experience; for this relation is positive only in the case of the Euclidean geometry."[29]

In connection, however, with the new development of physics in the general theory of relativity, this epistemological answer seems to become definitively untenable. Again and again the fact has been appealed to in the controversy concerning the epistemological justification of the different geometries that what determines value must not be sought in formal but in transcendental logic; that the compatibility of a geometry with experience is not involved but rather its "positive fruitfulness," *i. e.*, the " founding of

[29] *Cf.* Hönigswald (32); on the following *cf.* Bauch (1), p. 126ff.

experience," that it can give. And this latter was thought to be found in Euclidean geometry. The latter appeared as the real and unique "foundation of possibility of knowledge of reality," the others, on the contrary, always as only the foundations of the possible. But with regard to the extraordinary rôle that the concepts and propositions of Riemannian geometry played in the grounding and construction of Einstein's theory of gravitation, this judgment cannot be supported. Supported by the same logical criterion of value, one now seems forced rather to the opposite conclusion: non-Euclidean space is alone "real," while Euclidean space represents a mere abstract possibility. In any event, the logic of the exact sciences now finds itself placed before a new problem. The fact of the fruitfulness of non-Euclidean geometry for physics can no longer be contested, since it has been verified, not only in particular applications, but in the structure of a complete new system of physics; what is in question is the explanation to be given to this fact. And here we are first forced to a negative decision, which is demanded by the first principles of the theory of relativity. Whatever meaning we may ascribe to the idea of non-Euclidean geometry for physics, for purely empirical thought, the assertion has lost all meaning for us that any space, whether Euclidean or non-Euclidean, is the "real" space. Precisely this was the result of the general principle of relativity, that by it "the last remainder of physical objectivity" was to be taken from space. Only the various relations of measurement within the physical manifold, within that inseparable correlation of space, time, and the physically real object, which the theory of relativity takes as ultimate, are pointed out; and it is affirmed that these relations of measurement find their simplest exact mathematical expression in the language of non-Euclidean geometry. This language, how-

ever, is and remains purely ideal and symbolic, precisely as, rightly understood, the language of Euclidean geometry could alone be. The reality which alone it can express is not that of things, but that of laws and relations. And now we can ask, epistemologically, only one question: whether there can be established an exact relation and coördination between the symbols of non-Euclidean geometry and the empirical manifold of spatio-temporal "events." If physics answers this question affirmatively, then epistemology has no ground for answering it negatively. For the "a priori" of space that it affirms as the condition of every physical theory involves, as has been seen, no assertion concerning any definite particular structure of space in itself, but is concerned only with that function of "spatiality" in general, that is expressed even in the general concept of the linear element *ds* as such, quite without regard to its character in detail.

If it is seen thus, that the determination of this element as is done in Euclidean geometry, does not suffice for the mastery of certain problems of knowledge of nature then nothing can prevent us, from a methodological standpoint, from replacing it by another measure, in so far as the latter proves to be necessary and fruitful physically. But in either case one must guard against taking the "pre-established harmony between pure mathematics and physics," that is revealed to us in increasing fulness and depth in the progress of scientific knowledge, as a naïve copy theory. The structures of geometry, whether Euclidean or non-Euclidean, possess no immediate correlate in the world of *existence.* They exist as little physically in things as they do psychically in our "presentations" but all their "being," *i. e.,* their validity and truth, consists in their ideal *meaning.* The existence, that belongs to them by virtue of their definition, by virtue of a pure logical act of

assumption is, in principle, not to be interchanged with any sort of empirical "reality." Thus also the applicability, which we grant to any propositions of pure geometry, can never rest on any direct coinciding between the elements of the ideal geometrical manifold and those of the empirical manifold. In place of such a sensuous congruence we must substitute a more complex and more thoroughly mediate relational system. There can be no copy or correlate in the world of sensation and presentation for what the points, the straight lines and the planes of pure geometry signify. Indeed, we cannot in strictness speak of any degree of similarity, of greater or less difference of the "empirical" from the ideal, for the two belong to fundamentally different species. The theoretical relation, which science nevertheless establishes between the two, consists merely in the fact, that it, while granting and holding fast to the difference in content of the two series, seeks to establish a more exact and perfect correlation between them. All verification, which the propositions of geometry can find in physics, is possible only in this way. The particular geometrical truths or particular axioms, such as the principle of parallels, can never be compared with particular experiences, but we can always only compare with the whole of physical experience the whole of a definite system of axioms. What Kant says of the concepts of the understanding in general, that they only serve "to make letters out of phenomena so that we can read them as experiences" holds in particular of the concepts of space. They are only the letters, which we must make into words and propositions, if we would use them as expressions of the laws of experience. If the goal of harmony is not reached in this indirect way, if it appears that the physical laws to which observation and measurement lead us cannot be represented and expressed with sufficient exactitude and simplicity by a given system

of axioms, then we are free to determine which of the two factors we shall subject to a transformation to reëstablish the lost harmony between them. Before thought advances to a change of one of its "simple" geometrical laws it will first make the complex physical conditions that enter into the measurement responsible for the lack of agreement; it will change the "physical" factors before the "geometrical." If this does not lead to the goal and if it is seen, on the other hand, that surprising unity and systematic completeness can be reached in the formulation of the "laws of nature" by acceptng an altered conception of geometrical methods, then in principle there is nothing to prevent such a change. For if we conceive the geometrical axioms, not as copies of a given reality, but as purely ideal and constructive structures, then they are subjected to no other law than is given them by the *system* of thought and knowledge. If the latter proves to be realizable in a purer and more perfect form by our advancing from a relatively simpler geometrical system to a relatively more complex, then the criticism of knowledge can raise no objection from its standpoint. It will be obliged to affirm only this: that here too "no intelligible meaning can be gained" for empiricism in geometry. For here, too, experience does not *ground* the geometrical axioms, but it only makes from among them, as various logically possible systems, of which each one is derived strictly rationally, a certain selection as to their concrete use, as to the interpretation of phenomena.[30] Here, too, Platonically speaking, phenomena are measured by Ideas, by the foundations of geometry, and these latter are not directly read out of the sensuous phenomena.

But when one grants to non-Euclidean geometry in this sense meaning and fruitfulness for physical experience, the general methodic difference can and must be urged, that

[30] On this relation of the problem of metageometry to the problem of "experience," *cf.* esp. Albert Görland (28, p. 324ff.)

still remains between it and Euclidean geometry. This difference can no longer be taken from their relation to experience, but it must be recognized as based on certain "inner" moments, *i. e.*, on general considerations of the *theory of relations*. A special and exceptional logical position, a fundamental simplicity of ideal structure, can be recognized in Euclidean geometry even if it must abandon its previous sovereignty within physics. And here it is precisely the fundamental doctrine of the general theory of relativity, that, translated back from the language of logic and general methodology, can establish and render intelligible this special position. Euclidean geometry rests on a definite axiom of relativity, which is peculiar to it. As the geometry of space of a constant curvature O, it is characterized by the thorough-going relativity of all places and magnitudes. Its formal determinations are in principle independent of any absolute determinations of magnitude. While, *e. g.*, in the geometry of Lobatschefsky, the sum of the angles of a rectilinear triangle is different from 180° and indeed the more so, the more the surface area of the triangle increases, the absolute magnitude of the lines enters into none of the propositions of Euclidean geometry. Here for every given figure a "similar" can be constructed; the particular structures are grasped in their pure "quality," without any definite "quantum," any absolute value of number and magnitude, coming into consideration in their definition. This indifference of Euclidean structures to all absolute determinations of magnitude and the freedom resulting here of the particular points in Euclidean space of all determinations and properties, form a logically positive characteristic of the latter. For the proposition, *omnis determinatio est negatio,* holds here too. The assumption of the indeterminate serves as the foundation for the more complex assumptions and deter-

minations, that can join on to it. In this sense, Euclidean geometry is and remains the "simplest," not in any practical, but in a strictly logical meaning; Euclidean space is, as Poincaré expresses it, "simpler not merely in consequence of our mental habits or in consequence of any direct intuition, which we possess of it, but it is in itself simpler, just as a polynomial of the first degree is simpler than a polynomial of the second degree." (*72*, p. 67.) This logical simplicity belonging to Euclidean space in the system of our intellectual meanings wholly independently of its relations to experience, is shown, *e. g.*, in the fact that we can make any "given" space, that possesses any definite curvature, into Euclidean by regarding sufficiently small fields of it from which the difference conditioned by the curvature disappears. Euclidean geometry shows itself herein as the real geometry of infinitely small areas, and thus as the expression of certain elementary relations, which we take as a basis in thought, although we advance from them in certain cases to more complex forms.

The development of the general theory of relativity leaves this methodic advantage of Euclidean geometry unaffected. For Euclidean measurements do not indeed hold in it absolutely but they hold for certain "elementary" areas, which are distinguished by a certain simplicity of physical conditions. The Euclidean expression of the linear element shows itself to be unsatisfactory for the working out of the fundamental thought of the general theory of relativity, since it does not fulfill the fundamental demand of retaining its form in every arbitrary alteration of the system of reference. It must be replaced by the *general* linear element ($ds^2=\sum_1^4 g_{\mu\nu}dx_\mu dx_\nu$), which satisfies this demand. If, however, we consider infinitely small four-dimensional fields, it is expressly demanded that the pre-

suppositions of the special theory of relativity, and thus its Euclidean measurements shall remain adequate for them. The form of the universal linear element here passes over into the Euclidean element of the special theory when the ten magnitudes $g$, which occur in this as functions of the coördinates of particular points assume definite constant values. The physical explanation of this relation, however, consists in that the magnitudes $g_{\mu\nu}$ are recognized as those which describe the gravitational field with reference to the chosen system of reference. The condition, under which we can pass from the presuppositions of the general theory of relativity to the special theory, can accordingly be expressed in the form that we only consider regions within which abstraction can be made from the effects of fields of gravitation. This is always possible for an infinitely small field and it holds further for finite fields in which, with appropriate choice of the system of reference, the body considered undergoes no noticeable acceleration. As we see, the variability of the magnitudes $g_{\mu\nu}$, which expresses the variation from the homogeneous Euclidean form of space, is recognized as based on a definite *physical* circumstance. If we consider fields in which this circumstance is absent or if we cancel it in thought, we again stand within the Euclidean world. Thus the assertion of Poincaré that all physical theory and physical measurement can prove absolutely nothing about the Euclidean or non-Euclidean character of *space,* since it is never concerned with the latter but only with the properties of *physical reality in space* remains thus entirely in force. The abstraction (or, better expressed, the pure function) of homogeneous Euclidean space is not destroyed by the theory of relativity, but is only known as such through it more sharply than before.

In fact, the pure meaning of geometrical concepts is not limited by what this theory teaches us about the conditions of measurement. These concepts are indeed, as is seen now anew, neither an empirical datum nor an empirical *dabile,* but their ideal certainty and meaning is not in the least affected thereby. It is shown that in fields where we have to reckon with gravitational effects of a definite magnitude, the preconditions of the ordinary methods of measurement fall aside, that here we can no longer use "rigid bodies" as measures of length, nor ordinary "clocks" as measures of time. But this change of relations of measurement does not affect the calculation of space, but the calculation of the physical relation between the measuring rods and rays of light determined by the field of gravitation. (*Cf.* 83, p. 85ff.) The truths of Euclidean geometry would only be also affected if one supposed that these propositions themselves are nothing but generalizations of empirical observation, which we have established in connection wth fixed bodies. Such a supposition, however, epistemologically regarded, would amount to a *petitio principii.* Even Helmholtz, who greatly emphasizes the empirical origin of the geometrical axioms occasionally refers to another view, which might save their purely ideal and "transcendental" character. The Euclidean concept of the straight line might be conceived not as a generalization from certain physical observations, but as a purely ideal concept, to be confirmed or refuted by no experience, since we would have to decide by it whether any bodies of nature were to be regarded as fixed bodies. But, as he objects, the geometrical axioms would then cease to be synthetical propositions in Kant's sense, as they would only affirm something that would follow analytically from the concepts of the fixed geometrical structures necessary to measurement. (30a, II, 30.) It is, however, overlooked by

this objection that there are *fundamentally synthetic forms of unity* besides the form of analytic identity, which Helmholtz has here in mind and which he contrasts with the empirical concept as if the form of analytic identity were unique, and that the axioms of geometry belong precisely to the former. Assumptions of this sort refer to the object in so far as in their totality they "constitute" the object and render possible knowledge of it; but none of them, taken for itself, can be understood as an assertion concerning things or relations of things. Whether they fulfill their task as moments of empirical knowledge can be decided always only in the indicated indirect way: by using them as building-stones in a theoretical and constructive system, and then comparing the consequences, which follow from the latter, with the results of observation and measurement. That the elements, to which we must ascribe, methodologically, a certain "simplicity," must be adequate for the interpretation of the laws of nature, can not be demanded *a priori*. But even so, thought does not simply give itself over passively to the mere *material* of experience, but it develops out of itself new and more complex *forms* to satisfy the demands of the empirical manifold.

If we retain this general view, then one of the strangest and, at first appearance, most objectionable results of the general theory of relativity receives a new light. It is a necessary consequence of this theory that in it one can no longer speak of an immutably given geometry of measurement, which holds once for all for the whole world. Since the relations of measurement of space are determined by the gravitational potential and since this is to be regarded as in general changeable from place to place, we cannot avoid the conclusion that there is in general no unitary "geometry" for the totality of space and reality, but that, according to the specific properties of the field of

gravitation at different places, there must be found different forms of geometrical structure. This seems, in fact, the greatest conceivable departure from the idealistic and Platonic conception of geometry, according to which it is the "science of the eternally existent," knowledge of what always "is in the same state" (ἀεὶ κατὰ ταὐτὰ ὡσαύτως ἔχον). Relativism seems here to pass over directly into the field of logic; the relativity of places involves that of geometrical truth. And yet this view is, on the other hand, only the sharpest expression of the fact that the problem of space has lost all ontological meaning in the theory of relativity. The purely methodological question has been substituted for the question of being. We are no longer concerned with what space "is" and with whether any definite character, whether Euclidean, Lobatschefskian or Riemannian, is to be ascribed to it, but rather with what use is to be made of the different systems of geometrical presuppositions in the interpretation of the phenomena of nature and their dependencies according to law. If we call any such system a particular "space," then indeed we can no longer attempt to grasp all of these spaces as intuitive parts to be united into an intuitive whole. But this impossibility rests fundamentally on the fact that we have here to do with a problem, which as such stands outside the limits of intuitive representation in general. The space of pure intuition is always only *ideal,* being only the space constructed according to the laws of this intuition, while here we are not concerned with such ideal syntheses and their unity, but with the relations of measurement of the empirical and the physical. These relations of measurement can only be gained on the basis of natural laws, *i. e.,* by proceeding from the dynamic dependency of phenomena upon each other, and by permitting phenomena to determine their positions reciprocally in the space-time mani-

fold by virtue of this dependency. Kant too decisively urged that this form of dynamic determination did not belong to intuition as such, but that it is the "rules of the understanding" which alone give the existence of phenomena synthetic unity and enable them to be collected into a definite concept of experience. (*Cf.* above, p. 79.) The step beyond him, that we have now to make on the basis of the results of the general theory of relativity, consists in the insight that geometrical axioms and laws of other than Euclidean form can enter into this determination of the understanding, in which the empirical and physical world arises for us, and that the admission of such axioms not only does not destroy the unity of the world, *i. e.*, the unity of our experiential concept of a total order of phenomena, but first truly grounds it from a new angle, since in this way the particular laws of nature, with which we have to calculate in space-time determination, are ultimately brought to the unity of a supreme principle,—that of the universal postulate of relativity. The renunciation of intuitive simplicity in the picture of the world thus contains the guarantee of its greater intellectual and systematic completeness. This advance, however, can not surprise us from the epistemological point of view; for it expresses only a general law of scientific and in particular of physical thought. Instead of speaking ontologically of the being or indeed of the coexistence of diversely constituted "spaces," which results in a tangible contradiction, the theory of relativity speaks purely methodologically of the possibility of necessity of applying different measurements, *i. e.*, different geometrical conceptual languages in the interpretation of certain physical manifolds. This possible application tells us nothing concerning the "existence" of spaces, but merely indicates that by an appropriate choice of geometrical presuppositions certain physical relations,

such as the field of gravitation or the electromagnetic field, can be described.

The connection between the purely conceptual thought, involved in the working out of the general doctrine of the manifold and order, and physical empiricism (*Empirie*) here receives a surprising confirmation. A doctrine, which originally grew up merely in the immanent progress of pure mathematical speculation, in the ideal transformation of the hypotheses that lie at the basis of geometry, now serves directly as the form into which the laws of nature are poured. The same functions, that were previously established as expressing the metrical properties of non-Euclidean space, give the equations of the field of gravitation. These equations thus do not need for their establishment the introduction of new unknown forces acting at a distance, but are derived from the determination and specialization of the general presuppositions of measurement. Instead of a new complex of things, the theory is satisfied here by the consideration of a new general complex of conditions. Riemann, in setting up his theory, referred to its future physical meaning, in prophetic words of which one is often reminded in the discussion of the general theory of relativity. In the "question as to the inner ground of the relations of measurement of space," he urges, "the remark can be applied that in a discrete manifold the principle of measurement is already contained in the concept of this manifold, but in the case of a continuous manifold it must come from elsewhere. Either the real lying at the basis of space must be a discrete manifold or the basis of measurement must be sought outside it in binding forces working upon it. The answer to this question can only be found by proceeding from the conception of phenomena, founded by Newton and hitherto verified by experience and gradually reshaping this by facts that cannot be explained

from it; investigations, which, like the one made here, proceed from universal concepts, can only serve to the effect that these works are not hindered by limitations of concepts and the progress in knowledge of the connection of things not hindered by traditional prejudices." (77.) What is here demanded is thus full freedom for the construction of geometrical concepts and hypotheses because only thereby can physical thought attain also full effectiveness, and face all future problems resulting from experience with an assured and systematically perfected instrument. But this connection is expressed, in the case of Riemann, in the language of Herbartian *realism*. At the basis of the pure form of geometrical space a real is to be found in which is to be sought the ultimate cause for the inner relations of measurement of this space. If we carry out, however, with reference to this formulation of the problem, the critical, "Copernican," revolution and thus conceive the question so that a real does not appear as a ground of space but so that space appears as an ideal ground in the construction and progress of knowledge of reality, there results for us at once a characteristic transformation. Instead of regarding "space" as a self-existent real, which must be explained and deduced from "binding forces" like other realities, we ask now rather whether the *a priori* function, the universal ideal relation, that we call "space" involves possible formulations and among them such as are proper to offer an exact and exhaustive account of certain physical relations, of certain "fields of force." The development of the general theory of relativity has answered this question in the affirmative; it has shown what appeared to Riemann as a geometrical hypothesis, as a mere possibility of thought, to be an organ for the knowledge of reality. The Newtonian dynamics is here resolved into pure kinematics and this kinematics ultimately into

geometry. The content of the latter must indeed by broadened and the "simple" Euclidean type of geometrical axioms must be replaced by a more complex type; but in compensation we advance a step further into the realm of being, *i. e.*, into the realm of empirical knowledge, without leaving the sphere of geometrical consideration. By abandoning the form of Euclidean space as an undivided whole and breaking it up analytically and by investigating the place of the particular axioms and their reciprocal dependence or independence, we are led to a system of pure *a priori* manifolds, whose laws thought lays down constructively, and in this construction we possess also the fundamental means for representing the relation of the real structures of the empirical manifold.

The realistic view that the relations of measurement of space must be grounded on certain physical determinations, on "binding forces" of matter, expresses this peculiar double relation one-sidedly and thus, epistemologically regarded, inexactly and unsatisfactorily. For this *metaphysical* use of the category of ground would destroy the *methodological* unity, which should be brought out. What relativistic physics, which has developed strictly and consistently from a theory of space and time measurement, offers us is in fact only the combination, the reciprocal determination, of the metrical and physical elements. In this, however, there is found no one-sided relation of ground and consequent, but rather a purely reciprocal relation, a correlation of the "ideal" and "real" moments, of "matter" and "form," of the geometrical and the physical. In so far as we assume any division at all in this reciprocal relation and take one element as "prior" and fundamental, the other as "later" and derivative, this distinction can be meant only in a logical, not in a real sense. In this sense, we must conceive the pure space-time manifold as the logi-

cal *prius;* not as if it existed and were given in some sense outside of and before the empirical and physical, but because it constitutes a principle and a fundamental condition of all knowledge of empirical and physical relations. The physicist as such need not reflect on this state of affairs; for in all the concrete measurements, which he makes, the spatio-temporal and the empirical manifold is given always only in the unitary operation of measurement itself, not in the abstract isolation of its particular conceptual elements and conditions.

From these considerations the relation between Euclidean and non-Euclidean geometry appears in a new light. The real superiority of Euclidean geometry seems at first glance to consist in its concrete and intuitive determinateness in the face of which all "pseudo-geometries" fade into logical "possibilities." These possibilities exist only for thought, not for "being"; they seem analytic plays with concepts, which can be left unconsidered when we are concerned with experience and with "nature," with the synthetic unity of objective knowledge. When we look back over our earlier considerations, this view must undergo a peculiar and paradoxical reversal. Pure Euclidean space stands, as is now seen, not closer to the demands of empirical and physical knowledge than the non-Euclidean manifolds but rather more removed. For precisely because it represents the logically simplest form of spatial construction it is not wholly adequate to the complexity of content and the material determinateness of the empirical. Its fundamental property of homogeneity, its axiom of the equivalence in principle of all points, now marks it as an abstract space; for, in the concrete and empirical manifold, there never is such uniformity, but rather thorough-going differentiation reigns in it. If we would create a conceptual expression for this fact of differentiation in the sphere of

geometrical relations themselves, then nothing remains but to develop further the geometrical conceptual language with reference to the problem of the "heterogeneous." We find this development in the construction of metageometry. When the concept of the special three-dimensional manifold with a curvature O is broadened here to the thought of a system of manifolds with different constant or variable curvatures, a new ideal means is discovered for the mastery of complex manifolds; new conceptual symbols are created, not as expressions of things, but of possible relations according to law. Whether these relations are realized within phenomena at any place only experience can decide. But it is not experience that grounds the content of the geometrical concepts; rather these concepts foreshadow it as methodological anticipations, just as the form of the ellipse was anticipated as a conic section long before it attained concrete application and significance in the courses of the planets. When they first appeared, the systems of non-Euclidean geometry seemed lacking in all empirical meaning, but there was expressed in them the intellectual preparation for problems and tasks, to which experience was to lead later. Since the "absolute differential calculus," which was grounded on purely mathematical considerations by Gauss, Riemann and Christoffel, gains a surprising application in Einstein's theory of gravitation, the possibility of such an application must be held open for all, even the most remote, constructions of pure mathematics and especially of non-Euclidean geometry. For it has always been shown in the history of mathematics that its complete freedom contains the guarantee and condition of its fruitfulness. Thought does not advance in the field of the concrete by dealing with the particular phenomena like pictures to be united into a single mosaic, but by sharpening and refining its own means of determination while

guided by reference to the empirical and by the postulate of its determinateness according to law. If a proof were needed for this logical state of affairs, the development of the theory of relativity would furnish it. It has been said of the special theory of relativity that it "substituted mathematical constructions for the apparently most tangible reality and resolved the latter into the former." (38, p. 13.) The advance to the general theory of relativity has brought this constructive feature of it more distinctly to light; but, at the same time, it has shown how precisely this resolution of the "tangible" realities has verified and established the connection of theory and experience in an entirely new way. The further physical thought advances and the higher universality of conception it reaches the more does it seem to lose sight of the immediate data, to which the naïve view of the world clings, so that finally there seems no return to these data. And yet the physicist abandons himself to these last and highest abstractions in the certainty and confidence of finding in them reality, *his* reality in a new and richer sense. In the progress of knowledge the deep words of Heraclitus hold that the way upward and the way downward are one and the same: ὁδὸς ἄνω κάτω υίη. Here, too, ascent and descent necessarily belong together: the direction of thought to the universal principles and grounds of knowledge finally proves not only compatible with the direction to the particularity of phenomena and facts, but its correlate and condition.

## VII. The Theory of Relativity and the Problem of Reality

WE HAVE attempted to show how the new concept of nature and of the object, which the theory of relativity establishes, is grounded in the form of physical thought and only brings this form to a final conclusion and clarity. Physical thought strives to determine and to express in pure objectivity merely the natural object, but it thereby necessarily expresses itself, its own law and its own principle. Here is revealed again that 'anthropomorphism" of all our concepts of nature to which Goethe's wisdom of old age loved to point. "All philosphy of nature is still only anthropomorphism, *i. e.,* man, at unity with himself, imparts to everything that he is not, this unity, draws it into his unity, makes it one with him himself. . . . We can observe, measure, calculate, weigh, *etc.,* nature as much as we will, it is still only our measure and weight, as man is the measure of all things." Only, after all our preceding considerations, this "anthropomorphism" itself is not to be understood in a limited psychological way but in a universal, critical and transcendental sense. Planck points out, as the characteristic of the evolution of the system of theoretical physics, a progressive emancipation from anthropomorphic elements, which has as its goal the greatest possible separation of the system of physics from the individual personality of the physicist. (68, p. 7.) But into this "objective" system, free from all the accidents of individual standpoint and individual personality, there enter

those universal conditions of system, on which depends the peculiarity of the physical way of formulating problems. The sensuous immediacy and particularity of the particular perceptual qualities are excluded, but this exclusion is possible only through the concepts of space and time, number and magnitude. In them physics determines the most general content of reality, since they specify the direction of physical thought as such, as it were the form of the original physical apperception. In the formulation of the theory of relativity this reciprocal relation has been confirmed throughout. The principle of relativity has at once an objective and a subjective, or methodological meaning. The "postulate of the absolute world," which it involves according to an expression of Minkowski, is ultimately a postulate of absolute method. The general relativity of all places, times and measuring rods must be the last word of physics, because "relativization," the resolution of the natural object into pure relations of measurement constitutes the kernel of physical *procedure,* the fundamental cognitive function of physics.

If we understand, however, how, in this sense, the affirmation of relativity develops with inner consequence and necessity out of the very form of physics, a certain critical limitation of this affirmation also appears. The postulate of relativity may be the purest, most universal and sharpest expression of the physical concept of objectivity, but this concept of the *physical* object does not coincide, from the standpoint of the general criticism of knowledge, with reality absolutely. The progress of epistemological analysis is shown in that the assumption of the simplicity and oneness of the concepts of reality is recognized more and more as an illusion. Each of the original directions of knowledge, each interpretation, which it makes of phenomena to combine them into the unity of a theoretical

connection or into a definite unity of meaning, involves a special understanding and formulation of the concept of reality. There result here not only the characteristic differences of meaning in the objects of science, the distinction of the "mathematical" object from the "physical" object, the "physical" from the "chemical," the "chemical" from the biological," but there occur also, over against the whole of *theoretical* scientific knowledge, other forms and meanings of independent type and laws, such as the ethical, the aesthetic "form." It appears as the task of a truly universal criticism of knowledge not to level this manifold, this wealth and variety of forms of knowledge and understanding of the world and compress them into a purely abstract unity, but to leave them standing as such. Only when we resist the temptation to compress the totality of forms, which here result, into an *ultimate* metaphysical unity, into the unity and simplicity of an absolute "world ground" and to deduce it from the latter, do we grasp its true concrete import and fullness. No individual form can indeed claim to grasp absolute "reality" as such and to give it complete and adequate expression. Rather if the thought of such an ultimate definite reality is conceivable at all, it is so only as an Idea, as the problem of a totality of determination in which each particular function of knowledge and consciousness must coöperate according to its character and within its definite limits. If one holds fast to this general view, there results even within the pure concepts of nature a possible diversity of approaches of which each one can lay claim to a certain right and characteristic validity. The "nature" of Goethe is not the same as that of Newton, because there prevail, in the original *shaping* of the two, different principles of form, types of synthesis, of the spiritual and intellectual combination of the phenomena. Where there exist such diversities in fun-

damental *direction* of consideration, the *results* of consideration cannot be directly compared and measured with each other. The naïve realism of the ordinary view of the world, like the realism of dogmatic metaphysics, falls into this error, ever again. It separates out of the totality of possible concepts of reality a single one and sets it up as a norm and pattern for all the others. Thus certain necessary formal points of view, from which we seek to judge and understand the world of phenomena, are made into things, into absolute beings. Whether we characterize this ultimate being as "matter" or "life," "nature" or "history," there always results for us in the end confusion in our view of the world, because certain spiritual functions, that coöperate in its construction, are excluded and others are over-emphasized.

It is the task of systematic philosophy, which extends far beyond the theory of knowledge, to free the idea of the world from this one-sidedness. It has to grasp the *whole system* of symbolic forms, the application of which produces for us the concept of an ordered reality, and by virtue of which subject and object, ego and world are separated and opposed to each other in definite form, and it must refer each individual in this totality to its fixed place. If we assume this problem solved, then the rights would be assured, and the limits fixed, of each of the particular forms of the concept and of knowledge as well as of the general forms of the theoretical, ethical, aesthetic and religious understanding of the world. Each particular form would be "relativized" with regard to the others, but since this "relativization" is throughout reciprocal and since no single form but only the systematic totality can serve as the expression of "truth" and "reality," the limit that results appears as a thoroughly immanent limit, as one that is

removed as soon as we again relate the individual to the system of the whole.

We trace the general problem, which opens up here, no further but use it merely to designate the limits, that belong to any, even the most universal, *physical* formulation of problems, because these limits are necessarily grounded in the concept and essence of this way of formulating the question. All physics considers phenomena under the standpoint and presupposition of their measurability. It seeks to resolve the structure of being and process ultimately into a pure structure or order of numbers. The theory of relativity has brought this fundamental tendency of physical thought to its sharpest expression. According to it the procedure of every physical "explanation" of natural process consists in coördinating, to each point of the space-time continuum, four numbers, $x_1$, $x_2$, $x_3$, $x_4$, which possess absolutely no direct physical meaning but only serve to enumerate the points of the continuum "in a definite, but arbitrary way." (18, p. 64.) The ideal, with which scientific physics began with Pythagoras and the Pythagoreans, finds here its conclusion; all qualities, including those of pure space and time, are translated into pure numerical values. The logical postulate contained in the concept of number, which gives this concept its characteristic form, seems now fulfilled in a degree not to be surpassed; all sensuous and intuitive heterogeneity has passed into pure homogeneity. The classical mechanics and physics seeks to reach this immanent goal of conceptual construction by relating the manifold of the sensuously given to the homogeneous and absolutely uniform time. All difference of sensation is hereby reduced to a difference of motions; all possible variety of content is resolved into a mere variety of spatial and temporal positions. But the ideal of strict homogeneity is not reached here

since there are still always two fundamental forms of the homogeneous itself that are opposed to each other as pure space and pure time. The theory of relativity in its development advances beyond this opposition also; it seeks to resolve not only the differences of sensation but also those between spatial and temporal determinations into the unity of numerical determinations. The particularity of each "event" is expressed by the four numbers $x_1$, $x_2$, $x_3$, $x_4$, whereby these numbers among themselves have reference to no inner differences, so that some of them $x_1$, $x_2$, $x_3$, cannot be brought into a special group of "spatial" coördinates and contrasted with the time coördinate" $x_4$. Thus all differences belonging to spatial and temporal apprehension in subjective consciousness seem to be consistently set aside in the same way that nothing of the subjective visual sensation enters into the physical concept of light and color.[31] Not only are all spatial and temporal values exchangeable with each other, but all inner differences of the temporal itself, unavoidable for the subjective consciousness, all differences of *direction,* which we designate by the words "past" and "future," are cancelled. The direction into the past and that into the future are distinguished from each other in this form of the concept of the world by nothing more than are the + and — directions in space, which we can determine by arbitrary definition. There remains only the "absolute world" of Minkowski; the world of physics changes from a *process* in three-dimensional world in which time is replaced as a variable magnitude by the imaginary "ray of light" (*Lichtweg*) $(x_4 = \sqrt{-1}\, c\, t)$.[32]

This transformation of the time-value into an imaginary numerical value seems to annihilate all "reality" and qualitative determinateness, which time possesses as the

[31] On this latter point *cf.* now Planck, *Das Wesen des Lichts* (71).
[32] *Cf.* Minkowski (54, p. 62ff.); Einstein (18, p. 82f.).

"form of the inner sense," as the form of immediate experience. The "stream of process," which, psychologically, constitutes consciousness and distinguishes it as such, stands still; it has passed into the absolute rigidity of a mathematical cosmic formula. There remains in this formula nothing of that form of time, which belongs to all our experience as such and enters as an inseparable and necessary factor into all its content.[33] But, paradoxical as this result seems from the standpoint of this experience, it expresses only the course of mathematical and physical objectification, for, to estimate it correctly from the epistemological standpoint, we must understand it not in its mere result, but as a process, a method. In the resolution of subjectively experienced qualites into pure objective numerical determinations, mathematical physics is bound to no fixed limit. It must go its way to the end; it can stop before no form of consciousness no matter how original and fundamental; for it is precisely its specific cognitive task to translate everything enumerable into pure number, all quality into quantity, all particular forms into a universal order and it only "conceives" them scientifically by virtue of this transformation. Philosophy would seek in vain to bid this tendency halt at any point and to declare *ne plus ultra*. The task of philosophy must rather be limited to conceiving this *meaning* in its logical dependency by recognizing fully the logical *meaning* of the mathematical and physical concept of objectivity. All particular physical theories including the theory of relativity receive their definite meaning and import only through the unitary cognitive will of physics, which stands back of them. The moment that we transcend the field of physics and change not the means but the very goal of knowledge, all particular concepts assume a new aspect and form. Each of these concepts means something different, depending on the general "modality"

[33] *Cf., e. g.,* J. Cohn (14, p. 228ff.).

of consciousness and knowledge with which it stands and from which it is considered. Myth and scientific knowledge, the logical and the aesthetic consciousness, are examples of such diverse modalities. Occasionally concepts of the same name, but by no means of the same meaning, meet us in these different fields. The conceptual relation, which we generally call "cause" and "effect" is not lacking to mythical thought, but here its meaning is specifically distinct from the meaning that it receives in scientific, and in particular, in mathematical and physical thought. In a similar way, all the fundamental concepts undergo a characteristic intellectual change of meaning when we trace them through the different fields of intellectual consideration. Where the copy theory of knowledge seeks a simple identity, the functional theory of knowledge sees complete diversity, but, indeed, at the same time complete correlation of the individual forms.[84]

If we apply these considerations to the concepts of space and time, then it is obvious what the transformation of these concepts in modern physics means, in its philosophical import, and what it cannot mean. The content of physical deductions cannot, without falling into the logical error of a μετάβασις εἰς ἄλλο γένος be simply carried over into the language of fields whose structure rests on a totally different structural principle. Thus, what space and time are as immediate contents of experience and as they offer themselves to our psychological and phenomenological analysis is unaffected by the use we make of them in the determination of the object, in the course of objective conceptual knowledge. The distance between these two types of consideration and conception is only augmented by the theory of relativity and thus only made known more distinctly, but is not first produced by it. Rather it is clear

[84] I am aware of the fragmentary character of these suggestions: for their supplementation and more exact proof I must refer to some subsequent more exhaustive treatment. *Cf.* also the essay *Goethe und die mathematische Physik* (11).

that even to attain the first elements of mathematical and physical knowledge and of the mathematical and physical object we assume that characteristic transformation of "subjective" phenomenal space and of "subjective" phenomenal time, which leads, in its ultimate consequences, to the results of the general theory of relativity. From the standpoint of strict sensualism too, it is customary to admit this transformation, this opposition between the "physiological" space of our sensation and presentation and the purely "metrical" space, which we make the basis of geometry. The latter rests on the assumption of the equivalence of all places and directions, while for the former the distinction of places and directions and the marking out of one above the others is essential. The space of touch, like that of vision, is anistropic and inhomogeneous, while metrical Euclidean space is distinguished by the postulate of isotropism and homogeneity. Compared with "metrical" time, physiological time shows the same characteristic variations and differences of meaning; one must, as Mach himself urges, as clearly distinguish between the immediate sensation of duration and the measuring number as between the sensation of warmth and temperature.[35]

[35] Mach (50, p. 331ff., 415ff.). If, with Schlick (79, p. 51ff.), one would call the psychological space of sensation and presentation the space of intuition, and contrast with it physical space as a conceptual construction, no objection could be made against this as a purely terminological determination; but one must guard against confusing this use of the word "intuition" with the Kantian, which rests on entirely different presuppositions. When Schlick sees in the insight that objective physical time has just as little to do with the intuitive experience of duration as the three-dimensional order of objective space with optical or "haptical" extension, "the kernel of truth in the Kantian doctrine of the subjectivity of time and space," and when he, on the other hand combats, on the basis of this distinction, the Kantian concept of "pure intuition," this rests on a psychological misunderstanding of the meaning of the Kantian concepts. The space and time of pure intuition are for Kant never sensed or perceived space or time, but the "mathematical" space and time of Newton; they are themselves constructively generated, just as they form the presupposition and foundation of all further mathematical and physical construction. In Kant's thought, "pure intuition" plays the rôle of a definite fundamental *method of objectification;* it coincides in no way with "subjective," *i. e.*, psychologically experienceable time and space. When Kant speaks of the subjectivity of space and time, we must never understand experiential subjectivity but their "transcendental" subjectivity as conditions of the possibility of "objective," *i. e.*, of objectifying empirical knowledge. (*Cf.* also the significant remarks of Selliens against Schlick; 81, p. 19, 39.)

This contrast between subjective, "phenomenal" space and time, on the one hand, and objective and mathematical space and time, on the other, comes to light with special distinctness, when one considers a property which seems at first glance to be common to them. Of both we are accustomed to predicate the property of *continuity,* but we understand thereby, more closely considered, in the two cases something wholly different. The continuity, which we ascribe to time and processes in it on the basis of the form of our experience, and that which we define in mathematical concepts by certain constructive methods of analysis, not only do not coincide but they differ in their essential moments and conditions. The experiential continuity affirms that each temporal content is given to us only in the way of certain characteristic *"wholes,"* which can not be resolved into ultimate simple *"elements";* analytic continuity demands reduction to such elements. The first takes time and duration as "organic" unities in which according to the Aristotelian definition, "the whole precedes the parts"; the second sees in them only an infinite *totality* of parts, of particular sharply differentiated *temporal points.* In the one case, the continuity of becoming signifies that living flux, that is given to our consciousness only as a flux, as a transition, but not as separated and broken up into discrete parts; in the other, it is demanded that we continue our analysis beyond all limits of empirical apprehension; it is demanded that we do not allow the division of elements to cease where sensuous perception, which is bound to definite but accidental limits in its capacity for discrimination, allows it to end, but that we follow it purely intellectually *ad infinitum.* What the mathematician calls the "continuum" is thus never the purely experiential quality of "continuity," of which there is no longer possible any further "objective" definition, but it is a purely conceptual construction, which he puts in the place of the latter. Here

too he must follow his universal method; he must reduce the quality of continuity to mere number, *i. e.,* precisely to the fundamental form of all intellectual *discreteness.* (Cf. 6, p. 21.) The only continuum he knows and the one to which he reduces all others, is always the continuum of *real numbers* which modern analysis and theory of groups seek, as is known, to construct strictly conceptually with renunciation in principle of any appeal to the "intuition" of space and time. The continuum thus considered, as Henri Poincaré especially has urged with all emphasis is nothing but a totality of individuals, which are conceived in a definite order and are given indeed in infinite number, of which each one is opposed to the others as something separate and external. We are here no longer concerned with the ordinary view, according to which there exists between the elements a sort of "inner bond" by which they are connected into a whole, so that, *e. g.,* the point does not precede the line, but the line the point. "Of the famous formula, that the continuum is the unity of the manifold," concludes Poincaré, "there remains only the manifold,—the unity has disappeared. The analysts are nevertheless right when they define continuity as they do, for in all their inferences they are concerned, in so far as they claim rigor, only with this concept of the continuous. But this circumstance suffices to make us attentive to the fact that the true mathematical continuum is something totally different from that of the physicist and the metaphysician." (72, p. 30.) In so far as physics is an objectifying science working with the conceptual instruments of mathematics, the physical continuum is conceived by it as related to and exactly correlated with the mathematical continuum of pure numbers. But the "metaphysical" continuum of the pure and original "subjective" form of experience can never be comprehended in this way, for the very *direction* of mathematical consideration is such that, instead of leading to this

form, it continually leads away from it. The critical theory of knowledge, which does not have to select from among the different sorts of knowledge, but merely to establish what each of them "is" and means, can make no normative decision as to the opposite aspects under which the continuum here appears, but its tasks consists in defining the two with reference to each other in utmost distinctness and clarity. Only by such a delimitation can be reached, on the one hand, the goal of phenomenological analysis of the temporal and spatial consciousness, and on the other hand, the goal of the exact foundation of mathematical analysis and its concepts of space and time. "With regard to the objection," a modern mathematical author concludes his investigation of the continuum, "that nothing is contained in the intuition of the continuum of the logical principles that we must adduce in the exact definition of the concept of the real number, we have taken account of the fact that what can be found in the intuitive continuum and in the mathematical world of concepts are so alien to each other, that the demand that the two coincide must be rejected as absurd. In spite of this, those abstract schemata, which mathematics offers us, are helpful in rendering possible an exact science of fields of objects in which continua play a rôle. The exact temporal or spatial point does not lie in the given (phenomenal) duration or extension as an ultimate indivisible element, but only reason reaching through this can grasp these ideas and they crystallize into full determinateness only in connection with the purely formal arithmetical and analytical concept of the real number." [86]

If we bear in mind this state of affairs, the deductions of the theory of relativity in its determination of the four dimensional space and time continuum lose the appearance of paradox, for it is seen that they are only the final conse-

[86] Weyl, 84, p. 83, 71.

quence and working out of the fundamental methodic idea on which rests mathematical analysis in general. But the question as to which of the two forms of space and time, the psychological or the physical, the space and time of immediate experience or of mediate conception and knowledge, expresses the *true* reality has lost fundamentally for us all definite meaning. In the complex that we call our "world," that we call the being of our ego and of things, the two enter as equally unavoidable and necessary moments. We can cancel neither of them in favor of the other and exclude it from this complex, but we can refer each to its definite *place* in the whole. If the physicist, whose problem consists in objectification, affirms the superiority of "objective" space and time over "subjective" space and time; if the psychologist and the metaphysician, who are directed upon the totality and immediacy of experience draw the opposite conclusion; then the two judgments express only a false "absolutization" of the norm of knowledge by which each of them determines and measures "reality." In which direction this "absolutization" takes place and whether it is directed on the "outer" or the "inner" is a matter of indifference to the purely epistemological judgment. For Newton it was certain that the absolute and mathematical time, which by its nature flowed uniformly, was the "true" time of which all empirically given temporal determination can offer us only a more or less imperfect copy; for Bergson, this "true" time of Newton is a conceptual fiction and abstraction, a barrier, which intervenes between our apprehension and the original meaning and import of reality. But it is forgotten that what is here called absolute reality, *dureé rèelle,* is itself no absolute but only signifies a standpoint of consciousness opposed to that of mathematics and physics. In the one case, we seek to gain a unitary and exact measure for all objective process, in the other we are concerned in retain-

ing this process itself in its pure qualitative character, in its concrete fulness and subjective inwardness and "contentuality." The two standpoints can be understood in their meaning and necessity; neither suffices to include the actual whole of being in the idealistic sense of "being for us." The symbols that the mathematician and physicist take as a basis in their view of the external and the psychologist in his view of the inner, must both be understood as *symbols.* Until this has come about, the true philosophical view, the view of the *whole,* is not reached, but a partial experience is hypostasized into the whole. From the standpoint of mathematical physics, the total content of the immediate qualities, not only the differences of sensation, but those of spatial and temporal consciousness, is threatened with complete annihilation; for the metaphysical psychologist, conversely, all reality is reduced to this immediacy, while every mediate conceptual cognition is given only the value of an arbitrary convention produced for the purposes of our action. But both views prove, in their absoluteness, rather perversions of the full import of being, *i. e.*, of the full import of the *forms* of knowledge of the self and the world. While the mathematician and the mathematical physicist stand in danger of permitting the real world to be identified with the world of their *measures,* the metaphysical view, in seeking to narrow mathematics to practical goals, loses the sense of its purest and deepest *ideal* import. It violently closes the door against what, according to Plato, constitutes the real meaning and the real value of mathematics; that, namely, "by each of these cognitions an *organ of the soul* is purified and strengthened, which under other occupations is lost and blinded; for its preservation is more important than that of a thousand eyes: for by this alone is the truth seen." And been the two poles of consideration, which we find here, there stand the manifold concepts of truth of the different concrete sciences—

and therewith their concepts of space and time. *History*, to set up its temporal measure, cannot do without the methods of the objectifying sciences: chronology is founded on astronomy and through this on mathematics. But the time of the historian is nevertheless not identical with that of the mathematician and physicist, but possesses in contrast to it a peculiar concrete form. In the concept of time of history, the "objective" content of knowledge and the "subjective" experiential content enter into a new characteristic reciprocal relation. An analogous relation is presented, when we survey the aesthetic meaning and shaping of the forms of space and time. Painting presupposes the objective laws of perspective, architecture the laws of statics, but the two serve here only as material out of which develops the unity of the picture and of the architectural spatial form, on the basis of the original artistic laws of form. For music, too, the Pythagoreans sought a connection with pure mathematics, with pure number; but the unity and rythmical division of a melody rests on wholly different structural principles than those on which we construct time in the sense of the unity of objective physical processes of nature. What space and time truly *are* in the philosophical sense would be determined if we succeeded in surveying completely this wealth of nuances of intellectual meaning and in assuring ourselves of the underlying formal law under which they stand and which they obey. The theory of relativity cannot claim to bring this philosophical problem to its solution; for, by its evolution and scientific tendency from the beginning, it is limited to a definite particular motive of the concepts of space and time. As a physical theory it merely develops the meaning that space and time possess in the system of our empirical and physical measurements. In this sense, final judgment on it belongs exclusively to physics. In the course of its history, physics will have to decide whether the world- picture of

the theory of relativity is securely founded theoretically and whether it finds complete experimental verification. Its decision on this, epistemology cannot anticipate; but even now it can thankfully receive the new incitements which this theory has given the general doctrine of the principles of physics.

## AUTHOR'S NOTE

The above essay, of which this is the concluding section, does not claim to give a complete account of the philosophical problems raised by the theory of relativity. I am aware that the new problems presented to the general criticism of knowledge by this theory can only be mastered by the gradual and common work of physicists and philosophers; here I was merely concerned with beginning this work, with stimulating discussion, and, where possible, guiding it into definite methodic paths, in contrast to the uncertainty of judgment which still reigns. The purpose of this writing would be attained if it succeeded in preparing for a mutual understanding between the philosopher and the physicist on questions, concerning which they are still widely separated. That I was concerned, in purely epistemological matters, also, to hold myself in closest contact with scientific physics and that the writings of the leading physicists of the past and present have everywhere essentially helped to determine the intellectual orientation of the preceding investigation, will be gathered from the exposition. The bibliography, which follows, however, makes no claim to actual completeness; in it only such works are adduced as have been repeatedly referred to and intensively considered in the course of the exposition.

Albert Einstein read the above essay in manuscript and improved it by his critical comments; I cannot let it go out without expressing here also my hearty thanks to him.

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# THE PSYCHO-GENESIS OF SPACE

DE MORGAN declares in the introduction to his *Budget of Paradoxes* that the sense or nonsense displayed by a paradoxer "will not depend upon what he maintains, but upon whether he has or has not made a sufficient knowledge of what has been done by others, *especially as to the mode of doing it,* a preliminary to inventing knowledge for himself." We may well accept this test of a veteran student of paradoxes to guide us aright in the face of the flood of paradoxes let loose in the name of "relativism" by a hitherto sober physical science. If we take a paradox in De Morgan's sense as "something which is apart from general opinion," then we are indeed confronted by the master paradox of all in the treatment of time as a fourth dimension. There is plenty of warrant for the suspicion that Minkowski, largely responsible for this development, has merely uttered the paradoxes of Zeno over again with the important difference that he believes them, whereas Zeno did not. Zeno employed his paradoxes to show that motion is inconceivable in terms of added states of rest; whereas Minkowski seeks to show that not only motion but the time it involves may be treated as a dimension. It would be difficult to invent a paradox that would so completely contradict the daily experiences of men. Yet, we must remember Copernicus, and with him the lesson that paradoxes may become commonplaces. It is the purpose of this paper to consider the grounds upon

which the paradox of Minkowski may look forward to such a happy outcome.

We needed to realize at the outset that the issue is a far more significant one than we ordinarily take paradoxes to be. We shall see that here is a conflict that can be traced back to that uncertain dawn when man first sought for the meaning of his experiences. Throughout the history of philosophy, where the external conflicts recorded by the historian, are seen as internal conflicts in the souls of men, the strife between two opposed conceptions of time are never abated. The *time of the clock* and *time lived* form the central issues in the conflict between mechanism and vitalism in biology, between continuity and relativity in physics, and between intellectualism and humanism in philosophy.

What are these opposed conceptions of time? Consider the wholly opposed nature of time *as lived* and time *as measured* in the simple experience of drinking a cup of tea. If thirsty, we impatiently wait for the lump of sugar to dissolve, and the time it requires *seems* to us far greater than the *actual* time recorded by the placid clock on the wall. When, however, the lump is finally dissolved, the time *seems* to pass quickly with the pleasure of drinking; but the clock is placid as before. Curiously enough, however, when these events are recalled later on, it will appear that the interval during which we waited will seem short and the succeeding interval of drinking will seem long. Hitherto, in deference to the clock, we have considered its time as *real* and our own time as *illusory*. It will be the purpose of this paper to ask if the very reverse is not the truth, namely: that the time lived is *absolute* whereas that of the clock is only *relative*. Second, it will be my aim to show that the paradox of Minkowski has grown out of the error of supposing that relative time, the time of the clock, is the only time there is.

Time as lived, far from resembling the divisions of a scale, cannot be represented by a line or any conceivable arrangement of them. Time as lived does not consist of separate moments in juxtaposition, but is a uniform inter-penetration. The time of the clock is reversible: for the purposes of the laboratory it is immaterial whether the clock runs backwards or forwards; and the fact that it repeats itself after every twenty-four hours is a great convenience. An astronomer, observing the transit of a star, may well turn back the hands of his clock or set his pendulum swinging anew, but the time he has wasted in wrong computations cannot be recovered so easily. Time as lived is irreversible, and in just this does the stinging quality of its reality consist.

In truth, there is no spatial figure that can possibly represent time in its fullness; and we can now see that the above differences are only the consequences of a more deep-seated difference between them. Eventually, the fundamental difference between the two conceptions inheres in the circumstance that in mathematical time it is assumed that creation is impossible whereas in the opposed conception it is assumed that creation is unavoidable. It is because science holds that the events with which it deals do not profit by their experiences that it is able to hold to its doctrine of conservation. The power of science to predict rests in its turn on the doctrine of conservation. In short, prediction in science proceeds by eliminating psychological time with the train of the unforeseen that it drags into all calculation. Mechanics is a mathematical science in which prediction is so perfect that time may be considered as non-existent. We can predict the "future" of a machine because it does not "learn" no matter how frequent its repetitions. Conversely, the very fact that a machine and a machine alone is capable of perfect repetition is because it is in capable of improving by practice. On the other hand,

a living organism cannot help but improve by practice, and this is because the past is conserved and to it is added the increased power brought by every new experience. It is clear that we might equally well have considered our own time as *real* and that of the clock as *appearance,* for who knows at first hand but that the rate of physical changes go now slow and now fast, leaving the *real* time within us pleased or impatient as the case may be? It thus appears that the problem is first a problem of definition; and it must be said that not a little of the disputation that the problem has exacted in contemporary speculation is the outgrowth of assigned mixed meanings to the terms involved.

What term is properly applicable to the experience of time as lived? This is the sole problem facing us. I believe we prepare the way for insoluble difficulties if instead of this simple ideal we suppose ourselves gauging the truth or falsity of a view from their harmony, not with our experience, but with some supposed "truth" underlying it. We shall not say that what we are seeking is knowledge that will correspond with "reality" unless by reality we mean no more than our experiences. The objection to the term *reality* is that it viciously conceals assumptions that in the metaphysical thinking of the past has given rise to endless trouble. Idealism has taken the term to refer to a universe of truth supposed to subsist without relation to human knowledge. Thus, they are able to save the possibility of knowledge only by the assumption that this reality is really known, and it forthwith declares that reality to be in the nature of an eternally complete and ideal universe. From this position, aside from the flat contradiction it contains, it will be forever impossible to comprehend the nature of human experience, which now becomes the unknown. Of matter, which is not ideal, and finite events, which are certainly not "complete," absolute idealism can give only two accounts. It may call pyramids

*unreal* or "appearances" because they are not *per se* ideas, but if this is taken as it is intended, to mean their non-being it is absurd. Ontologically speaking, "appearances" are just as real as anything we can know, and seem some how to get themselves talked about much more than the eternal verities that are supposed to expose their ghostly nature. On the other hand, if it is meant that pyramids are less real than their eternally ideal counterparts we insist that the proper terms *actual* or *potential* be employed. Materialism, on the other hand, believing as Flint says that "the seen is more potent than the unseen" only rivals the jargon of idealism by its profitless distinction between a *real* phenomenal universe and an *unreal* or epiphenomenal one. The term *unreal* has here been used to mean at once a dual implication of *non-existence* and *insignificance*.

What idealism and materialism plainly do is to demand that experience conform to preconceived ideas about it. This leaves all discourse the egoistic preoccupation of each man proving his case by denying all negative evidence; and history has shown that in such situations the supposed custodians of truth prefer to convince the heads of others by breaking them. Thought, instead of solving problems misses its function by denying their existence. Conceptions indeed form the substance of knowledge, but it is the business of discourse, not to force experience into conceptions already given but to find such conceptions in proper number, as will represent experience in its true unity or diversity.

I shall now point out how the above considerations will concern us in discussing space and time as forms of experience. It is a mistake to suppose that even were time proved to be "relative" that it therefore becomes unreal in any sense. What relativists must mean when they call time "unreal" is *that time conceived as an absolute* is unreal, i. e., not verified by experience. If time is truly so

relative that all its characteristics can be represented by a dimension then it is in every sense as real as any or all of the other three. In fine, the issue is solely to decide whether the experience of time can in all respects be considered relative or whether it has characteristics that demand the opposite category, the *absolute*. We may next inquire into the necessary meanings of these latter terms.

By the term relative we mean to indicate the dependence of the signification of anything by its comparison with something else. In order to have such a comparison at all it must be supposed that the terms of the comparison are alike in kind. In short, their comparison can only be in *quantitative* terms. The term *relative,* then, is of necessity inapplicable to the *qualitative* aspect of experience. It may be said that qualities are surely comparable in terms of their *intensity,* but reflection will show that in every idea of intensity there is hidden at once the notion of real and of implied space: it illegimately conveys two contradictory ideas, the idea of an actual space and the idea of a potential one. Space itself is none other than the field where *relations* are found: this is its actual service to science. It is misleading to say that space is the sum total of all points for points *per se* may be anywhere and actually are nowhere until they are given a locus by a relation. Space is a field of "anywhereness" rendered concrete by a relation, but science rightly holds that relations when empirically established are more than the "mental constructs" of Kant. The substance of the relation consists of the distance it measures between points. This distance, to be sure, may be construed to mean either the sum total of the unit distances comprising it or the effort that is required to traverse it. In exact science distances are always understood in the first sense, and are expressed by numbers. Thus, there is saved from the meaure taken, only its quantitative aspect—unless we inquire too closely into the inner

meaning that numbers have for us. So long, however, as numbers are regarded as symbols of pure quantity, those quantities may be decreased or increased at will so long as these changes take place proportionally, hence the accuracy of maps. As geographers, the difference in size between the map and the area it represents are very nearly negligible—not quite negligible, because however minute the map its various points still have to be traversed and hence it will still consume time, though far less time than it would take to traverse the surface it represents. As mathematicians, however, we reach the apex of all possible achievement in map-making for now distances may be represented by figures and any figure may represent any distance desired. In short, we have sacrificed all quality for the sake of schematism, but the sacrifice is only temporary for the reason that scheme making is not the whole of life. As wayfarers we cannot treat the qualitative aspects of life with such lordly indifference. In practice, we are bound to consider distances in terms of the effort it will require to traverse them. We are now interested, not in the surface of the earth *en toto* but in the particular stations we must reach in our wayfaring. The scale of miles we made as geographers will show the same numbers of miles as we shall find in traveling. The great advantage of the scale is that it will enable us to traverse the space with an amount of fatigue so small that we may fail to see that it is a substitute miniature for the real experience. Thought, therefore, does not save us entirely from the "trial and error method" of lower organisms. With humanity, the trial and error method has been transferred to *thought* instead of action; and in a sense, thought is life in miniature, not a quantitative miniature, but a qualitative one, if the phrase may be permitted. The scale of miles is a qualitative replica of the original because it is stored in the memory,

and its only disadvantage is that it is necessary to remember when the time for action arrives that each number shall have gradually restored to it, to the very end of the journey, the full measure of quality taken from it during the computation. This full measure must be returned in a varying proportion of *intension of effort* and of *extension of time*. During the journey we may save effort by sacrificing more time, and time may be saved by the intension of the effort: but a complete reduction of one to the other is inconceivable. What is the role played by time and effort during the period of calculation? We shall see that here there is a transposition of the factors involved, where there is an intension or contraction of time and *an extreme extension of effort*. But here, as before, there can be no complete reduction of the one to the other. *In other words, there can be no complete elimination of effort (and, therefore, of time) out of computations, any more than there can be a complete elimination of space out of action.* So inexorably are both facts of experience that a monistic philosophy could only gain credence by eliminating either the one or the other from experience; and the attempts and failures to do this are equally notorious in the history of thought. It is the business of thought, not to deny the experience of space or time but to show how they can concur in the same experience. What the nature of that concurrence is can already be vaguely seen: it is certainly functional whatever else it may be.

We may next concern ourselves about the necessary meaning of the term *absolute*. If it is not to encroach on the meaning of the word *relative* this term must hold that the signification of a thing depends on itself alone. As Bergson has it, a thing is absolute by being absolutely what it is. It is at once obvious that the term is misapplied when referred to a world of space whose points offer only relations. The most common misuse of the term is to take it

to refer to a thing in the aspect of its "wholeness" or "allness." We cannot say that a single atom or the whole universe of them are absolute for this would involve the meaningless comparison of a thing with itself, and comparison belongs to the relative and not to the absolute. The only aspect of our experience to which the term absolute may be legitimately applied is to *time*, for *time alone is undivided and this alone forbids relations*. Our very inability to conceive limits to the largeness of the universe or the smallness of it arises because we take it to be a universe of space, and space by its very definition is infinitely divisible or extensible; hence, infinitely relative. It will be objected that successive intervals of time may be related and compared, but in truth such comparison are always made of time already flown and quantified. When we say that the time now passing is more intense or less intense than heretofore we take advantage of the dual signification of the term *intensity;* and it is necessary to rescue this word from equivocation. It has misled psychological speculation because it implies at once the ideas of quality and quantity. It has thus been illegimately employed as a means of measuring qualitative experiences, a goal that has long been dear, not to Weber and Fechner alone, but to the scientists of all ages too enamoured of mathematics. We quantify a sensation as a measurable intensity because we prefigure its result in action even while experiencing it. There is in every sensation a reflexion of the extensity which it must meet, but to endure in memory they must be purely qualities, and as such, they are not relative. Time in a relative sense has no experiential meaning.

However, time is an absolute in a quite different sense than the word was employed by Newton. In Newtonian physics time is considered an absolute in the sense of a uniform flow. In the literature of modern physical relativism, it is common to refer to the Newtonian conception

of time as "the relativism of Newton" to distinguish it from the "relativism of Einstein." The relativism of Newton is supposed to inhere in the inference that since time is uniform in its flow, its actual rate, like the actual extent of distances is negligible. This form of relativism is supposed to be distinct from Einstein's in that the latter considered the flow of time as relative to bodies while Newton considered its flow independent of them. Examination will show that the difference is only a verbal one. Underlying both conceptions is the notion that time is a literal flow, like that of water. Now, it is only a verbal difference whether we say that this flow is relative to bodies, as does Einstein, or that the motion of bodies is relative to the flow of time, as does Newton. If we once say, as does Newtonian physics, that time is absolute, we cannot then say that its *absolute rate* does not matter, for we do so only by smuggling in some other absolute by means of which its rate is determined. Such a conception of time should perhaps be described as "relatively absolute" and would not be one whit different from the time of Einstein which is "absolutely relative." Both of them, De Morgan would surely say, go beyond all serious paradoxing, and belong in the class with "round squares" and "square circles." Clerk Maxwell is the author of that other paradox which holds that should all bodies at the same time receive a blow that would increase their motion by a proportionate amount, the change would go undiscovered. Poincare has invented a similar one in which it is held that should the universe expand and contract uniformly, man would not be able to discover it. In either case, the reason why the changes are not discovered is because they are supposed to occur *instantaneously*, that is, without the change from one state to the other occupying an actual duration. Now, an instantaneous change is as bad a paradox as any. If no time is permitted for the "changes" of which they speak, then indeed, no change has

occurred. The seeming paradoxes arise because we imagine ourselves at once to be subject to the change, but that we still carry with us the memories of the previous universe.

Let us apply some of these considerations to the details of the physical conceptions of Newton and Einstein. Both seek an eternally conserved universe, and this is one, as we have seen, that must be unaffected by time. All changes are to be purely quantitative, and the vehicle in which they occur is the ether. Not only this, but ether is intended to solve that other persistent problem of the problem as to how in a timeless universe there can be an exchange of influences between mutually external objects. To illustrate, we will allow *A* and *B* to represent any objects in juxtaposition, two electrons, or, if you please, two stars. How is one to impart its motion to the other? *A* cannot displace *B* until *B* moves, and *B* will not move until *A* moves it. Not only are we left in this deadlock but the whole concept of contact between bodies is hardly imaginable. We can only say negatively that two indivisible atoms are in contact when they are neither separated nor invading. The miracle of transmitting the motion of one to the other occurs on contact, for manifestly that transfer could not occur while they are completely separated. But how are we to conceive the contact to accomplish the transfer? If the contact is a passive one it will not do so. On the other hand, how can a contact be active without involving at once the two contradictory notions of a simultaneous separation and invasion? Such a contact would moreover have to be instantaneous, which, again, leaves it merely passive. But physics has found in the concept of elasticity an escape from the difficulty. The property of elasticity will allow *B* to yield to *A* and we are able to image the rebound of B. But since both *A* and *B* are by hypothesis indivisible, we cannot say that they become distorted for

this supposes that they are not indivisible but are themselves compositions of parts into which distortions may enter. The problem of interaction would thus not be solved by the concept of elasticity but only transfers it to other *A's* and *B's* within the first *A* and *B* with which we began. Thus, when we logically follow the simple impacts of objects, constantly witnessed by the eye, we are left in a mystifying quandary. The difficulty we have hit upon is, stating it generally, the difficulty of conceiving how there can be a change of one state to another without an intermediate time evolution. This factor atomism alone cannot provide, so the physics of Newton has invoked ether to supply the defect. We shall now inquire whether the ether theory unties the Gordian knot or only cuts it.

The ether is the medium of exchange when energy is sent from body to body; and during the transmission, serves as the storehouse of potential energy. In other words, it is a substitute for an evolution in time without which a change from one state to another is unthinkable. The theory of ether is therefore at once used by science to embody the contradictory notions of an eternally conserved universe and an eternally evolving one. By this means science is enabled to regard its utterances as not only *useful* but as *true* in the philosophical sense. In short, atomism would be at once the description and the explanation of experience. But we have already seen that a harmony between the idea of the reality of development and the idea of the unreality of time is impossible; and science has managed to secure the appearance of such a synthesis by shifting the incongruity to the concept of the ether. Ether, accordingly, embodies a group of contradictory properties: it is at once matter and non-matter; it exists as do other bodies but offers no resistance; it is imponderable but still transmits energy in the form of waves as real as those of an unquiet sea; it is at once perfectly rigid and perfectly

elastic. That Michaelson, Morley, Miller and others should fail to find objective evidence of the existence of such an ether is hardly surprising! The great weakness of Newtonian physics is that it fails to provide an image of how energies are transferred in an eternally conserved universe. But not only has it failed here, but further inquiry will show that its failure is completed by its failure to image the universe as at all eternally conserved.

In Newton's view the universe is infinitely extended and the result is that when we wish to determine the gravitational effects at any point within it we are unable to arrive at a final sum, for these effects will not terminate in a given amount since they are eternally added to from gravitational effects coming from the ever deepening removes of space. The only solution for this difficulty is to suppose that our universe is finite and measurable. But this offers difficulties as great as the last for now we are again without a quantitative universe for its average density will approach zero. The universe will in this view be eventually dissipated by radiation in straight lines into infinite space and would eventually cease to exist, a result far from the doctrine of conservation.

The failure of Newtonian physics offers cues to the relativists for a better solution. To save the situation it is necessary to invent a scheme of things giving us both the properties of space and time without actually admitting the latter as in the nature of a growth or maturation. There is necessary an image of the universe in which it will appear as eternally conserved but still with a history. Now, to have a natural history it is necessary to provide against the possibility of a stable equilibrium. A stable equilibrium, however, is inevitable to a finite universe or to any "closed system." Science must therefore suppose, in order to have a continued dynamism, that infinite recesses of space exist from which such influences come,

since it denies creation. On the other hand, to have an eternally conserved universe it is necessary to have one which is in some sense finite, given once for all. But its finiteness must not be in the sense of a universe surrounded by immeasurable gulfs of emptiness for such a universe is already unhappily attenuated to zero or inexorably approaching such an extinction. To save the situation, there is required an image of the universe in which it will appear both finite and infinite.

It was the genious of Einstein that has made the last plausible attempt at such an accomplishment. Let us briefly state his hypothesis of "spherical space" which has been hailed everywhere as astounding both in its logical simplicity and consistency. We must not confuse his conception of a spherical universe with the ancient notion of a ponderous sphere beset with the fixed stars or the spherical universe of Copernicus. The universe is spherical to Einstein in the sense that straight lines do not exist, but when followed up will invariably return to their starting point. The universe is composed of the sum total of such lines of force, moving in immense circular paths of which the solar expeditions undertaken to verify Einstein's hypotheses have discovered objective evidence. The world is thus at once complete in itself but yet without boundaries. It is limited but it still constitutes the entire universe. In short, we have here a universe that is apparently at once finite and infinite; infinite in its dynamic aspects and finite in its amount; it is a universe with four dimensions.

Before criticising Einstein's solution, let us restate what we shall require of it. It must succeed where Newtonian physics has failed, and this failure was two-fold. It failed to show how mutually external forces can interact, and it failed to show how forces can be conserved.

Is the problem as to how $A$ can impart its motion to $B$ simplified if we suppose them to be moving in curvilinear

fashion instead of in straight lines? There certainly will be no gain for this view if we conceive of circular motion as atomism conceives it. Atomism will either say (in the language of the calculus) that a circle is a succession of infinitely short lines constantly changing their direction, or that it is a row of points behaving in the same way. Since lines cannot be infinitely short we must conclude that at the instant of impact between *A* and *B* they are moving in rectilinear fashion and in this case we meet with all the objections confronted in Newton's view. Since points cannot be conceived as real without possessing diameters, they are equally inadequate. Einstein's universe, therefore is constituted of an infinite number of circles as mutually external to each other as the points in the Newtonian scheme. In both systems, each circle or point is a universe by itself without any relation to the others except it be in the mind of some onlooker.

In the last statement we have the answer also to the question as whether Einstein's universe is a conserved one. It may be said of both the points in Newton's system and the geodetic lines of relativism that neither are conserved since their conservation depends on a relation, and unless that relation is itself a generative one it must be a relation that is *understood* by an onlooker. *Thus the universe to be preserved must first be preserved in time; it must have a duration of its own.* If we set aside the logical difficulty of interaction in Einstein's universe, and allow the curvilinear forces therein to interact they will at once become involved in an interpenetration during which every force will modify every other and will itself be modified by all just as a falling pebble in Newton's system will jar the entire universe and will express in its reaction the momentum that the universe possesses. In Einstein's scheme, the total effect would be as that of a sphere which rotates at once in every conceivable direction, an impossible image!

Such a universe would be essentially a living one, before which all images fail. If the number of his forces were finite they would end eventually in a completely homogenous stability; and would be none other than Newton's absolute space, in short, would be non-existence. Einstein only saves the continuation of his scheme by supposing the number of his forces as infinite in number: for Newton's infinite space he substitutes infinite relations. And just as Newton's infinite space can be rendered intelligible only by Aristotle's view that it is pure possibility, a potter's clay for the creative spirit, so in Einstein's view the infinite relations are saved only by an onlooker who preserves them by remembering them. Aristotle held that matter arises when spirit impresses pure possibility with form, and Einstein's view will be inconsistent unless it holds that matter arises not when a relation is *seen* but when a relation is *achieved*. We are here in a position to see what space really signifies. It is because the spirit continually renews its achieving that old tasks are conceptualized as the points of an imaginary space stretched beneath them. To the spirit every form it discerns is an instrument of its freedom; and the ideal of life were to make of the entire universe a simple lever that will yield too rather than intercept the will. Every form is a worth only because it enhances life; and for it to become of greater worth it must enter with other forms into a higher construction, a more specialized instrument of freedom. This means that in order to become a higher form any given form must for the sake of calculation become a pure possibility else calculation will be crippled by being limited. Hence it must become a point in an ideal space retaining of its former character only its quantitative aspect. In the idea of space is at once the idea of utter barrenness and of infinite possibility, and of these simple experience gives us no wisdom. It is matter alone, embodying at once a promise and an

achievement that is thus able to mirror the moral discovered in daily life that though life is irreversible, the choice of it is reversible. Space, therefore, is time stratified. *It is a momentary laying side by side of the many victories of life so that a major achievement may be projected from them.* It is the function of perception to cut the surrounding world into unit objects, each of which have been partly but not wholly subjected to the human will. The properties of space which they possess express the extent of their submission; and were they purely spacial they would indeed be no more than ideal, and the universe would spring miraculously into existence at the behests of thought: it would be a universe completely conquered, completely passive: it would be the pure space, the pure possibility of Aristotle. But *real* objects possess other properties than that of pure space—the property of resistance. As we have seen, resistance cannot obtain without time in which it occurs. *It is in this direction that interactions may be solved—by seeking to understand the very experience in which they occur, the experience of strife.* It is in this way that the inert must be explained by the living, and not the living by the inert. Relativism may mark another mile-stone in physics apparently because it has saved the principle of conservation. But what is the inner meaning of this achievement? It is, that relativism marks an epoch in the history of science because it saves the creative power of science by thus saving the possibility of prediction. That is, relativism is destined to be a tool in the hands of a creative force in the universe it so expressly denies. Nay, relativism was itself born of the creative imagination of man.

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# CRITICISMS AND DISCUSSIONS

## THE REALISM OF TONGIORGI

IN THESE DAYS of interest in what is called the New Realism, and its dealings with Being as wider than Reality, it is of interest to recall the positions of that distinguished thinker of last century, Tongiorgi on some of the issues now engaging attention. This particular purpose is the reason for my not discussing certain matters in Tongiorgi, such as the *sensus fundamentalis* described by him and by Rosmini, and critically reviewed by certain scholastic writers of distinction. The moderate-minded Scholasticism of Tongiorgi gives him the greater claim on our attention.

In his "Institutiones Philosophicae" (3 vols., Rome, 1861), Tongiorgi says: "Whatever is conceived as having some reality (*aliquam realitatem*), is conceived either as actually existing, or at least after the manner of something existing; for it is such that there is in it no repugnance to existence. Nay, more, by the very fact of its being an object of thought, it has a certain existence"—of the ideal or 'intentional' order, as he goes on to explain (Vol. II, *Ontol*, p. 8). Further on, he says: "Whatever has some reality, by which it can be an object of thought, has it either as existing, extra-mental (*extra mentem*) reality, or as not existing, but capable of becoming existent." This latter is *en in potentia* or *possible*, the former is *ens in actu* or *existens* (p. 23). This discrimination of ideal being is not really new, as often supposed, having had explicit recognition at least as far back as the time of Wyclif and Occam; neither is it, I would add, a discrimination peculiar to Scholastic philosophers, but made by many other philosophers, especially, but not exclusively, by those of realistic turn. They have held that whatever thought must be thought as though it had being or existence, and that possible being must be thought, as it would be, if it existed. That is to say, it must be thought under the attribute of existence.

Even Bradley has spoken of what "is real either inside of our heads or outside of them. And thus it always stands for exists." But I must leave others aside, as I have elected to speak of Tongiorgi.

Of the wide comprehension of Being opened by the recognition of ideal being, Tongiorgi says the *ens in potentia* or possible essence is privative in character, and belongs to the intelligible and metaphysical order, not to the order physical and actual. As later realists would say, it subsists, not exists. Possibility is, to Tongiorgi, "intrinsic" or "extrinsic." Intrinsic possibility is where there exists no repugnance to being in the constitutive nature of the thing, as, e. g., the possibility of a statue of gold. Intrinsic impossibility means a contradiction, that is, in its constitution, as, e. g., a quadrilateral triangle. Extrinsic possibility exists where there is fit or suitable cause to produce the thing, as, e. g., the possibility of a statue by an artificer. Intrinsic possibility is absolute, having no limit but contradiction; extrinsic possibility is relative. "Adequate" possibility embraces both (II, *Ontol,* p. 27). Elsewhere, Tongiorgi says that intrinsic repugnance means metaphysical impossibility (Vol. I, *Log.,* p. 225). Thus the unreal becomes reduced to what is intrinsically impossible, or to what has no existence outside our minds, for which latter he claims, as we have seen, a certain ideal being as an object of thought. As present day realists would say, it has being, not existence, as being timeless. Tongiorgi's discussion of possibility is in keeping with the unwonted, but merited, attention devoted to this subject by many Scholastic philosophers. Not that appreciation of the category of possibility has been confined to the Scholastics, for others, like Leibniz and Weisse, have made high use of possibility. Thus Bertrand Russell speaks of philosophy as "the science of the possible," as an "inventory of possibilities," and as a "repertory of abstractly tenable hypotheses." But is not this, it might naturally be asked, to assume for the knowledge of abstractions a higher dignity than for knowledge of the world in which we live? Is it not to assume an all too complete independence for purely conceptual entities and thought-possibilities? The concept is not a quid or simple entity which can be apprehended immediately outside us. in the mode assumed, without any need of involved mental process in the unity of a consciousness. Consciousness is not an aggregate of facts that are only exteriorly connected. The concrete unity of the conscious subject is involved in every act of thought, however simple. Can we so easily discard or shuffle

off the world of experience for a hypothetical world of ideas, conceived as in complete independence of the former? It has been the precise and peculiar claim of moderate realism—which strikes me as having been marked by great good sense in realisms of the past—that it keeps the most abstract metaphysical speculation in wholesome contact with the actual world in which we live and move and have our being. Being, not possibility, was the primal idea to Tongiorgi, who maintained that the notion of possibility is an efflorescence of our knowledge of things existing.

I may here notice Tongiorgi's discussion of the objective reality of ideas. To him the first idea is that of entity. First principles are formed from this idea. From these first principles ideas are, in upspringing of experience, acquired or formed. The objective reality of the ideas either immediately follows, or, can at least he thinks, be demonstrated. His position as to the objective reality of ideas is that a thing, which is an object of the mind, is an entity (*aliquod ens.* I, Log., p. 278). For what, he asks, is the *ratio* of entity, if not thing and object? He thinks that if ideas lacked this objective value (*objectivus idearum valor*), the first principles also would be inane. Such a theory of ideas is a long anticipation of certain recent theories; it differs, equally with Russell's, from Plato's theory of ideas, which Russell calls "one of the most successful attempts hitherto made," in that it posits no simple reminiscence of the ideas, but a direct apprehension of them.

Relation, Tongiorgi distinguishes as real, and as logical. He thinks real relation exists between things independently of the thinking or comparing power of the intellect. He instances, in this exteriority of relations, the relation of cause and effect. Logical relation is made by the intellect, as in the case of the relation of identity with itself. Of things logically identical, one can be affirmed of the other; but this affirmability does not obtain in the case of real identity. He thinks it the mark of real relations that they exist in the nature of things. The whole order, harmony and beauty of the world exist for him as real relations. When things are really identical. he says, that means they are really relative, are really opposed, and so far are really distinguished. Identity differs, in his view, from unity, which is absolute. in that identity is relative, as supposing *plura* which, compared *inter se,* are reduced to a certain unity. In real identity, two things which are identical with a third according to a common concept, are identical *inter se* according to

this very concept. In logical identity, two concepts which are identical with a third according to the object, are identical *inter se* according to this very object. But I doubt whether Tongiorgi would have countenanced the tendency to erect the logical independence of two facts into their real separation, or to treat two terms and the relation between them as separate entities. Moderate realism, at any rate, has not regarded the formal element in relation—its *esse ad* —as a distinct and separate reality. Tongiorgi retains a very clear sense of whatever is *mera distinctio rationis*. It is one thing to distinguish, in the course of experience, forms and qualities, and another thing to erect these into separate entities. Tongiorgi holds, as objects of the intellect, not only the essences of substances, but the essences of the qualities of substances, adding that such qualities are *quaedam entia*, and have their own essence (III. *Psychol*, p. 162). The context shows that such matters as color, sound and figure, are what he has in mind. But although he takes *omne ens*, which is the object of intellect, in a wide sense, he goes on to say that, nevertheless, what is offered *"primo* ac *per se"* to the intellect, is, the essence of external objects or material things. He does not, however, wish acts of the knowing subject to be excluded, but points out that it is by the intellect these become object. Thus he differentiates. Distinguish as we may and must the various and diverse aspects of reality, we yet cannot separate or rend them asunder, but must hold them in the living synthesis of thought, the concrete unity of consciousness. This is the unity of reason, "quae universalium propria est, nec extra mentem invenitur in rebus, sed fit a ratione per cognitionem abstractivam et comparativam." (II, *Ontol.*, p. 45.)

Finally, as to the criterion of truth in Tongiorgi's realism. Truth is, to him, in true Scholastic fashion, an equation or the conformity of thought to thing. In a way that strongly reminds one of Aristotle, he takes evidence to be the criterion of truth.[1] Evidence, he says, may be immediate or mediate, may be absolute or hypothetical, for evidence does not pertain only to *à priori* truths. He explains that not ideas, but only judgments, are to be spoken of as evident, for ideas are not objects that we perceive. A judgment is called evident, in so far as it is objectively regarded. What is evident is true, he holds, precisely as certain other Scholastic philosophers have done. Evidence is taken to be the universal criterion of truth—

[1] Cf. my Discussion: "Aristotle and the Criterion of Truth," in THE MONIST, July, 1921.

*atque ultimatum certitudinis motivum* (I, Log., p. 361). He thinks this *quod est evidens* better than the *quod clare et distincte* of Descartes, with its vagueness and indefiniteness. Error, Tongiorgi thinks, is in the judgment; not in the senses, nor in intellectual apprehension. Falsity is the object of error—falsity under the guise or appearance of truth. He follows that notable metaphysician Suarez, in thinking that the intellect is not necessarily determined in its judgment, save by the evidence, and evidence cannot obey false judgment. The intellect can only be so determined by the will, when it is not free. Hence, he holds that every error has its origin in a free movement of the will. He thus puts a severe strain or responsibility upon the will. This idea has not been wholly absent from recent discussions on error, but it is safe to say that, neither in criticism, nor in furtherance of it, has this line of thought been so fully worked out as it could very well be.

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## THE FALLACY OF EXCLUSIVE SCIENTIFIC METHODOLOGY

BY THE *fallacy of exclusive scientific methodology* I mean the claim that science possesses the only valid method of knowledge, together with the denial or at least the ignoring of the existence of that which is incapable of being studied by the method of science. This is a conspicuous fallacy in the thinking of many at the present time who are devotees of scientific method to the exclusion of any other means of knowledge, and who fail to recognize that there are limitations to the scope of the sciences. In the field of psychology the fallacy is prominent in the work of the extreme behaviorists. It occurs also in the attempt of some to make scientific method the method of philosophy, which means the limitation of philosophy to such problems as are accessible to scientific investigation, with a denial, implicit at least, of the legitimacy of any other problems. Philosophers have been called "lords of the uncleared ground" of knowledge, concerned merely with matters that have not yet been dealt with by the sciences, and forced to content themselves with narrower domains after each new scientific conquest. Underlying such a conception of the unenviable position of philosophers there have lurked the ambition and the expectation of science eventually to dispossess philosophy completely by usurping the whole estate of learning. Were scientific method capable of dealing with all humanly significant problems, such a result would be inevitable and indeed "a consummation devoutly to be wished." I propose, however, to point out that there are problems which by their very nature are incapable of being reached by the scientific method and which, therefore, will remain indefinitely as distinct philosophical questions, questions, moreover, the importance of which is not surpassed by that of any of the problems of science.

In order to understand the limitations to the scientific method, it will be necessary first to state briefly what this method is. Its chief characteristics are observation, experimentation, and the application of mathematics in the formulation of the laws which observation and experimentation bring to light. It was Bacon who first clearly sounded the call to exact observation—to an accurate read-

ing of the book of nature without any prejudiced anticipation of what its contents might be. Experimentation is simply observation of events carried out under conditions of control such that the events may be isolated and repeated under identical or varied conditions, as the experimenter's wish may be. Then the conclusions of observation and experimentation are not only arranged in orderly form but they are given mathematical expression so far as this is possible. Kant said that a body of knowledge is scientific only to the extent to which it may be cast in mathematical form. Though there are other factors involved in scientific procedure, and though common usage of the term "science" justifies its application in fields that are not so mathematically exact as, for example, physics or chemistry, there is unanimity of emphasis in all the sciences upon observation of facts as the basis. As Professor Titchener says, "Scientific method may be summed up in the single word, 'observation.'" (*A Text-Book in Psychology*, p. 19.)

Scientific observation implies the existence of objects and the occurrence of events which are capable of being seen, weighed, and measured, or at least of being inferred from their sensible effects, not merely by one observer but by the whole body of scientific investigators who may take the trouble to examine the facts in question. As Professor Royce has said, "Successful description, made with any scientific purpose, seems to involve the possibility of comparing together the various attempts at description made by different observers in view of the same facts." (*Outlines of Psychology*, p. 5.)

The behaviorists deserve credit for having recognized and applied this principle. Their criticism of introspection as a scientific method and of introspective psychology as a branch of natural science is wholly justified. Mr. Watson has clearly stated the case against introspective psychology as a science in the following sentences (*Behavior*, pp. 6, 26, 27): "Psychology has failed signally during the fifty odd years of its existence as an experimental discipline to make its place in the world as an undisputed natural science." "It has enmeshed itself in a series of speculative questions which . . . are not open to experimental treatment." Mr. Watson and other behaviorists are determined that their science shall be really scientific, based solidly upon observation and experimentation, without resort to introspection. For the first time in history, in the hands of Mr. Watson and other behaviorists, psychology (for behaviorism still retains this name) has become a genuine science. As

Mr. Watson says: "The key which will unlock the door of any other scientific structure will unlock the door of psychology [i. e., of behaviorism]. The differences among the various sciences now are only those necessitated by the division of labor. Until psychology recognizes this and discards everything which cannot be stated in the universal terms of science, she does not deserve her place in the sun. Behavior psychology does make the attempt for the first time" (*Psychology from the Standpoint of a Behaviorist*, p. vii).

By what sleight-of-hand performance, however, it may be asked, has non-scientific psychology transformed itself into scientific behaviorism? It has done so by ceasing to be a study of consciousness, which requires introspection, a non-scientific procedure, and by becoming merely a study of behavior, which is indeed open to strictly scientific observation and experimentation. Thus Mr. Watson, in criticizing introspective psychology, speaks of the "mistaken notion that its field of facts are conscious phenomena and that introspection is the only direct method of ascertaining these facts" (*Behavior*, p. 26). He says (*Behavior*, pp. 7, 9): "The time seems to have come when psychology must discard all reference to consciousness; when it need no longer delude itself into thinking that it is making mental states the object of observation." "It is possible to write a psychology, to define it . . . as the 'science of behavior,' and never to use the terms consciousness, mental states, mind, content, will, imagery, and the like." And in his latest book (*Psychology from the Standpoint of a Behaviorist*, p. viii), he says, "the reader will find no discussion of consciousness."

If Mr. Watson and other behaviorists of his type did not continue to apply the term "psychology" to their science, few people would have any quarrel with them. We would agree that introspection is not a scientific method; we would agree that consciousness cannot be studied otherwise than through introspection; and we would therefore agree that the study of consciousness cannot become a science. We are justified in objecting, however, to the application of the term "psychology" (which means the study of consciousness if it means anything) to the new science when defined explicitly as being ***not*** a science of consciousness. And we are justified still more in objecting to the tendency of some behaviorists to deny the existence of consciousness from the fact that it is incapable of being studied by the method of science. In some of the above quotations from Mr. Watson consciousness is recognized as a fact, but as a

fact to be ignored since it is outside the possible field of scientific investigation. In the following passage, however, Mr. Watson asserts his faith that sometime behaviorism will study even consciousness itself. This would mean an implicit denial of the existence of consciousness in itse essential and unique nature. "Psychology as behavior," Mr. Watson says, "will, after all, have to neglect but few of the really essential problems with which psychology as an introspective science now concerns itself. In all probability even this residue of problems may be phrased in such a way that refined methods in behavior (which certainly must come) will lead to their solution" (*Behavior,* p. 28). To say that the observational method of behaviorism can eventually solve the problems of introspective psychology is to deny by implication the existence of consciousness, since consciousness, as has been indicated, and as will be shown more fully below, is something which can never be studied as an object among objects by the scientific method. Mr. Watson and other extreme behaviorists are guilty of what I have called the fallacy of *exclusive scientific methodology.*

My position will be made clearer by a further discussion of the meaning of consciousness and of the reason why it cannot be studied by the scientific method. So far as objects in the outside world are concerned, including other persons, there is no reality of which we are certain except such as is capable of common observation. We observe the behavior of animals and of men, but we are unable to *prove* that they are conscious. Even if our powers of observation were infinitely magnified so that the activity of each brain cell in a person under observation were capable of being inspected, we should observe nothing except what is the subject-matter of physiology and, in the last analysis, of physics and chemistry. As Professor Paulsen has said: "Let us imagine with Leibniz the skull of an animal or man to be as large as a mill. Suppose one could walk around in it and observe the processes of the brain as one can observe the movements of the machinery and the cogging of the wheels in the mill. . . . One would see as little of psychical processes, of ideas and thoughts, as in the movements of the mill

(*Introduction to Philosophy,* p. 84). Does this fact, however, that scientific observation is limited to physical processes prove that there is nothing except the physical in existence? On the contrary, the existence of consciousness is proved in the experience of each individual by the fact of his own awareness. As Professor Royce has expressed it: "Were physiologists better endowed with sense organs and with instruments of exact observation, we can, if we choose, conceive them as, by some unknown device, coming to watch the very molecules of our brains; but we cannot conceive them, in any possible case, as observing from without our pains or our thoughts in the sense in which physical facts are observable. . . . No microscope could conceivably reveal them. To me alone, would these states be known. And I should not see them from without; I should simply *find* them, or *be aware* of them. And what it is to find them, or to be aware of them, I alone can tell myself" (*Outlines of Psychology,* pp. 4, 5).

Thus, whoever asserts that scientific observation is capable of studying all of what properly goes under the name of psychology is refuted by the experience of each individual. There is consciousness and it is known directly only through introspection, which is not a scientific method inasmuch as its objects are not objects of common observation. What I refer to in saying, "I am conscious," or "I experience a sensation," is beyond the reach of a purely objective scientific study. To assert that scientific method is the only method, and especially to assert that nothing exists which scientific observation is incapable of reaching, is to commit the *fallacy of exclusive scientific methodology.*

The objection might be raised, however, that consciousness may be capable of becoming an object of scientific investigation through its sensible effects, just as in the case, for example, of electricity. Scientific hypotheses regarding electricity may be tested through observation of its sensible effects. There is not a correct analogy here, however. Few of those who assert that consciousness is a fact would admit that it has any observable effects in the outer, physical world. To assert that it does influence physical events

would be to affirm the theory of psycho-physical interaction, which is contrary to the principle of the conservation of energy. The existence of electricity and of similar scientific entities "makes a difference" in the occurrence of observable external events, but consciousness, in the sense in which the term is employed in this article, cannot "make a difference" in the phenomena of physics and of physiology.

Behaviorism may limit itself to an objective study of behavior merely, without using terms referring to psychical contents, while admitting that consciousness is real but accessible only to a philosophical study; and when it does this I classify myself as a behaviorist. It seems to me probable that there is a mechanistic, i. e., physico-chemical, basis of all human behavior; including language and the processes involved in such complex responses, for example, as those of Shakespeare in writing his plays or in the work of a mathematician in formulating the principles of mathematics. And I am in hearty accord with the general spirit of the work of such a biologist as Mr. Jacques Loeb in his studies of behavior. Mr. Loeb, however, commits the *fallacy of exclusive scientific methodology* when he comes to a discussion of consciousness. Objecting to the term "consciousness," which he correctly calls a metaphysical concept, he substitutes the term "associative memory." (See *Physiology of the Brain*, pp. 214, 15, 17, 32; *The Mechanistic Conception of Life*, p. 73.)

He then proceeds to define associative memory" in purely objective terms, as docility, or the capacity of the organism to learn new responses and consequently to modify inherited forms of response. He says, for example: "By associative memory I mean that mechanism by which a stimulus brings about not only the effects which its nature and the specific structure of the irritable organs calls for, but by which it brings about also the effects of other stimuli which formerly acted upon the organism almost or quite simultaneously with the stimulus in question" (*The Mechanistic Conception of Life*, pp. 73, 74). This is a case merely of the "conditioned reflex," and in his most recent book Mr. Loeb employs the latter term

(*Forced Movements, Tropisms, and Animal Conduct*, p. 167). The fallacy involved in substituting the term "associative memory" for the term "consciousness," and then in defining associative memory purely in terms of behavior, is that of denying by implication the existence of consciousness in its proper sense, as a fact of inner experience inaccessible to a scientific study.

There is a philosophical type of behaviorism which, if its metaphysical basis could be proved correct, would study consciousness itself objectively, thus escaping the fallacy of ignoring or of denying the existence of consciousness. I refer to such a metaphysical theory of consciousness as that, for example, which William James has so well expressed in his philosophy of radical empiricism, and which has been incorporated in the philosophy of American neorealism. If one goes beyond both "common sense" and science, and asserts that the content of each personal stream of consciousness is identical with that portion of the outer environment to which the organism reacts selectively, then one may say that consciousness itself, conceived in the manner that such a theory presupposes, becomes an object of common observation. James, for example, maintained that "a given undivided portion of experience, taken in one context of associates, plays the part . . . of a state of mind, of 'consciousness'; while in a different context the same undivided bit of experience plays the part of a thing known, of an objective 'content'" (*Essays in Radical Empiricism*, pp. 9, 10). On this assumption consciousness itself may become an object of common observation, but this is a metaphysical assumption that no behaviorist merely as such can accept; nor can scientific method establish the truth of such an assumption.

In the field of general philosophy the *fallacy of exclusive scientific methodology* is committed by those naturalistic philosophers who claim for reality at large what extreme behaviorism claims for its special portion of reality, namely, the exclusive validity of the scientific method and the denial or the ignoring of the existence of anything beyond the reach of this method. A materialistic philosophy consisting of a generalization from the sciences may be refuted in

the same way as that in which an extreme behaviorism may be refuted. My personal awareness is, as has been shown, beyond the reach of scientific observation, and yet it is the thing that is most indefeasibly real for me. I thus am certain of at least one bit of reality of which not only extreme behaviorism but also a general philosophy of materialism, based exclusively on scientific method, denies the existence.

Whether or not some sort of an idealistic system of philosophy may be built up from this initial certainty of individual consciousness is another question, of which I will merely suggest a possible answer. The fact that man as viewed outwardly by the scientist is only a complex machine not different in kind from the other mechanisms in nature, while we acknowledge, though we do not observe, in every person a conscious life, which is purposive as purpose is experienced by ourselves—an inner freedom which is outwardly determined,—this fact suggests that the universe at large, though it is purely mechanical as the sciences describe it, no less so and no more so than the human organism, may also have an inner purpose, even a free and conscious purpose, no less real and perhaps infinitely more significant than finite human purposes. Just as I ascribe to other persons an inner, conscious life which objective observation does not reveal to me, while at the same time viewing them outwardly as mechanisms explainable in terms of stimulus and response, so to the totality of existence, explainable outwardly in terms of mechanistic science, I at least cannot deny the possibility of an inner life analogous to my own inner life and to that of my fellow men. To deny the possibility of such a reality on the ground that it is not an object observable by scientific method would be to commit the *fallacy of exclusive methodology.* If there be such a conscious purpose in the universe at large, it is by its very nature as inacessible to scientific observation as is my own consciousness.

It seems strange that even many philosophers should have been so much dominated by the prestige of modern science. That the popular mind should fall under the spell of science is easily enough

accounted for. Science is practical. As Bacon said, scientific knowledge is power over nature. As a result of modern science, nature has been turned to man's uses to an extent that earlier generations would not have believed possible. Applied science has yielded innumerable satisfactions to man's desires for physical health and comfort and achievement. The progressive conquest of nature by science even stirs the blood by its dramatic venturesomeness. In contrast with the picturesque story of modern science, the story of philosophy seems uninteresting to the type of mind that does not care to exercise its human prerogative of abstract thinking. Philosophy is not practical; it can bake no bread: and yet it deals with problems that are, in a sense, the most practical of all problems in so far as their solution can help mankind to feel at home in a universe that seems outwardly so alien to man's inmost interests.

While one can understand why the popular mind should think the term "scientific" always a complimentary one, and the term "philosophic" almost a reproach, one can hardly forgive this attitude in those of greater discernment. Why should the student of the mind object to the admission that a part of his field is not science but philosophy? Why not take the epithet, "arm-chair psychologist" as a genuine compliment (even though it be not intended as such)? The psychologist who does not go beyond the scientific method of the laboratory to the reflective method of philosophy fails to cover the whole field of psychology. Behavior can be studied scientifically and behaviorism can be applied in such practical fields as advertising, salesmanship, pedagogy, and the like. The scientific study of behavior can be of great assistance also, in an indirect way, to the understanding of purely theoretical problems of consciousness itself. The philosopher should make what use he can of scientific result. But psychology will always be in part a *philosophy* of consciousness or else it will be an incomplete psychology. The sciences may and ought to aim at a complete description of the physical world; and yet there will always remain, over and above the most complete scientific descriptions that are possible, philosophical problems such, for example, as the one briefly touched upon above. By its very

nature such a problem cannot be dealt with by the scientific method. The philosopher, if he rightly conceives of his problems and of his method, can never be displaced through the advances of science. He should co-operate with the scientists, welcoming each new scientific achievement, while being proud at the same time to be a worker in the important and distinctive field of philosophy.

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## ETHICS, MORALITY, AND METAPHYSICAL ASSUMPTIONS

IF WE START with the supposition that ethics and metaphysics are two distinct sciences, there are four possible relationships which they may have to one another. Either ethics is based on metaphysics, or metaphysics is based on ethics, or they are mutually independent.

It has been the rule, perhaps, rather than the exception, to assume, often without proof, the first relationship, namely, that ethics is based on metaphysics. Ethics becomes a kind of appendage, though an inseparable appendage, to metaphysics; logically and often temporarily posterior to it. Examples of this are to be found in rationalistic systems like Spinoza's, in naturalism, such as Spencer's or Stephen's, or in idealism such as Bradley's. We say "often temporally posterior," because it is not always or necessarily so. Bradley's ethics, for example, were not temporally posterior, but temporally prior to, his metaphysics, but his idea of harmony and individuality demand for their completion, and so really presuppose, his metaphysical absolute Idealism.

The second alternative, the basing of metaphysics on ethics, we shall not be concerned with here, to any extent. Kant is the great classical example of this second alternative, but even Kant, although he asserted the primacy of the practical reason, worked out his enquiry into the nature of metaphysics (which resulted in his denying the possibility of metaphysics) before his ethics proper, and his

stress on the fact that the metaphysical ideas of God, Freedom, Immortality, are simply practical postulates, is an outcome of his examination, his criticism, of metaphysics. If it is not strictly true to say that Kant's metaphysics is prior to his ethics, since he denied metaphysics, it is true to say that his views, properly epistemological, on the possibility and character of metaphysics, are prior both logically and temporally to his ethics.

The most modern attempt definitely to construct metaphysics upon an ethical basis is to be found in Professor Sorley's Gifford Lectures, "Moral Values and the Idea of God."

We shall come to certain conclusions about the third possibility, the mutual dependance of ethics and metaphysics, by an examination later of the first alternative.

The fourth possible alternative mentioned was that metaphysics and ethics should be mutually independent, standing alone as separate sciences. The plea for independent metaphysics is seen in the claim for a more scientific philosophy, rid of ethical prejudice, made by thinkers such as Mr. Bertrand Russell. A philosophy of this kind, and the claim for it, is the outcome of an emphasis on the physical and material rather than on the spiritual problems of the universe. The demand that ethics should be an independent science is also "the prevailing doctrine of the Intuitional moralists and may be found in the Scholastics before them. Certain ethical propositions—such as those that affirm that justice, veracity, and the common welfare are good—are held to be self-evident, not derived from mathematical, causal, or any other purely theoretical propositions. Ethical truths, and truths of theoretical philosophy will be regarded as arrived at in the same way, . . . but there will be no primacy of one over the other; if metaphysics is not a result of ethics, neither is ethics derived from metaphysics. And this method, as far as

regards ethics, has often been employed by writers like Richard Price, who have not worked out any metaphysical system, as well as by others—Reid, for example—whose ethical doctrine is part of a general philosophical view."[1] The best modern example of ethics as based on intuitions is perhaps Sidgwick, who holds that "there are certain absolute practical principles, the truth of which, when they are explicitly stated, is manifest."[2] Examples of some of these principles are, the Golden Rule, which Sidgwick restates this: "It cannot be right for A to treat B in a manner in which it would be wrong for B to treat A, merely on the ground that they are two different individuals, and without there being any difference in the natures or circumstances of the two which can be stated as a reasonable ground for difference of treatment."[3] The truth of this, he says, "so far as it goes, appears to me self-evident."[4] Again, the principle of Rational Self-love or Prudence—that one ought to aim at one's good on the whole; and the principle that the good of any one individual is of no more importance, from the point of view of the Universe, than the good of any other; are regarded by Sidgwick as ultimate, self-evident; and requiring no metaphysical or other justification. Summing up, Sidgwick says, "The axiom of Prudence, as I have given it, is a self-evident principle, implied in Rational Egoism as commonly accepted. Again, the axiom of Justice or Equity as above stated—'that similar cases ought to be treated similarly'—belongs in all its applications to Utilitarianism as much as to any system commonly called Intuitional: while the axiom of Rational Benevolence is, in my view, required as a rational basis for the Utilitarian system."

[1] Sorley, *Moral Values and the Idea of God*, p. 12.
[2] *Methods of Ethics*, 6th ed., p. 379.
[3] Op. cit., p. 380.
[4] Op. cit., pp. 386-7.

This view, indeed, any view, which regards ethics as independent as based on intuitions, we must reject in the last instance as dogmatic. Maciver,[8] discussing ethics, which he is here assuming to be an independent study, asks, "Is there a science of ethics? If we turn to the authoritative works on ethics we find they are devoted primarily to the question, what is the supreme good or the supreme good for man? . . . But we discover soon enough that there is no body of accepted doctrine in respect to that problem, and that in the nature of the case there can be none. For if I say . . . that what man *ought* to seek is happiness, how can that statement be controverted except by an equally dogmatic statement that they *ought not* to seek it . . . ethics is . . . concerned with the question of ought, the question of right and wrong, good and bad. It is concerned, that is, with a question lying beyond the bounds of scientific procedure, beyond verification, beyond induction, beyond actuality. . . . All ethical claims are claims of worthfulness, and we can neither confirm nor refute them save by our own estimate of their worth." And this is true, most certainly, if we accept the veiled assumption that ethics is an independent study. Only our whole contention will be that it need not be, and indeed must not be, so regarded, that judgments of worth or value must be tested for their truth, just like any other kind of judgment, by their coherence or consistency with the whole scheme of knowledge regarding nature, man, and his place in the cosmos. We may hold certain intuitive beliefs about values, but if these beliefs cannot be supported by cold clear reason, bringing to them everything relevant from science and other knowledge, then we have no right at least to base a *theory* of ethics upon them, however valuable they may be in the guidance of our practical life.

[8] *Community, a Sociological Study*, pp. 52 and 53.

The attempt, then, to work out a moral scheme for man, apart from any consideration of his origin, his psychological nature, his relation to his environment—and therefore, to some extent the general nature of his environment—seems to us to narrow, quite unreasonably, the sphere of morals, and indeed to render any worthy answer to its questions impossible. To rest ethics upon an intuition unsupported by grounds of reasonable knowledge, is to render it subjective and limited to the beliefs of a particular time, place, society, or even individual. The test must, in the first place, be one of consistency, and in the second place, it must be cosmic rather than local, eternal rather than merely temporal. Man in his deepest nature is an active member in this wider polity of the cosmos, and it is in the demands and experiences of his deepest nature that we must seek for ethical principles. Man regarded thus, as on the rim of the vast cosmic whirl, may appear a little, a solitary, perhaps a lonely spectacle; but this solitariness, this aloneness, even this loneliness, of man, is as ultimate a fact of his nature (a fact too often forgotten when his sociality is stressed), as is his dependence for his very self upon his fellowmen. Man has desires, though too often he does not know it, that cannot be satisfied except in this aloneness with the infinite, when his spirit is filled with what the religious call religion and the holy-minded call worship. Ethical systems hitherto have been based almost solely upon the fact of man's community with his fellows: the idea which we shall here attempt to work out will emphasize, though it must never, never, emphasize exclusively, the fact of man's solitary relationship with the wider cosmic whole. In the very realization of this wider relationship is to be found, we believe, the key to the nature of goodness, whereby can be unlocked the doors of these problems of man's no less important relationships with his fellows. And in so far as discussion of man's place in the

cosmos involves questions as to the ultimate nature of the cosmos, so far it will be necessary to make, dogmatically perhaps (since this cannot be a metaphysical essay as well), one or two assumptions about the general nature of reality.

There are certain problems surrounding the relationship of ethics to metaphysics, which require, perhaps, a preliminary treatment. One of the most usual objections to the view that ethics must be based upon metaphysics, is that the "ought" can never be based upon the "is" because there can in the nature of things be no passage between the two. And in a sense this is true. From the "is" as such, it is not possible to deduce the "ought," if "ought" implies a standard above that which is. But as soon as we examine it we see that this is true only in the narrowest possible sense, in so narrow a sense indeed, that it is nothing more than tautology. If the "ought" and the "is" are defined negatively in terms of one another (i. e., "ought"= something which (at least) *is not,* and "is"= something which (at least) is *not "ought"*) then by definition it is impossible to find the positive meaning of one in terms of the other. But there is, in fact, a sense in which the "ought" can be discovered in the "is," and that is, when what ought to be does actually exist. Aristotle, for example, took as his standard of what *ought* to be the actual choice of the morally wise man. So, although in the very narrow, logical, even tautological, sense we have mentioned above, it may be true that if I *am* just in a particular case there is no meaning in telling me that I ought to be just in that particular case, yet it is in a broader sense palpably true that if one ought to be just in such and such a particular way, and I am just in such and such a particular way, then in that case at least the "ought" has become "is."

It is indeed the "ought" in the larger sense of the moral ideal as a whole, that philosophers have alluded to, when they have said that it is not derivable from what is. But

if it is in no sense derivable from what is, how is it possible to justify its claims in the scheme of knowledge, how is it possible to assert it otherwise than, as we said, dogmatically? Further, how can it be given any *content* at all, if we are not allowed to derive its content from what is? Or, what is the same thing from a slightly different point of view, how can the moral ideal which is in no sense and in no degree realized, have any meaning? It may be a fact that moral ideals, what ought to be, are never fully realized, never, in the full sense, actually *are*. So it may not be possible to discover the ideal as an actual existent *fact* in what is. But what ethics is concerned with mainly, is, not the discovery of a fact, as such, but a value which is a standard by which oughts may be measured. And there is no inherent impossibility, we believe, in the finding of real values, and more especially the direction in which real values lie, in reality as it actually exists.

But the view that the problem of the relation of ethics to metaphysics is the problem of the relation of "ought" to "is," is indeed a somewhat narrow one. In reality the question is much bigger, as we hinted in our last paragraph. It is the question of the relation between judgments of value and judgments of fact. A judgment of value need not be in the form of "ought," indeed in so, and directly, it cannot be in the form of "ought." Judgments which contain "ought" are really dependent upon judgments of value which do not contain "ought" themselves. "This is good, therefore I *ought* to strive to attain it." Here the second clause is directly dependent upon the first. And the first, notice, is a judgment of fact of a particular kind (justifiable, often, in terms of a wider realm of fact), "this is (as a matter of fact) good. . . . A judgment of value may then, clearly enough, be a judgment of fact, and vice versa. There is no unbridgeable gap between them, as is often supposed. The reason why there is so often sup-

posed to be such a gap, is, we think, just because the relationship between fact and value is confused with that between "is" and "ought," in the narrow sense alluded to on page 6. If "is" and "ought" are defined negatively in terms of one another, then it follows by definition that "ought" can never be discovered in what is, just because "ought" is (explicitly or implicitly) defined as something which *is not*. And the supposition that an absolute "ought" can never in *any* degree be or exist, the supposition which seems to make it imperative for some moralists to say that "ought" should be defined as (at least) what is not, seems to be grounded in a frantic conviction that if we ever admit that the "ought" or ideal is in any degree attainable, we have destroyed completely its ideal nature. So we must preserve, like Moses, the bounds of the moral Sinai, lest the people, breaking through to gaze, should perish.

The gap which exists as a matter of fact between what is and what ought to be is not for a moment to be denied, of course. We are only maintaining that it is a relative and not an absolute one, and further that the existence of this gap is no warrant for any assertion that there is a gap necessarily existing between any given fact and any given value. A value may be a fact, and vice versa, as we saw. It is indeed the task of the moral life to make values into facts, and for the individual it is essential that he should be conscious of the gap which lies between the fact of his life and the further values which he desires to realize in himself. It is because the desirable thing is just beyond our present reach that we must strive to bridge the gap by our efforts. What ought to be and what is, value and fact, are in this sense, and only in this sense, separated from one another. But the very fact of the moral life is a witness to their constant abridgement by human effort, and although the abstract Ought (with a capital letter) may theoretically never become *is*, or moral effort would

cease, yet the concrete, particular ought is always and continuously being brought into existence.

We have been concerned in the preceding paragraphs with showing that within the strictly moral sphere, fact and value are not necessarily opposed, that even "ought" and "is" themselves are not necessarily opposed, unless we define them in a narrow and restricted way. We have established, with some trouble, what is perhaps an obvious fact when it is once reflected upon. It has been necessary to establish this fact because of the undue stress laid in ethical text books upon the opposition between what is and what ought to be, which is usually assumed to be parallel to the distinction between a natural and a normative science.

But in confining our issue to a somewhat narrowly moral example, we have not yet really faced the fundamental difficulty whch besets the problem of the relation between ethics and metaphysics. That difficulty is, not, can values be facts? or, can the "ought" exist as a matter of fact? but, can we pass from a non-ethical proposition to an ethical one? If we can, then, says the argument, ethics is derivable from metaphysics; if we cannot, then it is not, and the ground of ethics must be found elsewhere, presumably in some sort of intuition.

There is a supposition in the assumed antithesis between ethics and metaphysics which requires careful examination. The supposition is simply that, while ethics has to do with ethical propositions, metaphysics is thought to be concerned with non-ethical ones. And the question arises, is it a valid assumption? May not our third alternative stated in the first paragraph, be truer? May not ethics and metaphysics overlap, and so be mutually dependent? Is the subject matter of metaphysics definitely and necessarily non-ethical, in the sense of excluding judgments of moral value?

To say that it is, seems, at first, anyhow, to imply an unnecessarily narrow view of the scope of metaphysics. If metaphysics means what Descartes, for instance, thought it to mean, or if it means what can be proved by the empirical methods of natural science, then it is perhaps true, that from this brand of metaphysics, there can be obtained no judgments of value. If metaphysics is so defined (implicitly or explicitly) that it excludes judgments of value, then of course it follows by definition that no ethical results can follow therefrom. Ethical text books are right when they oppose ethics to the natural sciences, for from these as such can be derived no ethical propositions. But metaphysics, though it may deal with, and be influenced by, truths which it is the business of the scientist to discover, is certainly very much more than a summation of their truths. It is, rather, being the attempt to understand *experience* as a whole, in itself a valuation of the truths of science. In setting them in their place in the cosmic scheme, it cannot avoid interpreting them according to some principle of value or another, and any such unifying interpretation is in itself a valuation. That is what, in the end, Plato's "Form of the Good" meant. It is then not merely that we must take into account in our metaphysics what are usually called moral values, in the sense of values relating to human conduct. We must in addition pronounce judgments of value in terms of some dominating value upon facts and truths which are in themselves non-ethical.

But the question still remains unanswered. Admitting that metaphysics must include strictly moral facts in its survey, and that it must arrange truths under some dominating conception of value, is it true that this value is of an *ethical* kind? The answer to such a question would of course depend upon the type of metaphysics which is assumed. Certain theologies, for example, have conceived

of the universe in a strictly moral or ethical sense, "moral" and "ethical" being taken here in their etymological meaning, as related to human custom and tradition and conduct. The orthodox Christian theory of the Atonement is an instance. It is based upon a notion of justice or requital more or less savage (according to the way in which it is interpreted) which is grounded quite definitely in a pre-Christian morality which had its salvation in the blood of goats, and which however it is interpreted is never less than crudely "moral" in a definitely anthropomorphic sense.

This is an extreme example, and it may be maintained that it is possible to explain the universe in strictly moral terms without falling into the nets of anthropomorphism. Perhaps it is so, and our disagreement with such a view may in the end turn out to be a matter of terminology. It seems to us, however (and it is a strong thing to say), that the terms "moral" and "ethical" can never, without distortion be made large enough to include the universe, or to be made its supreme predicate, although interpreted in a large sense they may well be one of several fundamental predicates. The terms "morality" and "ethics" can never wholly escape from their own etymological significance, and even if they could, it still remains true that morality is *not* the whole of man's nature, though everything he does or thinks or feels is capable of moral valuation. And a metaphysics, to be true, must, as Mr. Bradley has said, satisfy *all* sides of our nature. Our own view is that the aesthetic side of our nature has received too little consideration from metaphysics. A complete view of the world must indeed give complete justice to the fundamental values, beauty and truth as well as goodness.

It is not our purpose here to construct or defend any such complete view of the world. We must return to our original question—as to whether the subject matter of

metaphysics is definitely and necessarily non-ethical—answer it, and so see more clearly the relation of ethics to metaphysics, and from that proceed to examine the general nature of the effect upon the moral life of a definite view of, and attitude towards, reality. The meaning of this very abstract programme will perhaps grow clearer as we proceed.

Our answer to the question as to whether the subject matter of metaphysics is ethical, depended, we saw, on the way in which it was taken. Metaphysics must, being an account of experience in general, take into account the facts of moral as well as other kinds of experience, and it must be a valuation of the facts of experience. On the other hand, although metaphysics is a valuation, we found we could not hold it to be a moral valuation, in the strict sense of "moral," because that tended to involve an anthropomorphic view of the universe. And now, as we have accepted provisionally the view that ethics must be grounded in metaphysics (the grounding of metaphysics in ethics can only mean that metaphysics must take account of ethical values), we have to face the problem of how ethics, the science of moral value, can be related to metaphysics, which uses some other conception of value, which we decided at least could not, strictly speaking, be called moral. Can the moral judgment, "to do this is good" be shown to be true or false by reference not merely to the realm of morals itself, but by a reference to the nature of the larger cosmic reality of which man is a member as truly as he is a member of the community of men, and by reference to man's place and function in that larger community? Is moral good explainable in terms of a wider value which cannot itself be called "good" in the same sense? If it can, then ethics will be based on metaphysics which is not itself strictly ethical (though it must include and therefore be

profoundly influenced by ethical facts). A transition will be possible from the one science to the other.

As it is the practical aspect of the relationship which concerns us here, it will make the problem clearer if we approach it from that point of view. We shall not give any reasoned account of our view of the kind of metaphysic upon which ethics may be based, or study their theoretical relationships, but shall examine the practical effects which follow from a certain definite attitude to the universe, an attitude which no doubt does involve very far reaching metaphysical assumptions and consequences which it is impossible here to elaborate.

The problem is then now, not so much, "is moral good explainable in terms of some other kind of wider value?" as, "is my conduct affected morally, for good or ill, by my attitude towards the wider objective world, and if so, how?" and, "what is the nature of my attitude, and what assumptions does it involve?"

Taking these questions in reverse order and answering them boldly and in loose fashion, we may say, firstly, that we presuppose that the universe is at least a good universe; secondly, that a man's attitude towards, or relationships with, a universe which he presupposes, and finds by experience, to be good, can involve a definite emotional tone, to be described; and thirdly, that this attitude or relationship having emotional tone can so affect character that conduct is influenced, and made morally good, thereby. In a word, and still more loosely, it is the emotion which we experience when we perceive, sometimes in a flash, universal good in and through the objects (or persons) of our experience, which is the inspiration of the good life. This is necessarily vague, and in some measure incorrectly stated. It is impossible within the scope of this article to justify it fully. For the present, let us examine certain difficulties which appear on the surface.

In the first place, we have used the forbidden term "good" of the universe. Our excuse is lack for the present of a better word. In order to understand what we do mean by it, we must consider the kind of experience which we have when we make such a judgment as "it's a good world." For it is with an experience, and not with the validity of metaphysical judgments that we are now concerned. We are not referring to the sense of mere physical well-being which a man has, for example, after a comfortable meal, in a deep armchair before a warm fire with a glowing pipe, nor even such exhiliration as we suppose even the old must feel on a morning in the first springtime. These kinds of experiences do often give rise to utterance of optimistic judgments, but it is something bigger and deeper than this to which we refer. The experience may dawn slowly upon us when in contemplative vein, or it may flash suddenly like a divine light upon some incident, person, or thing which before had seemed trifling, even worthless. It is a glimpse of the universal through the particular, it is what men of religion have described as a vision of the ineffable in things of sense, and indeed it is religious experience of a kind, though it does not necessarily involve any formal theological creeds. It may be aroused by contact with natural objects, by intercourse with fellow human beings, humble or great, by joy in work, by creation, by discovery, and by adventure, by things of the mind or things of the body, by painting, music, poetry, or any of the arts, by, in fact, any human experience whatsoever, short of what is gross or hideous or immoral. It is a common experience of souls we are wont to call great, of those who have vision to see the universal spirit in little particular things, of men who are able to discern the true value of life and reality, who could say, like Jesus to his unseeing followers, "Suffer little children, and forbid them not to come unto me: for of such is the kingdom of

heaven," or, "Consider the lilies of the field how they grow; they toil not, neither do they spin: and yet I say unto you, that even Solomon in all his glory was not arrayed like one of these." It is the common experience of great men, but it is not their monopoly; and neither do they have these experiences because they are great first. Rather do they become great and greater because of the greatness which they are continually able to see in everyday things. And it is that vision which prompts them, and us through their influence, to exclaim, "it is a good world."

The goodness we may thus predicate cannot justly be called moral goodness, neither is it exactly beauty, even though it may be revealed both in human moral goodness and in the contemplation of beauty in art or nature. It seems to be simply intrinsic value itself, and indescribable in terms of any other particular kind of value. And yet on the other hand, it appears to partake both of the nature of goodness and of beauty. "Our appreciation of a beautiful sunset," says Professor Sorley,[6] "differs from our appreciation of a good deed or a good character. The former is admiration simply, the latter approval." And later,[7] "We do not speak of a sunset as good instead of beautiful, or, if we do, we recognize that we are not using the word 'good' in its ethical meaning. It is more common to apply the word 'good' to the work of human art, and still more common to apply it to the artist . . . moral approval is something superadded to aesthetic appreciation and not identical with it." It may be, therefore, when we say, "it is a good world," that there is a blending of two different kinds of judgment, one aesthetic, the other moral, either of which can predominate, according to circumstances. We may, even in contemplating nature, for example, experience a wave of sympathy with something definitely spiritual

[6] Op.cit., p. 32.
[7] Op. cit., p. 33.

which seems to reside there. There is a feeling of harmonious unity within ourselves due to the fact that we are "at one with nature" which is outside ourselves. In certain moods, and with certain types of mind, this attitude may become definitely religious; we may adore reverently, we may feel gratitude to a Person for the joy that is, not merely given to us from without, but which comes from a Being greater than, but intimately akin with our inner selves. In these moments we may say, not simply, "it is a good world," but, "God is good" and therefore, "it is good for us to be here." At other times when the vision comes with equal intensity, literally "the light that never was on sea or land," we may be so absorbed in the contemplation of beauty that we have neither ability for words nor need of them. In these moments it is still "good for us to be here," there is still a direct experience of communion with a universal spirit in particular things; but the experience itself is sufficient, and it is only afterwards, if at all, that we may make judgments of goodness or even of beauty. The first, the religious attitude, is one involving a consciousness of distinction of self from a Person, even whilst at one with that Person; in the second, the aesthetic attitude, we seem to be (that is not to say that we *are;* we are only analyzing the felt experience) so absorbed in, so projected into the object which we contemplate, that it is sometimes with a start that the spirit, after such an experience, realizes that it is, after all, attached to a physical body. Aesthetic contemplation in itself involves no desire to think or act in any way, though it may give rise afterwards to speculation about its meaning or to deep desires for artistic creation. The self-conscious religious attitude to a Being who is thought of as good in one in which it is easier to define the sort of value which is predicated, than the aesthetic attitude in which per se no predication is involved at all. If the aesthetic

experience simply died with itself, as it were, it would not even result in judgments and we could therefore say nothing about the view of the world which it engenders. It is because the man who has the aesthetic experience is also a remembering, thinking, acting being, that we can truly say that the judgment, "it is a good world," is reached through the avenue of the aesthetic experience as well as through the religious or moral one.

The judgment, "it is a good world," would indeed have an entirely different meaning for the man with religious, and the man with aesthetic tendencies, if either of the two were a complete type in himself, or existed in a water-tight compartment. It is because man cannot be solely interested in any one of the three, the good, the beautiful, or the true, that "good" in "it is a good world" cannot be given definitely moral, or aesthetic, or intellectualistic content. The artist (if we take him as representing roughly the man of aesthetic sensibility) may hold no overt religious beliefs, may even be, like Shelley, what is called an atheist, or of what are (again) called "loose morals." He proves, nevertheless, by his actions, that he believes in an ultimate value in reality worth striving to express. Again, the "moral" or religious man may regard definitely ethical or moral qualities as of the most fundamental importance in the universe, and yet he may be more susceptible than he knows to beauty. Most probably it will be to natural beauty that he will turn, since that satisfies his bias toward religion and enables him to think of a "Divine Creator."

We must, in fact, reassert that, if our judgment is to be true when we say, "it is a good world," the term "good" must indeed imply value unqualified; unqualified just because if explored into its depths, it must bring us face to face with any and *all* the values that reside in the universe. "All the great values are cognate with each other,"

says Bosanquet,[8] "and any of them can be reinforced and vitalized from any other as a point of departure." And each man has his own interests, which to a large extent determine his environment. This environment displays in its character some dominant value, whether intrinsic or instrumental, and it is as a rule this value which seems most important to those who perceive it. To the philosopher it may be truth, to the aesthetic beauty, to the practical moralist goodness. These are intrinsic. To the social worker the most important value may seem temperance, or housing, or sexual morality: to the tradesman it may be honesty or money or leisure; to the public man it may be power, and so on. The dominating object of desire in a man's life is for him the dominating value. But for philosophy it is essential to rid oneself of personal bias, even with regard to the fundamental intrinsic values. "It is a good world," may mean, for me, that the world is good because in men's lives or in nature we see shining a larger, wider goodness, or it may mean that it is good because I experience delight in a labor for truth, or because beauty is there, and beauty seems enough. But for such a judgment to be true in the completest sense, it must hold nothing less than the fullest meaning which is in and beyond all men's minds when they make it.

That is the difficulty inherent in the word "good." "My ways are not your ways, saith the Lord," and finite man the moralist has no right to attribute his little human concepts born of human relationships to the Cosmic Spirit. Moreover, even his finite brother-man the artist would not agree with him. There are three senses in which the term "good" might be used. The first is the sense disputed above, in which it might be used of the world. The second can be predicated of a person, and might be divided into two classes, in one of which it is applied to God, in the

[8] *Some Suggestions in Ethics*, p. 234.

other to man. The third sense is used of actions which have intrinsic moral value, as when we say, "this is a good action." Some moralists would distinguish a good action from a right one, but we are not at present concerned with this. Now, when we speak of "a good action," or "a good person," if that person be human, the meaning is clear. When applied to God it is only clear if God be represented in some anthropomorphic way, as, for example, a Father. When applied to the universe, it has more or less definite meaning according as the Spirit of the Universe is identified or not with a personal God, but it seems that the more clear and definite the meaning of the word "good" the less true is it to apply it as a general predicate to the universe. Because good in this sense is clearly a value limited and defined in human lives and relationships (it is better that "good" should be taken to mean this) and although *moral* value may be regarded (in some greatly enhanced form) as belonging to the universe or reality as a whole, it would there become transfused with other values equally fundamental, and it would be the fusion of all these values which affects us when we have that vision of the universal through the particular, which we have described. The word "good" is then misleading, and our first conjecture (page 16), that it can be counted as nothing else but simply *value,* is perhaps most true in the end. If we are content to accept that it is in Universal Value (whatever that may mean) that all values meet, that the Universe is one, and that goodness, beauty, and truth, are but aspects of it (though they do not for that reason lose their identity), then we may begin to understand how it is that in a world of all sorts, all sorts and conditions of men may, if they will but gaze outwards from themselves, see Value in the infinite from the side of the finite, and be strengthened thereby in their own finite lives. It is in that sense that the one Value which before we called by the name

"good" may appear to ten thousand men in ten thousand different ways, though mainly by the great avenues, Goodness, Beauty and Truth.

It has not been our purpose, we may once again repeat, to establish one existence of such a value by metaphysical argument, but rather to describe, and without any attempt at psychological precision, the *experience* of it which may give rise to the judgment, whether spoken or implied, that it exists. The experience may come, we have said, mainly in one of three great ways. But whether it comes in the midst of practical life, or through speculation by the intellect, or through contemplation of beautiful objects, it must be something more than merely cognitive. It must be a felt experience, with a definite emotional tone which it is possible to describe.

It is important, then, to realize the significance of the value experience, for it is possible, we think, to base a whole moral theory upon it. Man, we believe, must, as he draws from the external world food and drink for his living body, draw from the cosmic value-experience the dynamic energy of his ever-growing moral life. And we can no more *explain* this moral life without reference to its cosmic environment, than we can form *theories* of physiological processes without reference to substances drawn from the wider realm of physical nature.

Our main metaphysical assumption has been that the Universe has Value which may at certain times be experienced by knowing, feeling, acting man.

It may seem perhaps that our assumption is too large a one to stand without some metaphysical proof, too easily optimistic to be accepted uncritically as the basis of a philosophy of morality. It may be said that we cannot, however we will, avoid considering facts so vital as pain and evil, or to take another aspect of the same thing, we cannot blind ourselves to the apparently non-moral, even the

anti-moral character, of the evolutionary process in nature. Pain has destroyed the moral life as well as made it. Surely if experience is to be the criterion, a pessimistic experience is just as real, just as much a fact as the Value experience which we have made so all important? We cannot here refute such a contention. Our only answer here to such a question would be that although it is possibly true in a sense that a pessimistic outlook on life is as much a fact as an optimistic one, the question arises as to whether it is possible to construct an adequate ethics upon such an experience as its base. If such an ethics could be constructed satisfactorily, then it would be its own justification, and optimism would stand condemned. We are of the opinion that the difficulties in the way of such a pessimistic ethics are insurmountable. As optimism must explain the fact of pessimism (which we think is largely the failure to distinguish between pain and evil) so would pessimism have to explain away the facts of the value experiences.

Even the fact of pain, so poignant in nature, is not, we can here only rather dogmatically assert, a fact that sets us at any real enmity with her. Our combat is but a harnessing of nature's forces to the chariots of the moral life; and the experience of value, even in pain, is the supreme testimony to the triumph of spirit over matter, of mind force over brute force. Once more, then, the felt knowledge, even through pain, that Value is in the world, can be a force revitalizing the whole texture of the moral life, the full realization of the importance of which must certainly, in its turn, most powerfully affect moral theory.

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# PERCEPTION AND NATURE

FORMERLY the philosopher's example was the table. One observer viewed it from one angle, another from a different, both at rest relatively, but in different axis systems. The inevitable question, "What is the real table?" followed. There is, of course, an assumption here, namely, that there is a real table, which we shall touch later. As a result of the extension of points of view, such, for example, as those made possible by the reduction of three-fold kinematics to four-fold statics, and dynamics to four dimensional geometry; and because of the "isolation of the epistemological problem," the question of perception and its relation to nature and to natural knowledge is forcibly brought to our attention.

Again, the foundations of geometry were investigated during the nineteenth century from the assumption of points as ultimate given entities in an absolute space. On such assumptions there is nothing more to be done in that field of research. However, on the assumption of relative space or of different space systems an investigation of the foundations of geometry will have to show how space and kinds of space originate from the relations of things given in perception, and what points are. "Thus the starting point for a discussion of the foundations of geometry is a discussion of the immediate data of perception."[1]

[1] Whitehed, A. N., *An Enquiry Concerning the Principles of Natural Knowledge*, p. 5.

The physicist has worked on the assumption that an explanation is complete when his subject matter is described in terms of mass, length, and time. His ultimate fact of nature was a distribution of matter in space at an instant of no duration. Such a conception of the ultimate nature of things is unable to account for such facts as momentum and velocity, which are matters of perceptual experience; nor can it account for any other natural facts which involve a space-time process. Processes are inexplicable on such assumptions and processes are facts of perceptual experience. Perception, therefore, becomes a central theme in the physical sciences and in mathematics, especially geometry.

* * *

Science takes its origin very late in the experience of the race, and the foundations of it come to the attention of the individual late in his experience if ever at all. Since this is the case science—the physical sciences are meant—begins with a world of objects partially defined. It does not inquire into the origin of objects as such so long as the purely scientific attitude is maintained, but the aim is to give an adequate description of things that are. What is an "adequate" description is not our purpose here to inquire. What we shall attempt to do is to offer an outline of a theory of perception that is adequate to the physical sciences.

To make a start at all is an assumption, but one must be made somewhere. Our start is with processes. This is not the starting point of the physicist who begins with fixity, i. e., matter spread out in absolute space at a durationless instant, and who is unable, therefore, to account for what is perceived. It occurs to me that, taking what is perceived, namely, processes, as fundamental, it might be shown how what we call "elements" in the process and

the relations that obtain between them are generated from other and earlier processes. For example, it may be that what we speak of as space and time are elements (not simple) that come to be as the result of processes—that space and time, in a significant sense, are ways of description in nature rather than nature as something in space and time to be described. Indeed, it may be that the "I" of perception and the "object" of perception are elements that have been generated from earlier processes. Still further, it is possible that perception is itself a process fundamentally. We shall investigate that view of it.

Before we consider perception as a process it is well to consider processes in general of which perception is a subclass. The following characteristics (or possibly postulates would be better) are fundamental in all processes; (a) All processes may be described in terms of the same kind of terms; (b) All processes involve relata and relations; (c) There is no "simple" process; (d) One process extends over another. We shall discuss each of these briefly.

By (a) is meant that the language of description is the same for all classes of processes. For example, the grammar for so-called mental processes is no different from that which prevails in the description of physical processes. There is no peculiar method which is uniquely applicable to any one field of discourse to the exclusion of all others, but in any description of any process the same method is applicable. Suppose we are making a map of any process. Now such a map is, in the nature of the case, static, fixed, determined, and represents in one aspect a cross-section of the process, or, what the physicist would say, the process at an instant. If the map is complete it represents not only the process at an instant, but the possibility of the process at any instant. At any stage of the process now going on, past, or to come, there will be dis-

played terms and their relations adequate to the process. Such a map can be constructed for all processes, and the method for constructing them is the same for all.[2]

(b) It is a description of processes as relata and relations that renders the description adequate. We can not here attempt a discussion of these phenomena but wish only to indicate the sense in which they are employed. Suppose we wish to indicate the position of a point in a plane. By a convenient fiction, such as a system of Cartesian coördinates, this can be done, and we may say that the point is defined by the pair of equations $5x - 4y = 0$, and $3x - 6y = 0$. In such examples it will be seen that the point is itself the cross-section of relations. There is a significant sense in which objects may be defined in terms of relational cross-sections, or, what is more nearly correct, objects *are* such cross-sections. Of the kinds of relations and of the notion of similarity between them we have no intention to discuss, but *assuming* them, we may be certain that a description is adequate when relations and their cross-sections are displayed.

(c) There is no unique process or unit operation in terms of which all others can be described. We may, of course, discuss the dynamics of a particle, and then of a system of particles, using our former discussion as a foundation for the latter; or we may take uniform and multiform functions in the same manner; or we may take sensory processes as "simples" in psychology and on them build up higher cognitive processes. Any process may be considered simple, but on further investigation it yields its con-

[2] This statement may appear too general. There are cases in which it will not hold provided certain almost unconscious and most "natural" assumptions are made. Since a map involves spatial connotations, it is easy to see that such a map could not be made for many processes. On the assumption of Euclidean space, for example, it is not possible to interpret certain arithmetical facts. It should be recognized that many of our "unsolvable" problems are such because of the assumptions which lie behind the statement of them. When we say a problem cannot be solved we mean that on the assumptions and with the method at our disposal a solution can not be made.

stituents. Simplicity is largely a matter of the purposes of the investigator.

If there were a unit process all phenomena, all the furniture of earth, heaven, and hell could be derived from it, given a sufficient technique. The universe would then be a universe; but in the present state of nature there are many unit processes which means there is no simple process. How these various unit processes are inter-related is one of the leading problems of philosophy; how they are discovered and the principle or principles upon which they are divided is the business of logic; and once the divisions are made (if it can be done) the investigator has the task of exploiting the subject matter of his field.

It may be impossible for logic to accomplish such a task as the one here suggested. It is certain that such a division as that proposed by John Stuart Mill, for example, leads to confusion. It may be that such a logical process of division and especially the search for principles of division involve circles; but circles do not seem as formidable as they once did. If all reasoning is circular, except as we define it otherwise, which can be done by taking a few terms as undefined and a few propositions as primitive, it is a weak charge against a bit of reasoning to assert it to be circular.

If all processes can be described in terms of the same kind of terms, does it not follow that there is a "simple" process? It is a cheap answer to say that the description of a process is not the process described. This may be true in some cases, but if the description is adequate there is a similarity between the two. There are cases, however, in which the description is the process, and the relation of similarity is then said to be "complete." Description is a technique, and what is meant by the statement that all processes may be described in terms of the same kind of terms is that the same technique is applicable to different

processes and not that since the technique is the same all processes may, therefore, be reducible to one simple process. Man is in possession of a method, more or less effective, for dealing with his world and this method is description (used in a wide sense). Description takes forms, is less accurate in some fields than in others, but in all fields the aim is the same, namely, to describe the facts of perception.

(d) The characteristic of processes by virtue of which they extend over each other is closely connected with that discussed in (c). The expression "extending over" is somewhat figurative, suggesting space and time, but these systems of order should be avoided at this point. They appear later as a resultant of the perceptual fact that one process extends over another, but our methods of description suggest the opposite view. It is this latter view that characterizes Newtonian mechanics, giving origin to absolute time which flows evenly on, and to the Kantian doctrine of the *a priori* nature of space and time. The notion that processes are the fundamental fact of nature demands that space and time be derived from them.

If processes are fundamental it would be strange if perception should be something else. It is not at all strange to believe that processes are given in perception, but it is not ordinary by any means to treat perception itself as another process. But a belief in the fundamentality of processes renders it difficult logically to stop short of this position. I am not saying that there are not certain invarients in processes, but perception certainly is not one of them.

The usual method of stating the problems of perception is such as to bring to the front the knower and the known, and the concomitant problem of primary and secondary qualities. A theory of perception which relegates a part of the world to mind and another part to nature fails,

it seems to me, to make any advance towards the solution of problems that have been in the philosophic atmosphere from the time of Descartes and Locke to our own time. Primary and secondary qualities are in the same boat. I believe the problem of perceptual errors and illusions, and the questions of primary and secondary qualities have been stated, and solutions of them given, in terms of a preconception of the relation between a knower and the known, *i. e.*, from a false emphasis on one or the other of the elements (or functions) in the perceptual process, issuing in answers analogous to that given to such a question as, "Why does a ball dropped from the front of a moving train reach the ground nearer the rear end of the train?"

It is true that the prevailing conceptions in any field determine the problems that can be solved as well as the method in which the problems are stated. If, for example, space is Euclidean is true, then the problems that can be stated and solved are predetermined; if imaginary numbers are not numbers is true, then it is useless to attempt a geometrical representation of the same; if disease is a matter of the possession of devils is true, medical technique is predetermined; if there is ether with certain characteristics, then the Fitzgerald equations have a place in nature. It seems that the statement of the perceptual problem and the solutions offered are no exceptions to this tendency.

Let us begin with perception as a process. By this is not meant what is commonly thought of as, "what is going on in the head." What is meant is that there is an event in nature which may be characterized as perceptual in the same way as when we speak of another event as political, or physical, or social.

There are at least three factors that enter into the determination of the kinds of process, namely, the terms, the resultant of the operations in the process, and the relations that obtain therein. It will be observed that terms

and relations are together. That is, terms imply relations and relations imply terms. It is possible, as suggested above, to reduce terms to the language of relations, but when this is done a relation is substituted for the original one, but the substituted one wears the garb of relations. We must possess these "solid" aspects of experience regardless of the garb they mask in.

It may appear that the whole "process" theory begs the question when it is asserted that the kind of objects (terms) which enter the process determine in part the kind of process. What is desired, it will be said, is a statement of processes which will determine the kind of object, and not, in advance, so to speak, a known kind of object which determines in part the kind of process. The difficulty in setting up such a statement is that in any reasoning process something is assumed, and all that can be expected of any theory is that it will find a place ultimately for the assumptions, *i. e.*, that they *yield,* as well as for the "facts" which it attempts to describe. It can be shown that objects actually are generated out of processes, but to describe the original process adequately requires the use of the very objects which issue from the process in question. Negative and imaginary numbers, for example, have been born out of operations on the fundamental operation of pure mathematics, namely, addition; but to describe adequately the operation of addition involves such a statement as would bring to light negative and imaginary numbers. No statement of the nature of addition is complete which does not make possible all the objects which may be generated out of it, and to render such a statement possible it is essential that the objects generated be known.

Given $x$, $y$, $z$ as terms in any process our problem is to determine as far as is possible by a consideration of terms alone, how processes are differentiated when constants are substituted for variables. The first point that

claims attention is that any process yields to schematization in some form as in the case above, *i. e.*, any process has formal properties. If we substitute for the variables oxygen, nitrogen, and argon there results a process which is characterized as physical; if we substitute length, mass, and time we find processes which issue in the body of knowledge known as classical mechanics; if length, breadth and thickness constitute our constants, and these are defined in certain ways, we have processes which are geometrical. Suppose we substitute for one of our variables a "living organism." The processes which result become more complex—they may be either physical, biological, or psychological. If other "living organisms" are substituted for our other variables complexity increases, so that it is impossible to reach definite ideas by a consideration of constants alone. The belief that constants are the only method by which processes can be differentiated has led to much confusion. This much, it seems, we may say safely from a consideration of terms alone, that such words as "psychological," "perceptual," "physical," etc., are adjectival in nature, names in a qualitative sense which are applicable to processes which are in reality the noun. From this standpoint perception is adjectival, descriptive of terms, products, and relations in process. It can be shown, I believe, that this method is applicable to the whole region of psychology, *i. e.*, that "consciousness," the science of which the psychologist claims as his field, is not one of what I have spoken of as the "solid" portions of processes, but is an adjective descriptive of relations, terms, and products in processes.

It is not practicable further to attempt to differentiate processes from terms alone. Indeed, not a great deal can be accomplished by this method alone—by the method of examining the terms only. For example, it is not possible to define order by considering the set of terms to be

ordered, for any given set of terms have many orders. The notion of order must be derived from the relations that obtain among the members of the set of terms.

In the same manner it is not possible to exhaust the details of any process by an examination of the objects that enter into it. There are factors in the process which escape description on the traditional methods of reasoning which confine valid logical operations to the forms of syllogism.

The most fundamental relation involved in the perceptual process is that of asymmetry. This is a relation which implies diversity, yet all diversity does not imply an asymmetrical relation. For example, *a* is different from *b* and *b* is different from *a,* yet this is a symmetrical relation; but *a* can not be greater than *b* and *b* be greater than *a.* It is out of such a relation as asymmetry that series are generated, though not wholly. The terms in such a relational complex are different, and can never be identified, *i. e.*, rendered identical. A common predicate can never be applied in case one should desire to substitute for relational propositions the substance-attribute type, the type common in Aristotelian logic.

The perceptual process involves another kind of relation which has been called "aliorelative."[3] This means a relation such that no term has the relation in question to itself. The importance of these relational aspects will be indicated presently.

Of the perceptual process analyzed into terms and relations which are found therein, we may say of the latter, using such descriptive functions as are common to the logic of relations, that the *domain* of the relation is any other process; the *converse domain* is the original process which we have characterized as "perceptual"; and the

[3] Russell, Bertrand, *Introduction to Mathematical Philosophy,* p. 32. See Chapters IV-VI for a brief treatment of relations.

*field* is both the domain and the converse domain, *i. e.*, all the subject matter of natural knowledge.

We may further say that the relational aspects of the perceptual process are one-many. This, however, is a weak differentia for all relations may be replaced by one-many relations. Such relations are descriptions, and it was with this fact in mind that the statement was made earlier that all processes could be described in terms of the same kind of terms. A term in such a complex in which the relation is, is described by the relation, *i. e.*, asyllogistically and not by the possession of qualities of any kind, not by the subject-predicate relation. For example, the discoverer of the doctrine of general relativity is described by that relation, and no other term (individual) has precisely the relation, the $r$ of $x$. The observer of this process (*this* meaning any process) is likewise described by the relation. But there may be *many* observers of this process, it may be said. Everyday language permits such statements but they are vague. The "many observers of the process" yield to $a, b, c, d$, each of which defined by the relation $r$ to $w, x, y, z$.

One of Eddington's dramatic examples[4] will illustrate the meaning here. The aviator who is moving past us at the rate of 161,000 miles a second believes that our cigar which we light at the same instant he lights his, lasts twice as long as his own; and we believe his lasts twice as long as ours. This is the case because he, in uniform rectilinear motion, believes himself at rest and that we on the earth are passing him at this great speed, while we on earth consider ourselves at rest. The perceptual process here may be analyzed into at least two terms (there may be any number) and the relation "observer." (We make the process as simple as possible for the purpose of the example.) Then each term which may be substituted for the

[4] Eddington, A. S., *Space, Time, and Gravitation*, p. 23.

first variable in the proposition *xry* is defined by the relation in question. It is some notion of perception very like this which is adequate for a relativistic physics.

Let us develop briefly the fact that the perceptual process involves an asymmetrical relation. Since such relations imply diversity all doctrines of "the self-identity of subject and object" become meaningless. The reduction of one of the terms in such a relational complex to the other is an impossibility. Such statements, therefore, as "Everything is mental," or "Everything is physical," "All is God, the Absolute," convey no information concerning anything in particular.

If *a* is the father of *b* it is not possible to reduce *b* to *a*. It may be that *b* is the father of *c* but he can not be the father of *a*. Again, if *a* is the observer of the process *b*, then *b* can not be the observer of *a*. This statement may be doubted in such a case as the following: *a* may be the observer of *b* and *b* may be the observer of *a* when both *a* and *b* are supplied with reaction machines, such as a nervous system. This difficulty is apparent when it is recognized that we are dealing with more than one process. Then, there is this significant difference that in the relation expressed in "*x* is the father of *y*" there is what we may call a "necessary" element which is lacking in the other case. In both cases, properly interpreted, the relation is asymmetrical.

That the perceptual process involves a kind of relation which is termed *aliorelative, i. e,* one such that no term has that relation to itself, is a fact which should drive a great deal of mysticism out of philosophy. Monisms of various kinds would doubtless profit from a consideration of these relations. From the standpoint of perception this relation is significant in that it points to the fact that it is impossible to equate the "knower" and the "known" (to use common terms that should be avoided). Since asymmetri-

cal relations are always aliorelative, what has been said under that heading need not be repeated. The point is that both relations imply diversity.

Processes involve products. There is always something produced, the minimum being the ideal course which is carved out by the process in its actual going-on; the maximum, the elaborate institutions of science, religion, politics, and philosophy.

A scientific psychology, *i. e.*, a quantitative psychology, has as yet not been completely developed. We are at about the same stage in its development as were the Pythagoreans in the development of numbers. Hence, the difficulty in speaking, from the standpoint of current psychology, with any degree of certainty concerning products which result from perceptual processes. Psychology has not developed a "fundamental" operation, and it knows no "operator" which can generate a set or a group. It speaks of sets and groups, but in the vague language of a science looking for a technique. True, it possesses "sensations" and speaks of other processes in their terms, but it has never been able to tell how other processes issue from them. The conic sections, for example, are produced from the circle by a single operation, but the psychologist has been unable thus far to formulate a statement of a process that results in any analogous set. Psychology has not as yet produced its Sophus Lie.

There may be a reason for this state of affairs, and that reason can not be that the psychologist has not had time to do this work, for "the science of the soul" is pretty old. It may be that he has been too busy investigating the "soul" and the "mind" and has let processes alone.

If such be the case any process *becomes* perceptual when the proper terms and relations are present. Perception, then, is not an "awareness," not something "present to mind, plus meaning," not "the consciousness of

something present to sense," but rather the name of a process involving a peculiar kind of term, a kind that possesses some form of reaction system, and a peculiar kind of relation, namely, asymmetrical and aliorelative. The products that result is a psychology in the sense of that term suggested above in the notion of "consciousness," best expressed today by the term "behavior" though not completely; and the vague region expressed by some such term as "nature" or "natural knowledge" which becomes possible through, and takes its origin from, the perceptual process.

The "facts" of perception must be interpreted in the light of the analysis made above. These facts are such as the perceiver or knower, the object known, the stimulus, the perceiving experience, perceptual errors, and illusions.

The first two of these facts have been shown to be terms in a complex. Difficulties have arisen by an emphasis on terms only, such as lead to subjectivism or materialism. The application of the logic of classes to material that is not exhausted by such an operation leaves a foggy view. These terms are related in a manner which is described as asymmetrical and aliorelative. From such a characterization consequences flow which render some views of the nature of objects and perceiver, which have held prominent places in philosophic literature, meaningless.

A misinterpretation of the nature of perception has led to many strange views concerning the stimulus. The chief of these is the Kantian notion of the "thing-in-itself."[5] Many of the doctrines that advocate an "unknowable" are bound up with a theory of perception in which is involved in some inexplicable manner the "mind." The stimulus, so it is said, sets in operation a mind, and there result objects of perception and knowledge. The object is cre-

[5] The "thing-in-itself" of philosophy when translated to physics becomes the "matter" of that science.

ated by the machinery of the mind, and the mind knows its own children. An erroneous view of the stimulus has led to the doctrine that knowledge is a copy of the object, and more indirectly, to the doctrine of primary and secondary qualities.

The stimulus in a perceptual process is not something that awakens a passive mind which in turn stamps its categories on the raw stuff of the senses, but is a term in a complex. However far it may be traced back it can always be described in this manner. Certain wave lengths in certain relations generate sound; others generate light. It is difficult to state this clearly due to our habits of thought about such matters. What is defined as a stimulus in one complex or process may be an object or term in another of a different kind. But in all processes involving what we call a stimulus this same stimulus becomes an object in a further perceptual process. That is to say, the stimulus in one of its chief rôles, if not the only one it takes, is what the psychologist speaks of as the "object of perception." Whatever can be treated as a stimulus is capable of treatment as an object in the perceptual process. It takes the same place in the world of nature that any other natural event takes, and instead of being the "unknown cause" of anything whatever, it submits to precisely the same treatment that any natural event takes.

It has been pointed out that there is a process which is "subjective" in nature, and which may be shown by some such diagram as the following: Object———Perceiver. Between the two there is a process of perceiving which is psychological, something "going on in my head." Let us assume that there is such a process which can be stated in some manner such as "$x$ is seeing a tree" when $x$ is not the seeing nor is the tree the seeing. How can a theory of perception as a process absorb such a fact? If the fact is capable of description it takes its place along with any

other of the same kind. That is, it is analogous to the fact that *x* is hitting a nail or the wind is blowing the leaves. When *x* describes the experience (process). "*x* is seeing a tree" he will proceed along the same lines as when *z* describes the experience of "*x* seeing a tree." If his description is adequate he will state the terms and relations in the process, for any fact that can be described must be described in the language that gives origin to the logic of classes and of relations, and in no other way.

It has been frequently argued that description of a process is fatal to the process, that science falsifies reality, that analysis gives only static views of processes which are *the* real. Touching our problem of perception it is said that description of it, or of any other "psychical" experience for that matter, falsifies the experience. The description of "*x* has the toothache" is different from "*x* has the toothache"—the immediate experience. But the description of "*x* has the toothache" differs in no essential respect from that of any process in which a constant is substituted for the subject and the predicate respectively of the original proposition. A "psychical" experience is no more falsified by description than any other experience (process). That is to say, there is nothing peculiar to "psychical" experiences which renders a description of them false when description may be true of some other process, such as the falling of a tree. Of course, this does not show that description does not falsify processes, but it indicates that if it falsifies "psychical" processes it also does it to "physical" processes.

We can not here attempt to answer the arguments of the falsifactory nature of description and analysis, but wish to say that there is no legitimate sense in which we may live in a world of "description" and one of "appreciation." The world of "appreciation" may be a little more difficult in yielding to our methods of treatment, but when it does

yield as parts have done from time to time, a new science is born. Indeed, the history of science is the story of how matters of "appreciation" have yielded to description and of how the technique of description has been refined.

Such beliefs (that science falsifies reality) are seen to be unwarranted in view of the fact that what science has to say about an event, process, or object is precisely what the event, process, or object is. If a mistake has been made it is always corrected in terms of another descriptive process, and not along lines which abandon description, unless we become mystics.

Errors and illusions have always been stumbling blocks to otherwise neat and attractive theories. They are usually believed to be "mental"—in some way the "mind distorts the real." It is strange, as has been pointed out by others, that a great many mechanical devices, such as the thermometer and the camera, have this uncanny habit along with the mind. Would it not be better, in all such cases of "distortion," to regard them as brought about by the interaction of various physical things? Every case can be described, and becomes, therefore, a part of the furniture of nature.

The belief that there is a unique *real* object or process, described by a few simple terms and relations has caused much difficulty in dealing with perceptual errors. Just as no set of terms has one order but as many as it is capable of, so is an object (any unit of discourse) not a unique something but everything it is capable of. Almost every day objects (in the sense above) are taking on complexity; new "orders," so to speak, are discovered, and the object is all the "orders" it is capable of. It seems to me that a

thorough recognition of this point will answer the difficulties of variability in perceptual experiences. Error, interpreted in the perceptual sense, is answered in the same manner. Interpreted as "falsehood" as against "truth," questions arise which are not within the province of an outline of a theory of perception.

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# A CRITICISM OF CRITICAL REALISM

CRITICAL REALISM occupies a half-way house between moderate Realism and moderate Idealism, and it is exposed to the adverse winds that blow from both quarters. It is a well known theory as to the machinery of what is called perception and the nature of the entities experienced in perception, which has been propounded by seven American professors as a counterblast to the six American Neo-Realists.

In theory, Critical Realism is one doctrine to which the seven American professors unanimously subscribe; in fact, it comprises at least two distinct doctrines, each of which contradicts the other. Whether this difference of doctrine has been detected and waived in the interests of unanimity, or whether the desire for unanimity has obscured the perception of difference, cannot be determined here—although the number of foot-notes contained in the volume entitled, *Critical Realism,* purporting to explain or resolve differences which are unimportant, suggests the latter view.

Of these two doctrines, the theories of Professor Drake and Professor Santyana may be taken as representative. I propose to consider these theories in turn, to suggest certain criticisms to which they appear to be open, and to indicate the reasons why it is impossible for a thinker who agrees with Professor Drake to subscribe to the doctrine of Professor Santyana, and vice versa.

According to Professor Drake there are involved in perception three separate existents: (a) the object of perception, (b) the mental state of the conscious organism which perceivies the object, (c) the intermediary processes such as ether waves, sense organs and neural correlates. In addition to these three existents there are also what he calls the data of perception, the entities which are actually perceived. These data are not existents; they are variously called character complexes or essences, and they are brought into being by the coming into contact of (a) the object of perception, and (b) the mental state of the conscious organism. The coming into experience of the datum is explained as follows: contact between (a) and (b) involves the exercise of a certain influence by (a) over (b); this influence is causal, what it causes among other things being—in true perception—the appearance of the characteristics of the object as the data of perception. These data are further described as projections of our mental states; "they are," in Professor Drake's words, "never *found* there by a sort of telephatic vision, but are *imagined* there by a mind," and we are told that there exist in, or in intimate connection with the brain, a series of mental states which have the qualities that make our data appear. Snce, however, it was the influence of the particular object (a), and not of any other object, that caused (b) to project the data, the data have a very definite reference to the object in question; and in fact, as has been already remarked, in true perception they *are* the actual characteristics of the object. Thus perception is a process of imagination, since in perception we experience data which the organism, affected by the outer object, "*imagines* as characteristics of the object in those vivid ways we call 'seeing,' 'feeling' (with our fingers), etc." We are also told that though the data may have being or 'subsistence' independently of the perceiver's consciousness of them, they

have not independent existence. We implicitly attribute existence to them when we imagine them as being *out* there in the world, and since in true perception the data or imagined character traits of the object really *are* the characteristics of the object, the attribution of existence is in such a case justified: in false perception it is not, and the data then have being or subsistence only.

Before proceeding to consider the view of Professor Santyana, I propose to submit certain objections to the theory of Professor Drake.

(1) The theory involves a relationship between two entities which exist and one which subsists. The entities which exist are the mental state of the knowing mind and the object: the entity which subsists is the datum or character complex which forms the content of the knowing mind. Now the analysis of perception is such that it requires us to hold that the object is never, and can never under any circumstances, be directly perceived. "Our data of perception," says Professor Drake, "are not actual portions or selected aspects of the objects perceived." But if this is so, in what sense are the objects perceived at all?

The difficulty here is simply the old difficulty that discredited the philosophy of Locke, the "object" of Professor Drake is the "substance" of Locke, and the theory of Professor Drake is the Representationalism of Locke.

For if we never know the object, but only character complexes which we like to think are, in veridical perception, the characteristics of the object, we cannot know anything about the object: we cannot know that the object exists, and we cannot know it is the cause of the occurrence of our data; while the belief that our character complexes sometimes correspond with its characteristics will remain a guess, which we shall regard as probable or improbable in so far as we already share or reject the beliefs of Pro-

fessor Drake, but cannot be cited as evidence of the truth of these beliefs.

(2) From the fact that we never know the characteristics of the alleged object, it follows that we can never know whether our character complexes correspond with them or not: hence we can never know whether our perception is accurate or not, and the Critical Realist criterion between true and false perception can never in practice be applied.

Professor Drake likes to think that perception is in the main accurate, and invokes at the beginning of his essay a number of Pragmatic considerations in favor of believing that it is so. But wishes father thoughts, they do not breed evidence: the fact that we would like a thing to be true does not mean that it is true; while the reference to the Pragmatic criterion as affording a meaning for truth is expressly disavowed by another Critical Realist.

Objections of this and of a similar character are in part considered by Professor Pratt in his essay on the "Possibility of Knowledge," in the course of which he endeavors to answer them. The gist of his reply consists in representing the view of the Critical Realist and that of the complete sceptic as the only possible alternatives, and then dilating upon the improbability of the sceptical view. "If the critic is right," says Professor Pratt, "we must suppose that by an incomprehensible collection of coincidences his own senses, the senses of all other observers, and the details of the prior and subsequent experiences of all concerned conspire to deceive us." But an expression of doubt as to the validity of the Critical Realist position does not surely involve the acceptance of so distressing an alternative. Professor Pratt presents us with a choice between accepting all our perceptions as accurate, or impugning them all as deceptive; but this dichotomy is an unreal one which ignores the real difficulty and the real

question, the difficulty being that in practice we know that our perceptions are neither all true nor all false, but sometimes true and sometimes false, and the question at issue how we are to distinguish the true from the false. It is the first requisite of any theory of perception that it should suggest some method of solving the difficulty involved in the answer to this question, and it is precisely this requisite that both Professors Pratt and Drake fail to supply.

There is, however, another item in Professor Pratt's defence which deserves mention. He appeals, in support of the accuracy of perception, to the uniform character of the testimony of the senses: each sense supports and bears out the other, and in so doing affords evidence of the truth of its testimony. Also there is the appeal to other persons. But here again Professor Pratt fails to appreciate the issue, which is not that of always true perception against always false perception, but between any true perception and any false one: and the trouble is that in a false perception the senses support one another just as frequently as in a true one, as for instance in the perception by a color blind person of a green apple. But in any event the attempt to bolster up the validity of one sense by another when it is the validity of sense perception as a whole which is questioned, will not work. The belief that my sense of sight which informs me that I am writing at a table is guaranteed by my sense of touch which also assures me of the table, is only reasonable if the validity of my sense of touch is established to begin with: but you cannot establish the validity of an A that is fallible, by an appeal to a B, which must itself be assumed to be fallible for just so long as the fallibility of A is itself in question. If A is fallible then it derives no support from an equally fallible B; if A is infallible it does not need it.

Let us suppose, however, that the testimony of our senses is always, and in every respect unanimous, does this

fact necessarily constitute a guarantee of their truth? Coherent error is by no means to be ruled out as a possibility, and is certainly not ruled out by Professor Pratt's analysis. The world of dreams and hallucination is frequently as coherent as that of every day life, and is not necessarily distinguished by contradictory deliverances on the part of the senses. And in any event, so long as the Critical Realist denies us all direct knowledge of the object perceived, the world of every day experience *need* possess no greater degree of connection or correspondence with the world of external reality than the nightmare phantasies of the injudicious diner; it may do of course, but the Critical Realist can neither prove that it must do, nor can he distinguish the occasions on which it does from those on which it does not.

I now proceed to a consideration of Professor Santyana's view, or rather to those aspects of it which differ from Professor Drake's.

The chief point of difference is contained in Professor Santyana's description of the datum. The datum is for him a logical essence, a quality which is permanent and given. It neither lapses nor moves forward, and it is therefore outside the flux of temporal events. We are expressly told that the circumstance that a datum is given is incidental only and does not affect the nature of the datum, from which it follows that the essence is not changed either by becoming a datum or by being abandoned for another. In experiencing a datum we are in fact becoming acquainted with an entity which subsists independently of our acquaintance, an entity which is immutable and eternal, and Professor Santyana proceeds to speak of the "datum" as Plato's τὸ ὄντως ὄν or as that which is intrinsic and essential.

The first point to notice about this conception is its wide divergence from that of Professor Drake. For Professor

Drake, as we have seen, the datum is literally a product of our imagination: it is "projected" or "imagined" as being out there. If this is so, it clearly cannot be out there before we projected or imagined it, and the circumstance of its being imagined becomes not an incidental attribute but a most essential fact about its nature, the very cause of its being. In short, the datum for Professor Drake is a mental construction which when we are lucky is identical with the characteristics of the so-called object.

But Professor Santyana's departure from the pure doctrine of Professor Drake raises its own crop of difficulties. In the first place it reduces the occurrence of perception to a mere accident. Thus Professor Santyana speaks of what happens "when our erring thoughts light up the intrinsic possibilities." Now Professor Drake attributed the occurrence of perception or the projection of data, to the emanation from the object of an influence upon the brain of the perceiver. But for Professor Santyana the experiencing of a datum happens by chance. We are not, as we might have been, told that for each object there subsists a corresponding datum or series of data, and that by some queer alchemy an influence exerted by the object makes us perceive not the object but the corresponding datum. Had we been told this, we should have a theory of perception that possessed some relationship with that of Professor Drake; but even so neither Professor Drake nor Professor Santyana could explain how we perceive erroneously. For if the cause, and the only possible cause of the perception of a datum is the influence upon the brain of an object possessing characteristics which are those of the datum perceived, how comes it that in error we perceive data which ex hypothesi are not the characteristics of any object? Whence come these data? What in fact is the starting point of the whole process of erroneous perception? If we conceive of perception with Professor

Drake as the projection of data which are the characteristics of the object, how can we project data which are not? If, on the other hand, we hold with Professor Santyana that the realm of essences is lying out there waiting to become the content of our experience, and that therefore "our object is simply what we happen to think of," we are forced to the conclusion that the objects of all possible perceptions, as for instance the objects of our perception of blue snakes when we are drunk, already subsist in their infinite multiplicity waiting to become our data when we light on them.

But if we are to adopt this latter view, we are faced with a refusal to apply Occam's razor of the most extravagant kind. It is a significant fact that in considering Professor Santyana's view we have necessarily drifted into speaking of the essence or datum as the object, and in so doing we have only followed his own terminology. But what has been happening all this time to the object in Professor Drake's sense of the word, that entity whose characteristics are or are the same as our (in Professor Drake's language) "imagined data"?

Well, it seems that the world is for Professor Santyana as for Professor Drake, peopled with these latter kind of objects, just as for Professor Santyana alone it is also peopled with independent logical essences, the relations between essence and object being such that when we perceive correctly we have hit on an essence that corresponds with the character of the object, when not, not. But what in the name of Occam's razor is the sense of peopling your universe with an infinite number of objects and essences, both of them lying out there, of which the object never can be an object of perception, while the essence always is an object of perception, whenever perception occurs; and both of them of such a nature that when a purely incidental and adventitious phenomenon such as perception, a phenome-

non which is irrelevant to the being of either, takes place and happens also to be correct, the essence is said to have the characters of the object, although since the object can never be perceived, it passes the wit of man to know whether the essence does have those characters or not.

And as a concluding comment on Professor Santyana's view, I should like to ask how it comes about that if essences become objects by accident, we all of us perceive what is approximately the same world. If the essence whose characteristics are those of an alleged object which is a motor car, is perceived by me by accident, and not because a motor car is really there, or is really perceived there, how comes it that an accident of precisely this same kind brings the same set of essences to the notice of my chauffeur at precisely the same time. Does not the repeated sequence of such accidents suggest that the process of perception is not an accident at all, but is dictated by some feature in the real which affects in much the same way, and affects directly the organisms of two similarly constituted persons?

It would be interesting to examine the essays of the other authors of Critical Realism, to see how they oscillate between the two views of the nature of the datum expounded by Professor Drake and Professor Santyana respectively. It would be interesting too to notice how the fact that the word "datum" is normally used in that one of its two senses which happens to be most convenient for the purposes of the argument at the moment, has not a little to do with the persuasiveness of their writing. It is, of course, unfortunate that his method may be inverted; and the critic who with equal authority adopts whichever of the two senses he pleases for the purpose of discrediting the argument, may achieve disastrous results. I will subject to this treatment one sentence from Professor Pratt's essay which typifies the ambiguity that so frequently results. "A sharp distinction must," he says, "be drawn

between object and content, between that which is before the mind and that which is within it." Now the word "object" here may be used either in the sense in which Professor Santyana speaks of the datum or essence as the object or in Professor Drake's sense of the word. If it is intended to refer to the datum according to Professor Santyana's use of the word "object," then we may suggest that as we always project or imagine our own data (Professor Drake), the mind can never contemplate anything but its own mental creations, and we are back on the subjective Idealism of Berkeley. If, on the other hand, "object" here means the real object which exists and not the datum which subsists, then the mental content must be the datum and the datum can be nothing but the mental content. But how can a mental content be a logical essence?

An analysis of Professor Roger's essay on the Problem of Error yields results which are not dissimilar. His theory leaves us with the same difficulties as those we have already experienced, the difficulty, namely, of explaining, (1) how we can ever tell a true perception from a false one, and (2) how an erroneous perception ever occurs. For as we never know the object we can never tell whether it has the qualities of the sense data we imagine, and as our attribution of those same qualities is generated by its influence of the object which is supposed to possess them, it is clear that we can never be stimulated to project or imagine qualities which the object does not possess.

A theory of error which fails to deal satisfactorily with the question of how we distinguish the true from the false, and the problem of the genesis of error, can only be termed inadequate.

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# THE SPIRIT OF RESEARCH

## I. The Four Types of Research

IN THE FAR SOUTHWEST lies a valley between two mountain chains. On the eastern side the mountains range from ten thousand feet down. At the north rise rather precipitously from the floor of the valley a number of granite peaks, conspicuous from its whole length. Mountain springs furnish brooks, which slowly vanish as they drop lower and lower down the thirsty canyons. Farther south the eastern wall is harsh and cut by deep erosions as the waters of the summer deluges of the centuries have gnawed away at their steep slopes. Two porphyry peaks rise abruptly, standing like Indian sentinels to warn the valley of approaching danger. Still farther south, the range becomes more massive, rises sharply in ragged pyramids that rest on a broad, sweeping base, where the tall yellow pine grows. At the extreme southern end of the chain the peaks become more rounded and are of quartzite. The chain that bounds the western side of the valley starts at the north in an extensive line of cliffs and jagged edges, granite and quartzite masses. It sinks into a low line of hills, and then rises again abruptly into a wall of upturned quartzite and marble, which the hand of Time has built into castles, towers, and fortifications of exceptional sharpness of outline. Farther south, the chain becomes a series of massive rounded mountains, containing many rich veins of copper ores.

The valley itself is of easy slopes, over which run the flood waters from the bordering walls and their canyons, but no river carries these waters back to the sea, as the valley is greedy for all the water it receives. The slopes are stream-built, and the entire valley is filled for thousands of feet with the débris of ages. A characteristic feature is the presence of many rounded buttes, standing like islands in the middle part, an archipelago of survivors, not yet completely buried, of the original igneous hills. The history starts with the Pre-Cambrian sedimentation, deformation, volcanism, and metamorphism, followed by erosion. In the Paleozoic time, the limestones were formed under the sea, shortly afterward were elevated, and with all the other strata were folded and crumpled like a handful of waste-paper. A dip to the south allowed the sea again to wear away the land for a short time. Since the beginning of the Quarternary period, the valley has filled up with the wash of the torrential rains, the conquests of the fierce charges of the wind, the wave-driven sands of a salt lake, while even underneath, the material has been re-deposited by underground channels.

Growing in isolated patches is the thorny mesquite, in others the cat's-claw, the grease-wood, the sage-brush. All are used to hard conditions, and can survive for many long weeks without water save such as may be in the slight humidity of the air. The ocatilla spreads its slender whips to the sky, tipped with scarlet. The kallstroemia and the evolvulus grow in profuse masses amidst the stretches of gramma grass, sacaton grass, galleta grass. Even on the edges of the alkali lake, lying torpid under the blazing sun, may be found saltbushes, which can exist under most adverse conditions.

In the shadows of the bushes hide the jack-rabbit, the small burrowing animals of the desert, the diamond-backed rattler, and various bronze and silver lizards. The cactus-

wren and song-sparrow flit about in search of food. Overhead wheel buzzards in search of victims of drought and starvation. Tall century-plants spread their honey-laden arms, and many forms of yucca and agave attract birds and insects. Under the stones hide centipedes and scorpions, and in the twilight the skulking shadow of the coyote glides silently through the landscape. In the canyons are forest-trees where the mocking-bird pours out his concert, while the flash of the Baltimore oriole lights the somber green. The crested quail with wary eye for fox and snake guards her young under the live-oaks, and high up in the indigo blue floats a solitary eagle.

In the depths of the mountains the metallurgist finds ores of gold, silver, copper, and lead. The tall smelter stacks evidence the activity due to these. In springs near some of the buttes, sulphur is a constituent, furnishing a name for the valley. The chemist finds in the soil and particularly in the alkali lake the chlorides, sulphates, and carbonates of potassium and sodium, and also salts of calcium and magnesium. The valley is even charted to show the amounts of these in the soil. The rise and fall of the ground-waters is a problem for the physicist. The drifting effect of the wind on the contours is a problem in mechanics. For the biologist there are other problems, for life exists here under hard conditions. Over a tract of fifty square miles there is practically no life at all, and few forms exist in the inhospitable and bitter waters of the lake. The valley is one of the spots where Nature intended to be alone.

This brief sketch indicates what scientific research is interested in when it studies the valley. A complete account would be quite voluminous. But it would not exhaust the description of the valley. There are other types of research as fundamental as that of science. There is for instance the mathematical. The flowing profiles of expo-

nential curves sweep down the stream-built slopes from the mountains. The helicoidal curves of vortices of dust that rise from the valley to the sky every hot summer day, wind their spirals in whirls that widen asymptotically upwards. The shimmering plane of the alkali lake reflects the distant mountain surfaces. These surfaces are bent and twisted like oriental rugs flung carelessly down. Edges of regression of developables, parabolic lines, hyperboloidal passes, pinch-points, conical points, nodal lines, the entire array of the geometry of surfaces, may be found in these deeply eroded, volcanic slopes, and jagged peaks. Fan-shaped draws spread out at the bases of canyons where they discharge their waters, and steep-walled arroyos break the continuity. The lines of level of old ocean shores, and of modern lake beaches can be discerned. In places, the cusps of the drifted sand-dunes limit the curves drawn by the fingers of the wind. In others, the fine dust lies in sweeping transcendental curves with centers, nodes, foci, fauces, lines of divergence or convergence, every type of curvature—evanescent congruences different every hour, yet all subject to the same differential equations. The wind blows and the shimmering lake breaks up into a thousand curved mirrors held by dancing fairies, but the partial differential equations have designed the ballet. The mountains grind the glittering crystals into finer and finer particles, but the dihedral angles are always the same, and under the laws of groups. All the projective geometry of optics may be studied in the valley atmosphere. Vector analysis sees its curl and divergence, vector lines and congruences every day. The algebra, geometry, and analysis of the valley is a wonderful story.

For a different group of investigators the valley does not consist of geological formations, minerals, chemicals, and strange forms of life, nor of twisted lines and crumpled surfaces. They see the valley as the home of the lean,

swarthy Apache. Before long-horned cattle had eaten all the grass, deer lived in the valley; game of many kinds was abundant; the streams furnished enough water for human life; mesquite beans and yucca pods furnished nutriment; the cactus furnished fruit; the maguey, dessert. The cougar and the coyote gave their skins, and birds their feathers. Stones furnished tips for his arrows, the sinews of the deer his cords. He lived continually outdoors, and could travel a hundred miles in a day, with no food, little water, and under a blazing sun, whose blue-white rays scorched his naked body. He climbed to the very tops of the impossible crags, for thence he could see the entire valley. His moccasined feet clung to the most precarious footing, and the grip of his lean hands was steel. When he lay down he melted into the ochre soil as if he could become invisible. He wove baskets from the grasses and other fibres, to hold his scant supplies, to carry water, and for other purposes. He knew the value of all the desert plants. On the mountain tops grew the laurel, whose leaves made men lose their minds. He knew where to find the cat-tail in the cienagas, with its sacred yellow pollen, with which he made his worship of the sun and moon. He carried a little deer-skin bag everywhere to hold this marvelous powder, for performing his rites in hunting, sickness, in spells, and all the affairs of life.

Lean and wiry, a function of the desert and the peak, of the sun and the wind, the lightning and tne cloud, tireless, abstemious, leading a life which had little humor or gaiety, facing the stern and pitiless vicissitudes of the valley, he became like the country itself. He had no grudge against the wild growths from which he wrested his sustenance, for he loved this land of burning sun and little rain. To him it was Chiricahua—the Beautiful. In return the land loved him and his grim mother always placed something within reach to maintain life. He lived in small

groups, never in towns or villages, since there was rarely enough in any one place to feed a large village. In consequence, he became solitary, taciturn, hiding his feelings tightly from everybody but the sun and moon, unless in the grip of some terrible calamity he held his spirit-dance to implore the gods for mercy and compassion.

After many centuries came invaders, tanned warriors from the south, with death-dealing fire-arms, mounted on horses, dressed in mail, a motley horde of warriors, adventurers, and priests. Some hunted gold, always chasing the end of the rainbow, some looked for excitement, some for new lands for the king, some to rescue the heathen from everlasting fire, even at the cost of his life. The years flowed along, there came more of the swaggering race, and a fight began for existence on one side and extermination on the other. The Apache began to hunt the Spaniard as he had hunted the deer, the cougar, and the coyote. All his cunning gained a new quality, all his harsh nature became more cruel, all his intelligence began to center on one aim, self-preservation. He knew that his mountain fastness, caves, inaccessible crags, huge bowlders, and obliterated trails were his only friends. The prickly cactus, the entangling cat's-claw, terrible cholla, stinging ant, the rattler, the centipede, the dazzling sun and dry watercourses, fought for him. He burned mission churches, slew men, women, and children, tortured to death all captives as a warning to others to stay out. Then came the pale-faced Anglo-Saxon. Although a few desperate struggles insured him the valley for a few years, the game was gone, the old life was impossible, and he was scattered on many reservations. Here he acquired new traits, still a function of his conditions, and his neighbors.

The valley is far from being exhausted, however, and another type of research presents it from a totally different aspect. These investigators are not hunting for the

structures and laws of science or mathematics; nor for the functional inter-relations of a people and its environment. They have in view a more intangible structure, more subtle laws. For them the colors laid on the long wrinkled slopes by the sun and the air, the ethereal ocean in which float the crags and domes, mean more than all the quartzite, porphyry, and granite, or the copper, gold, and silver ledges. The orange and cardinal cups, the little blue stars that shine among the dainty black feathers poised delicately on hair-like stems, mean more than classifications as Kallstroemia, Evolvulus, or Gramma. The lithe, slender, gracefully irregular whips, tipped with a scarlet flame, are more than Ocatilla. Against the western sky the serrated wall becomes a chief's war-bonnet of eagle feathers. The outline of the eastern wall is a perfect profile of the great Apache leader, sleeping on his own mountain chain, a mammoth statue wrought by his mother, the earth. His stern face is ironic as the new race tries to put fetters of civilization on this untamed wild creature, the desert. He might almost be thought to smile, when the mirage paints lakes, trees, shade and habitations, to lure the unwary newcomer to barren land, drought, burning heat and death. The epic of this vanished people is a research of different order from those mentioned above, but it is as much a part of the valley as the physics, chemistry, mathematics, and anthropology.

There are other artistic features besides the poetic. For when the painter sees the sun-washed surfaces of the Galiura mountains, with their translucent purple, heliotrope, lilac, amethyst, violet, and ultramarine; the ochre, saffron, gold, burnt gold, salmon, glowing orange, and scarlet; the ghostly beryl, the spirit emerald, the turquoise green, all the play of delicate color that cannot be named, making the mountain walls look like the vari-colored petals of some blossom created by the magic of a genie, he sees

a living beauty behind these silken veils, and paints a canvas which is the record of his researches. When he sees the long sunset shadows creep across the valley floor with its oriental carpets, bringing out the rose and violet of some scraggly yucca as it stands out against the indigo sky, while the distant mountains fade into pale aquamarine tints like dreams that steal away in the gathering twilight, his record contains a truth as great as any that may be written about the ores under the hill-tops. When he catches the dance of the sand-devil whirling in the arms of the hot wind, he puts the passion of this burning land on his canvas.

And the musician sits enraptured while the mocking-bird pours out his heart in an ecstacy of joy, and reports his research later in symphonic poems of great beauty. He will catch the elusive motif of the mountain-brook, the faint rustling of the moonlight on the grass. Even the alkali lake, tossed by the wind, beats a rythmic melody on its sands, and the long mathematical curls hold a faint harmony for him that hath ears.

The architect that built these towers and castles, spires, domes, and minarets, knew more than five orders, and left them on record for coming artists to copy. The statues sculptured by the elements have their own curves of beauty, and they will stand there for ages and ages.

The valley is filled with material for the research of the artist as it is for the anthropologist, the scientist, or the mathematician. And when we would comprehend the world in its entirety, we cannot neglect any of the results of any type of research. The world is an indivisible unit, not a parcel made up of laws and facts from science, anthropology, mathematics, or art. Each of these has a view which is only partial. Each is attentive only to certain

features, and the real world is far more than their fragmentary pictures of it.

Whether it be the Mathematician playing with his ideal geometries, algebras, and logics; the Artist in his warmer world of Aesthetics and Aspirations; the Anthropologist studying Man as interwoven with the World; or the Scientist who probes deep for eternal Laws; all are engaged in Research. For what is Research? It is more than the determination of a vapor-pressure, the calculation of a new function-table, or the study of some new-found pottery. It is the "passionate and disinterested curiosity of the human intelligence" (Picard). It is the young eagle trying his wings. The Universe has no terrors for Research. Its world contains the stern clutch of the polar cold, the burning languor of the tropics, the unclean leper, the black death, the vilest diseases, the most dangerous gases, the most violent explosives, the lightning's shafts, the sun's javelins, the depths of the South Sea, the heights of Mt. Everest, the fang of the snake, the sleep of the anesthetic, the hypnotic trance, the ravings of the insane. Alike for it are the wilderness, the city, the slum, the palace, the jail, the saint, the sinner. Research hunts for order, for beauty in the world, for imperturbable law, for the cloud-forms of Time, in nature, in man, in life, in death, in human woe and tears, or human joy and elation. Research holds nothing sacred. It looks for the origin of Man, of the World, of Religion, of God, of Institutions, of Society, of State, of Church, of Man's relations to Man, of Man's relation to Woman. It does not shrink from any depth, nor is it dizzy at any height. Research is not even content with what it finds, but would fain be a Creator, and make new worlds, new beauties, new loves, new hates, new passions, new things, new animals, new men, and new women.

Research is not only passionate and disinterested, it is audacious, ambitious, fearless. It has for its invocation:

"Eastward, where the Sun is kindled;
Northward, Cave where the Wind sleeps in Darkness;
Southward, Swamp where the Snake-Mist rises;
Westward, Plain where the Ghost-Trail goes."

Its benediction is:

"I bow myself to the Quarters,
I salute the Sun and Earth, my parents,
Once more the Song has gone forth,
Like smoke it has vanished in the Sunlight."

—(John Gould Fletcher.)

## II. Scientific Research

Scientific research has for its object the statement of laws. This is the same as saying that Science searches for what it considers to be uniformities in human experience. It hunts for the everlasting hills that hold up the four corners of the sky. It experiments over and over on like materials to ascertain if the same results follow. It analyzes the situation in every experiment and tries to ascertain what elements are necessary, what accidental. It endeavors to reduce dependence to a numerical statement, that is, it tries to base its conclusions on measurement. Of course, much of its present content is not altogether so based, perhaps never can be so based, but the attempt is made to reduce all data to numerical form. Science endeavors to be impersonal, to resort to self-registering apparatus, to the camera instead of the eye, to machines that draw their own graphs. Its first aim is a perfectly accurate record of the objective facts.

The facts determined beyond all question of error, self-deception, omission, or imagination, the next problem is to find the laws they imply. This is done first for the obvious connections between phenomena, then as the number increases they are synthetised into more general laws, and these with the assistance of fortunate hypotheses are included in more general statements, the whole: facts, laws, and hypotheses, ultimately constituting a Theory. The theory of celestial mechanics, or theory of electromagnetics, or theory of relativity, or theory of evolution are instances.

Science searches for the Invariants of the world. It desires to discover the permanent in the flowing stream of experience. It dreams of eternal verities, of a stable, essen-

tial structure of the universe, which is not subject to the vicissitudes of chance. Science does not find nor expect to find the capricious, the spontaneous, the free, the spiritual, for it assumes as its subject matter that which is determined. Given the state of the universe at a given instant, its history up to that instant, and the hope of Science is to be able to say what any succeeding state of the universe would be. Science sits patiently unraveling the most tangled threads that the web of phenomena produce, never tiring, never retrograding. Its hypotheses may give way to new ones, and its theories may be included in more satisfactory ones, but the essential results of Science are permanent, and it keeps what it gains. It saves the infant from death, the child from disease and malnutrition, the youth from dangerous ignorance, the adult from premature decay. It corrects educational systems, reduces drudgery, multiplies the yield for labor, increases the time for leisure, removes fear from the world. It gives humanity power over its surroundings, enhances comfort and well-being. It gives more time for reflection. It guarantees progressive evolution of the race. It gives Life and Life more abundantly.

This is a Golden Age for Scientific Research. Thousands are searching the heavens, the earth, the waters, and the waters under the earth, for jewels, small or large, with which to adorn the crown of Science. Since the time of Descartes—the founder of modern science—and his injunction to see, and to see more clearly, the votaries of Science have spread over the earth, examining the microscopic, the telescopic, every nook and cranny of the earth, investigating patiently and thoroughly what they found. Science is a Supreme Court of Last Appeal, in all questions of Natural Law and Invariancy, for it is simply the Human Mind perceiving these things with clear, unobstructed, sharply defined, immediate vision.

## III. Mathematical Research

When Kepler worked out his famous laws from observations he was doing scientific research. When Newton investigated the meaning of the laws he was engaged in mathematical research. The law of equal areas meant to him a central force. The elliptical orbits of the planets meant that the force varied as the square of the distance. The proportionality between the cubes of the distances and the squares of the periods meant the sway of this force extended to the confines of the solar system. By a flash of intuition he saw it also acting to the ends of the universe, and announced his law of gravitation. This was mathematical research.

Cayley looked at the diagrams of chains of atoms in organic molecules, and they became the theory of chemical "trees." The facets of crystals may all be found from the position of a single one by the laws of groups. The lines of descendants from living beings are threaded with the hereditary characteristics under the laws of combinations and averages. Statistics cannot lie when stretched on frequency curves and surfaces. The miniature solar systems in atoms are mathematical structures. Mathematical research permeates all these.

When Faraday filled space with quivering lines of force, he was bringing mathematics into electricity. When Maxwell stated his famous laws about the electromagnetic field it was mathematics. The relativity theory of Einstein

which makes gravity a fiction, and reduces the mechanics of the universe to geometry, is mathematical research. Wherever a hypothesis is set up and conclusions deduced from it, mathematics is at work. Wherever the scientist goes beyond his observed facts, introducing concepts such as energy, field, propagation of a state, line of force, action at a distance, action in a continuous medium, he is becoming a mathematician. Mathematics is a fundamental mode of thinking, impossible to evade.

Mathematics concerns itself with an ideal world of forms and relations. It constructs new worlds and studies their properties. It undertakes to draw all the necessary conclusions from given data and to point out what other propositions are consistent with the given data. It is not primarily concerned with the applicability of its worlds to that of everyday experience. But so far as this experience can be idealized, divested of its accidental features or its individual properties, mathematics includes the sublimated product among all the other more ethereal creations of its own. It has been defined as the Theory of Pure Forms, in which is meant to be included all the relations and properties of the forms. However, if so defined, it must be understood that the forms are sometimes made of flowing substance, and may in some sense become other forms, while yet retaining their essential properties. Although the definition is inadequate, it is yet quite useful in getting a view of the rôle of Mathematics as one type of Research. Mathematical research is the study of the Universal in the world. It endeavors to find the hidden spirit whose manifestations are numerous, but yet which is One despite its Protean character. It is impersonal, unemotional, not influenced by love or hate, joy or tears. Its worlds are eternal, even though they change like clouds of smoke in the wilful wind. Its additions to human thought are permanent, never decay, never explode, never

give way to others. It verifies laws, it destroys laws. It marks the boundaries of the realms in which theories may reign, it destroys theories. It humbles the arrogance of Logic and Philosophies, it equilibrates the claims of rival factions. It is the Supreme Court of Last Appeal, in all questions of reasoning, not on account of any merit of its own, but because it is after all, nothing but the Human Mind perceiving its own nexus of relations, with clear, unobstructed, sharply defined, immediate, vision.

## IV. Anthropological Research

Using the term broadly, Anthropological Research considers Man in every aspect as a function of the World, and the World as a function of Man. For the relation of functionality is symmetric, and if A is a function of B, B is a function of A. Anthropology is not concerned with the general but with the particular, with the human race, a unique product of the world. It studies man's form, structure, anatomy, physiology, neurology; it studies his origin, development, his customs, his traditions, his races, his classes, his societies, his groups, his family, each individual. Anthropology is interested in History and Education. It studies the State, the Church, Cosmopolitanism.

Anthropology is interested in the whole expanse of this tremendous function: Man and his dependence upon many variables. The regular part of the function it calls Civilization. The discontinuities and singularities it calls Revolution and Crises. The late war was a discontinuity in which the infinite ambitions of a few broke the continuity of the function. Anthropology seeks to know the value of this function, whether they all be only finite, or if perchance some may be infinite. It is interested in those ideals toward which man makes an approach but never reaches. It studies his oscillations about positions of equilibrium, his spiral paths, the winding surface of many dimensions which marks the progress of the race, the intersections of civilization with itself; its transition from one level to another. It studies the relations of the development of this function in isolated parts of the earth with

those in other parts. It studies man's reaction to his environment, how the sea makes laws of commerce, the mountains create power, the desert spirituality.

But equally it studies the world as function of Man. It is busy with the account of his inventions, of his increasing control over all the forces of Nature, of his adaptation of the world to his own comfort and better development. It studies the evolution of custom, of law, of order, of government, of the numerous compromises by which Man gains, inch by inch, a greater control over his environment. Sociology, Economics, Political Science, are merely names for some of the divisions of this enormous branch of human activity. For Civilization is a creation, it is not a product of nature.

There have always been investigators of this part of human experience, for mankind is interested in studying man. The first thinker, the first seer, the first priest, the first story-teller, must have had for his daily thought the question: How did I get into this world, and what is my relation to it? Many of the keenest minds today are studying this, the Great Function.

Anthropology has had its hypotheses. Read Plato's Republic, More's Utopia, Well's History, Buckle's History of Civilization. Read the History of Philosophy, of Art, of Mathematics, of Science, of Human Thought. Anthropology has witnessed many tremendous experiments, for they last for centuries. Experiments have been made in social schemes, in religious schemes, in schemes for the family, many inventions besides those of material things. Slavery was an invention, and experimented with. Marriage is an invention, and experimented with. The Church is an invention and experimented with. The State is an invention, and its experiments have been calamitous. Law is an invention, and has evolved into what we have today. Socialism and Bolshevism, Conservatism and Radicalism,

are inventions that are continually under experiment. The discoveries for the race have been numerous. The pages of History are eloquent with functional theorems: "The wages of sin is death!", "Persecution encourages a cause!", "Man develops immunity to some poisons, but is anaphyllactic to others," "The race and the opportunity must fit." Sociology, Political Science, Psychology, Education, Philosophy have all discovered theorems as fundamental as those of Science or Mathematics, but of a different type. Law and Universal Form are one thing, Functionality a different thing, each has its theorems. Anthropology is also a Court of Last Appeal, in all questions that concern the interwoven relations of Man and the World, for it too is the Human Mind observing the past and the future with clear, unobstructed, sharply defined, and immediate vision.

## V. Artistic Research

The Dreamer of dreams is also engaged in Research. He is not, like the Mathematician, looking for the Universal; nor like the Anthropologist, looking for the intertwined threads of the web of Man and the World; nor like the Scientist, searching for the uniformities of human experience; his object is the Romance of the world, the fascinating mystery that lives behind every veil. The Architect's touch makes stone float in air; the Sculptor's pliant fingers mould the invisible forms that are pleading for expression; the Painter's purple light and glowing mountains beckon man to the enchanted lands; the Musician's witchery speaks to the heart from incarnated forms of ethereal life; the Poet uses verse and prose to write the Divine Comedy and the Human Comedy. Each is engaged in research behind the phenomenal screen. Like other investigators the Artist is hunting reality.

He investigates the aspirations of man, his hopes, his desires, his dreams, his visions. His methods are his own. No rigorous reasoning with its fine steel net can catch the cloud-forms of his domain. No balance can weigh the character of Hamlet; no spectroscope can resolve the Ninth Symphony; no electrometer measure the potential in the Descent from the Cross. No Institution of society can imprison the subtle mist that bloweth where it listeth. But just as in Mathematics the imaginary becomes the guide to the common; just as in Anthropology the ideal of liberty, fraternity, and equality, emancipates woman, de-

thrones czars, educates the slave; so too the reveries of the Artist create new elements of life, give greater sustenance to the wings of the mind, infuse the spirit of man with new daring.

The Artist enunciates truths. They have often startled the race with their subtlety and profundity. Each school of art has seen a new vision, some new expression for these inner forms continually struggling for the right to be born, some new idea of sculpture, painting, music, of literary creation. They are experimented with, refined and finally accepted, just as are truths of mathematics, anthropology, or science. Not that every new creation of art has had its chance, any more than have all the new ideas of scientists, mathematicians, or anthropologists. The mind of man evolves but slowly.

"There was once a little girl, to whom an elderly friend gave a hyacinth bulb. "Little girl," she said, tossing up in her wrinkled hands the bulb, wrapped up in its balloon silks, "little girl, there is enclosed a miracle here; indeed, my child, life, joy, blowing color, perfume shut up in a jealous flask. It is a fine gift, a fairy present. Guard it well!" The little girl adored the hyacinth bulb, said her prayers while looking at it, nursed it like a doll, put it under her pillow at night, kissed it fondly upon waking, even took it out for a walk. But, alas! without water, without earth, without rest, the unhappy bulb withered, without ever permitting a tremor in a little green tongue, which even in the dark cellar it had put out at the face of life, as a matter of principle. And the disappointed little girl one day threw it in the face of the vexed old lady.

"Alas! Man is the child clinging to his hyacinth bulb, and with it withers his dearest desires, his hopes, his dreams." (Gérard D'Houville.)

Art searches for the spiritual, the ideal, the spontaneous, the free, knowing that these are the real, the immortal.

A tear and a laugh mean more to it than an atom, a curve, or an institution. Art also is a Supreme Court of Last Appeal, in all questions of the spontaneity of Man and the World, for it too is the Human Mind perceiving the things of the Spirit with clear, unobstructed, sharply defined, immediate vision.

## VI. The Spirit of Research

The Spirit of Research is the aspiring soul of man beating its wings against its limitations. In itself it finds the power of flight, the power of vision, the power of creation. It is not confined to what is in the field of view of a microscope, however minutely accurate. It is never content with the habitation it has built, however convenient the furnishings. It rises on the rarefied air of Mathematics that it may see the more distant horizons; it supports itself on the solid objectivity of Science that it may transmit its creations to posterity; it studies the intricate game of Man versus Nature in order that it may find the paths on which it may go farthest; it utilizes the creations of the Artist as patterns for its own spiritualized creations. Its characteristic is creative life, for it emanates from Life, not from mechanism. What it does is the spontaneous outcome of Activity.

The true scientific research is shown in Newton: "Playing like a boy on the seashore, diverting himself in now and then finding a smoother pebble or prettier shell than ordinary, whilst the great ocean of truth lies all undiscovered before him." It is shown in Poincaré: "Who was willing to work, to suffer, to pay for his seat at the show, in order that he might see, or at least that those who came after him might see." True mathematical research meets the criterion of Emerson: "We do not listen with the best regards to the verses of the man who is only a poet, nor to his problems if only an algebraist; but if a man is at

once acquainted with the geometric foundation of things and with their festal splendor, his poetry is exact, and his arithmetic musical." The true anthropological research meets the test of Henry Adams: "History has never regarded itself as a science of statistics. It is the Science of Vital Energy in relation with time; and of late this radiating center of its life has been steadily tending—together with every form of physical and mechanical energy—toward mathematical expression."

The Spirit of Research inspires the mind "to creep from fancy to the fact, and thus find progress, man's distinctive mark" (Browning). The Spirit of Research is like Socrates, who wished to know not only what seemed to be true, but what it meant for the Soul and how far it could carry man on his upward flight. The Spirit of Research is LIFE with its two wings: Intelligence and Sympathy. It is on its forward flight, impelled by the urge of an inner power. It rests but a moment on any crag, however solid; it drinks but an instant at any spring, however fresh; for from the beginning of the World it has had a Vision whose beauty, whose intoxication, whose smile, eternally say: COME.

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# A COMPARISON OF THE ETHICAL PHILOSOPHIES OF SPINOZA AND HOBBES

IF ONE could believe that a philosophical system emanates full-fledged from the mind of a single thinker, as Athene from the head of Zeus, he might venture to decide the extent to which one philosopher is indebted to another for the elements of his system. But, in point of fact, the task of awarding credit where credit is due varies directly in magnitude with the quantity of literary remains of the age under investigation. For philosophies originate out of the vague and ill-defined thoughts of multitudes of individuals. When a Plato or a Hobbes ultimately gives clearly and distinctly a systematic and classic expression to what previously existed more as an attitude than a way of life, more as a mood and a feeling than as a logical and intellectual formulation, we quite properly give homage. But as our acquaintance with the environment giving birth to this expression becomes more detailed and intimate, the more keenly do we realize that our Plato or our Hobbes has succeeded in stating better, more clearly, more consistently and more forcibly what weaker minds strove to convey. Indeed, it seems that philosophers like inventors build upon the trials and errors of their predecessors and contemporaries.

Consequently, we shall not attempt to determine precisely how far Spinoza is indebted to Hobbes, although we know he read him thoroughly. Some critics are so

rash as to consider Spinoza no more than a disciple of Hobbes, while, on the other hand, Duff, who has produced one of the most complete and careful studies thus far made of Spinoza's political philosophy, contends that Spinoza follows St. Paul more than Hobbes, and states, "a deeper influence than that of Hobbes was exercised by Hobbes' master, Machiavelli."[1] Duff's study, together with Pollock's excellent comparison of Hobbes and Spinoza, furnishes us with a thorough contrast of their political philosophies.

Our interest here is primarily in their moral philosophies, but since politics is one application of ethics, we shall have to treat of their political views in so far as these throw light upon their moral programs.

A suggestive approach to a study of the differences and similarities in the ethics of Hobbes and Spinoza is found in their opinions regarding the function of the state. According to Hobbes, *one* motive prompts men to institute a state—*fear*. The state of nature is a condition of war. Each man seeks to realize his desires, to enhance his power, and in so doing conflicts with others bent upon a like object, and the liberty (*Jus Naturale*), which he has "to use his own power, as he will himself, for the preservation of his own Nature"[2] and thus to appropriate the goods and services of other men is scant compensation for the dangers thus entailed. "In such condition, there is no place for Industry; because the fruit thereof is uncertain: and consequently no Culture of the Earth, no Navigation, nor use of the commodities that may be imported by Sea; no commodious Building; no Instruments of moving, and removing such things as require much force; no Knowledge of the face of the Earth; no account of Time; no Arts; no Letters; no Society; and which is worst of

[1] *Spinoza's Political and Ethical Philosophy*, p. 6.
[2] *Leviathan* (Everyman Edition), Ch. 14, pp. 66.

all, continual fear, and danger of violent death; and the life of man, solitary, poor, nasty, brutish, and short."[3]

Fear prompts reason to devise a condition of peace. . . . "As long as this naturall Right of everyman to every thing endureth, there can be no security to any man (how strong or wise soever he be), of living out the time, which Nature ordinarily alloweth men to live. And consequently it is a precept, or generall rule of Reason, *That every man, ought to endeavor Peace, as farre as he has hope of obtaining it; and when he cannot obtain it, that he may seek, and use, all helps, and advantages of Warre.* The first branch of which Rule, containeth the first and Fundamentall Law of Nature; which is, *to seek Peace and follow it.* The Second, the summe of the Right of Nature; which is, *By all means we can, to defend our selves.*"[4]

The motive then for establishing the state is fear, and the end sought is individual self-preservation. If man could live an ideal life, it would be, for Hobbes, a state of absolute subjection of others and absence of impediments to the desires of self. The absolute ruler most nearly embodies this ideal, for he alone enjoys the services of others without obligation to repay in kind. The average citizen, however, endures the state as a necessary evil. He assisted in its origin and helps to sustain it in order to avoid the worst possible calamity, a relapse into the state of nature. The contract then which creates the state is a renunciation of certain liberties or rights in return for protection and the liberty to gratify other desires. Nor does Hobbes believe he contradicts himself when he insists that in the event of a conflict between individual judgment and that of the monarch, the former shall yield. The individual must yield, he insists, because orignally he agreed to place the making of decisions in the hands of the state,

[3] *Ibid.*, Ch. 13, p. 65.
[4] *Ibid.*, Ch. 14, p. 67.

and, further, should men follow their own opinions society would disintegrate, and revert once more to "a warre, as is of every man, against every man."

The function of the state, as Hobbes views it, is thus essentially negative. As against anarchy or absolutism man selects the lesser of two evils—absolutism. And the laws of nature, which Hobbes calls the precepts or general rules of Reason, he sums up in a negative statement of the Golden Rule: "Do not that to another, which thou wouldest not have done to thy selfe."[5] Security is found in obeying the law, but a truly thoughtful and reasonable man must surely balance constantly in his mind the advantages of conformity to law as against a realization of his own desires.

Spinoza conceives the state otherwise. It is true there is a semblance of Hobbes in his account of its origin, for release from fear is one of the motives he mentions. But whereas Spinoza insists that at best fear is a poor motive, a passion in the individual, and a constant danger to the security of the state, Hobbes believes that life is never without fear,[6] and "the terrour of some Power" is the permanent basis of the commonwealth.[7] Hobbes, to be sure, realizes the advantages of co-operative endeavor, but for Spinoza mutual aid is the ultimate justification for social organizations and the indispensable means of realizing man's true happiness. Thus he writes in the Theologico-Political Treatise: "The formation of society serves not only for defensive purpose, but is also very useful, and, indeed, absolutely necessary, as rendering possible the division of labor. If men did not render mutual assistance to each other, no one would have either the skill or the time to provide for his own sustenance and preservation: for all men are not equally apt for all work, and no one

[5] *Ibid.*, Ch. 15, p. 82.
[6] *Ibid.*, Ch. 6, p. 30.
[7] *Ibid.*, Ch. 17.

would be capable of preparing all that he individually stood in need of. Strength and time, I repeat, would fail, if every one had in person to plough, to sow, to reap, to grind corn, to cook, to weave, to stitch, and perform the numerous functions required to keep life going; to say nothing of the arts and sciences which are also entirely necessary to the perfection and blessedness of human nature. We see that peoples living in uncivilized barbarism lead a wretched and almost animal life, and even they would not be able to acquire their few rude necessaries without assisting one another to a certain extent."[8] And again, when discussing the foundations of the state: "Nevertheless, no one can doubt that it is much better for us to live according to the laws and assured dictates of reason, for, as we said, they have men's true good for their object. Moreover, everyone wishes to live as far as possible securely beyond the reach of fear, and this would be quite impossible so long as everyone did everything he liked, and reason's claim lowered to a par with those of hatred and anger; there is no one who is not ill at ease in the midst of enmity, hatred, anger and deceit, and who does not seek to avoid them as much as he can. When we reflect that men without mutual help, or the aid of reason, must needs live most miserably, as we clearly proved in Chapter V, we shall plainly see that men must necessarily come to an agreement to live together as securely and well as possible if they are to enjoy as a whole the rights which naturally belong to them as individuals, and their life should be no more conditioned by the force and desire of individuals, but by the power and will of the whole body. This end they will be unable to attain if desire be their only guide (for by the laws of desire each man is drawn in a different direction); they must, therefore, most firmly decree and establish that they will be guided in everything by reason

[8] *Works* (Bohn Edition), Vol. I, p. 73.

(which nobody will dare openly to repudiate lest he should be taken for a madman), and will restrain any desire which is injurious to a man's fellows, that they will do to all as they would be done by, and that they will defend their neighbour's rights as their own."[9]

Spinoza saw clearly wherein he differed from Hobbes, and he states in a note to Chapter 16 of the *Theologico-Political Treatise*: "Now reason (though Hobbes thinks otherwise) is always on the side of peace, which cannot be attained unless the general laws of the state be respected."[10] And in Part IV of the *Ethics,* he writes, "Now, if men lived under the guidance of reason, everyone would remain in possession of this his right (his natural right) without any injury to his neighbour."[11] That is, whereas Hobbes considers the desires and wants of men inevitably bring them into conflict, Spinoza insists that it is only passion, the irrational and ill-informed opinions of their wants, which lead men to disagree. The true needs of men are in harmony and are realizable most fully in society. The state, for Spinoza, as for Hobbes, is a necessary evil; but it is a necessary evil, according to Spinoza, only because and in so far as it must resort to means which are a poor substitute for rational behavior. "Wherefore, in order that men may live together in harmony, and may aid one another, it is necessary that they should forego their natural right, and, for the sake of security, refrain from all actions which can injure their fellowmen. The way in which this end can be attained, so that men who are necessarily a prey to their emotions (IV., iv. Coroll.), inconstant, and diverse, should be able to render each other mutually secure, and feel mutual trust, is evident from IV., vii. and III., XXXIX. It is there shown, that an emotion can only be restrained by an emotion stronger

[9] *Ibid.*, Vol. I, p. 202.
[10] That is, of course, the laws of a democratic state.
[11] *Ethics*, IV, Prop. 37, note 2.

than, and contrary to itself, and that men avoid inflicting injury themselves." [12]

The best state then would be one which governs rationally; that is, one which establishes laws enabling men to develop and expand the potentialities of their nature. Consequently Spinoza opposes an absolutism and favors a democracy. "In a democracy, irrational commands are still less feared: for it is impossible that the majority of a people, especially if it be a large one, should agree in an irrational design: and, moreover, the basis and aim of a democracy is to avoid the desires as irrational, and to bring men as far as possible under the control of reason, so that they may live in peace and harmony: if this basis be removed the whole fabric falls to ruin." [13] As a necessary means to rational legislation, Spinoza pleads for the utmost freedom of thought and speech, distinguishing sharply between obedience to law and the expression of opinions regarding the wisdom of particular legislation.[14] "No," he exclaims passionately, "the object of government is not to change men from rational beings into beasts or puppets, but to enable them to develop their minds and bodies in security, and to employ their reason unshackled; neither showing hatred, anger, deceit, nor watched with the eyes of jealousy and injustice. In fact, the true aim of government is liberty." [15]

A difference in conception as to the function of the state carries with it a corresponding disagreement as to the nature and purpose of the individual. But here again, on

[12] *Ibid.*, IV, Prop. 37, note 2.

[13] *Theologico-Politiical Treatise. Works,* Vol. I, p. 206.

[14] See Ch. 20 of the *Theologico-Political Treatise.* A comparison of this chapter in Spinoza with chapter 29 in Hobbes' *Leviathan,* "Of those things that Weaken, or tend to the DISSOLUTION of a Common-wealth," will reveal the gap separating the two men. Contrasting what Hobbes has to say about the reading of the ancient writers with this title page of Spinoza's treatise: "Wherein is set forth that freedom of thought and speech not only may, without prejudice to piety and the public peace, be granted; but also may not, without danger to piety and the public peace, be withheld."

[15] *Theologico-Politiical Treatise. Works,* Vol. I, p. 259.

first reading, Spinoza seems to repeat Hobbes. Reason, says Hobbes, "is nothing but Reckoning."[16] And it is no more than a reckoning of consequences in terms of personal self-preservation, enhancement of vital motion and increase in power. Spinoza seems essentially to repeat Hobbes when he writes, "it is the sovereign law and right of nature that each individual should endeavor to preserve itself as it is, without regard to anything but itself."[17] And again, ". . . in no case do we strive for, wish for, long for, or desire anything, because we deem it to be good, but on the other hand we deem a thing to be good, because we strive for it, wish for it, long for it, or desire it."[18]

But it is no mere repetition of Hobbes, for Spinoza does not mean the same thing by reason, nor is his individual an insulated atom. Hobbes considers that reason recognizes little in common between men, nor does it seek to ascertain their mutual welfare. It serves rather to gratify the possessive impulses and to obtain individual advantage. When contrasting man with the bees and ants whose "Common good differeth not from the Private," Hobbes points out that "man, whose Joy consisteth in comparing himself with other men, can relish nothing but what is eminent."[19] Spinoza, however, believes that reason frees man from an isolated and miserable condition and in operating according to notions common to all men, it contributes to their mutual welfare. The rational life unites man to man. In the state of nature man has a natural right to gratify any and all desires, but this state of nature is not something actually prior to and apart from a social medium. The state of nature is merely a condition of subjection to passion and ignorance. Natural right means no more than a natural tendency to act under certain

[16] *Leviathan*, Ch. 5, p. 18.
[17] *Theologico-Political Treatise*. *Works*, Vol. I, p. 200.
[18] *Ethics*, III, Prop. 9, note.
[19] *Leviathan*, Ch. 17, p. 88.

conditions. Consequently, to say, "the ignorant and foolish man has sovereign right to do all that desire dictates, or to live according to desire," just as "the wise man has sovereign right to do all that reason dictates,"[20] is not to undermine sound morality; it is merely to say that if one lacks reason and is ruled by passion, he can act only in accordance with passion. Reason frees man from this hopeless state. It enables him neither to exploit another, nor to realize his desires at the expense of others—as it can very well do for Hobbes. As Spinoza conceives it, "men, in so far as they live in obedience to reason, necessarily do only such things as are necessarily good for human nature, and consequently for each individual man."[21] Reason thus supplies us with a criterion by means of which we can select those activities which at once aid us and assist others. Reason breaks down man's isolation; Hobbes' individual remains forever apart from others. In short, that deplorable state which Spinoza calls passion, the bondage of man, from which reason frees him, is for Hobbes the permanent condition of man. Human reason may, according to Spinoza, succeed in inaugurating an era of good will. Life, for Hobbes, is always a pugilistic encounter, and the best reason can do is to substitute gloves, a referee, and Queensbury rules for bare fists and go it as you please until the first man drops.

This difference in ultimate purpose applies as well to their conceptions of self-preservation. For each self-preservation is an increase in power, and power is stimulation of vital activity. Pleasure and pain, Hobbes defines in terms of motion.[22] But Spinoza will not object to describing emotions as modifications of Extension. He merely insists that we remember (what Hobbes denies) that vital motion and a thought activity are two aspects of one and

[20] *Theologico-Political Treatise*, Vol. I, p. 201.
[21] *Ethics*, IV, Prop. 35, demonstration.
[22] *Leviathan*, Ch. 6.

the same thing.[23] Had Spinoza chosen to treat emotion in the language appropriate to the Attribute of Extension he would not have profoundly disagreed with these statements from Hobbes: "The Endeavor, when it is toward something which causes it, is called Appetite or Desire." "And when the Endeavor is fromward something, it is generally called Aversion."[24] It is only when we inquire of each, "What is the final goal of endeavor, the ultimate end of self-preservation?" that we receive profoundly different replies.

Hobbes denies outright the existence of a Summum Bonum and contends, "that the Felicity of this life, consisteth not in the repose of a mind satisfied."[25] *"Continuall successe* in obtaining those things which a man from time to time desireth, that is to say, continuall prospering, is that men call Felicity; I mean the Felicity of this life. For there is no such thing as perpetuall Tranquility of mind, while we live here; because Life it selfe is but Motion, and can never be without Desire, nor without Feare, no more than without Sense."[26] "So that in the first place, I put for a generall inclination of all mankind, a perpetuall and restless desire of Power after power, that ceaseth onely in Death. And the cause of this, is not alwayes that a man hopes for a more intensive delight, than he has already attained to; or that he cannot be content with a moderate power; but because he cannot assure the power and means to live well, which he hath present, without the acquisition of more."[27] Carried out logically, this means that might makes right. The strongest desire, in the sense of the most vigorous and permanent desire, is the right desire in the individual's soul, as the strongest arm is the morally justified arbitrator of relations between men. We

[23] *Ethics*, II, Prop. 7.
[24] *Leviathan*, Ch. 6, p. 23.
[25] *Ibid.*, Ch. II, p. 49.
[26] *Ibid.*, Ch. 6, p. 30.
[27] *Ibid.*, Ch. II, pp. 49-50.

have observed above that the conflicts between men's interests necessitates the organization of the state; but the decrees of the state are right only because and only so long as the state can enforce its decisions. Hobbes calls upon no man to lay down his life for a lost cause. Self-preservation is the first and the last duty. "If a Monarch subdued by war, render himselfe Subject to the Victor; his Subjects are delivered from their former obligation, and become obliged to the Victor."[28] Right is the interest of the stronger. Such is the conclusion which Hobbes considers to be the dictates of reason.

Reason for Spinoza speaks a different tongue. We have already indicated that Spinoza considers natural law to be no more than a description of things as they are in the absence of organized relations between men. Natural right is not right in a moral sense. And when Spinoza states that "the law and ordinance of nature, under which all men are born, and for the most part live, forbids nothing but what no one wishes or is able to do and is not opposed to strifes, hatred, anger, treachery, or, in general, anything appetite suggests,"[29] he speaks not of what ought to be; he merely describes a fact. The laws of nature and natural rights are descriptions of conditions, not suggested programs for action. Spinoza's insistence upon viewing men's vices and imperfections dispassionately and scientifically should not blind us to his acceptance of right as an ideal, and as a valid, objective moral standard. Right is the reasonable. The right act involves in it more perfection and more power than a wrong act. In his study of human nature he means, "by 'good' that which we certainly know to be a means of approaching more nearly to the type of human nature, which we have set before ourselves; by 'bad,' that which we certainly know to be a hindrance to us in approaching the said type. Again, we

[28] *Ibid.*, Ch. 21, p. 117.
[29] *Political Treatise. Works*, Vol. I, p. 294.

shall say that men are more perfect, or more imperfect, in proportion as they approach more or less nearly to the said type."[30]

We have seen that Spinoza conceives the rational life as a social life, the life of co-operative endeavor and harmonious relations with one's fellows. Consequently the impulse towards self-preservation which, in Hobbes' opinion, sanctions and renders inevitable a personal aggrandisement and increase in power, becomes in Spinoza's ethics social action and the chief principle of social solidarity. If we act upon Hobbes' convictions we shall never transcend the selfish act; but if we heed Spinoza, in passing from passion to active emotion, we shall transform selfishness into altruistic action.

Indeed, as Spinoza conceives it, true happiness is found only when men act in accordance with rational endeavor. When he points the way to human freedom he insists, as the first condition of emancipation from passion, that we transform a passion into an active emotion, and this we may do by securing a clear and distinct idea of it. Thus, he writes in the note to Proposition 4 of Part V of the *Ethics*: "To attain this result, therefore (freedom from passion), we must chiefly direct our efforts to acquiring, as far as possible, a clear and distinct knowledge of every emotion, in order that the mind may thus, through emotion, be determined to think of those things which it clearly and distinctly perceives, and wherein it fully acquiesces: and thus that the emotion itself may be separated from the thought of an external cause, and may be associated with true thoughts; whence it will come to pass, not only that love, hatred, etc., will be destroyed (V. ii), but also that the appetites or desires, which are wont to arise from such emotion, will become incapable of being excessive (IV., lxi). For it must be especially remarked, that the appe-

[30] *Ethics*, IV, Preface.

tite through which a man is said to be active, and through which he is said to be passive is one and the same. For instance, we have shown that human nature is so constituted, that everyone desires his fellowmen to live after his own fashion (III., xxxi. note); in a man, who is not guided by reason, this appetite is a passion which is called ambition, and does not greatly differ from pride; whereas in a man, who lives by the dictates of reason, it is an activity or virtue which is called piety (IV. xxxvii. note i. and second proof). In like manner, all appetites or desires are only passions, in so far as they spring from inadequate ideas; the same results are accredited to virtue, when they are aroused or generated by adequate ideas. For all desires, whereby we are determined to any given action, may arise as much from adequate as from inadequate ideas (IV. lix)."[31]

Consequently, the impulse for self-preservation, which, in Hobbes' system, forever condemns the individual to "a perpetuall and restless desire of Power after power, that ceaseth only in Death," develops quite otherwise for Spinoza. On the plane of Imaginative Knowledge and of passion alone does it oppose self-interest to the good of others. When the impulse expands into active emotion and Rational Knowledge it leads to co-operative relations between men. Says Spinoza: "There are then many things outside ourselves, which are useful to us, and are, therefore, to be desired. Of such none can be discerned more excellent, than those which are in entire agreement with our nature. For if, for example, two individuals of entirely

[31] We should remember, however, that for Spinoza there is no distinction in kind between reason and passion. The idea of the good for him is merely the conscious aspect of an activity. When the activity gets its explanation from external objects acting upon the individual, it is passion. When it is self-directive activity it is active emotion. The increase in knowledge is not so much a cause for the transition from passion to active emotion as a description of the fact. In other words, Spinoza's doctrine of the necessary character of the universe robs the individual of genuine initiative and fundamentally renders inexplicable how on one's own account he can win freedom.

the same nature are united, they form a combination twice as powerful as either of them singly.

"Therefore, to man there is nothing more useful than man—nothing, I repeat, more excellent for preserving their being can be wished for by men, than that all should in all points agree, that the minds and bodies of all should form, as it were, one single mind and one single body, and that all should, with one consent as far as they are able, endeavor to preserve their being, and all with one consent seek what is useful to them all. Hence men, who are governed by reason—that is, who seek what is useful to them in accordance with reason—desire for themselves nothing, which they do not also desire for the rest of mankind, and consequently, are just, faithful, and honorable in their conduct."[32]

Spinoza's impulse of self-preservation leads men differently from the way Hobbes describes not merely because, in Spinoza's ethics, egoism, as the last quotation might suggest, is more farseeing than in Hobbes. In a measure such is the case. But Spinoza literally believes we gain our life by losing it. In so far as we live the life of reason we identify ourselves with God and thereby with what is permanent and common in all men. The distinctions which mark off man from man disappear, and as "the bases of reason are the notions which answer to things common to all,"[33] so the essence of individuality that remain when passion broadens out into active emotion is the force which "follows from the eternal necessity of God's nature."[34] "Whatsoever we conceive in this second way as true or real, we conceive under the form of eternity, and their ideas involve the eternal and infinite essence of God."[35]

[32] *Ethics*, IV, Prop. 18, note.
[33] *Ethics*, II, Prop. 44, Coroll. 2, demonstration.
[34] *Ethics*, II, Prop. 45, note.
[35] *Ethics*, V, Prop. 29, note.

And so the impulse of self-preservation properly leads man into the rational life. Only as a rational being does he increase his power and arrive at true independence. "Nay, inasmuch as human power is to be reckoned less by physical vigor than by mental strength, it follows that those men are most independent whose reason is strongest, and who are most guided thereby. And so I am altogether for calling a man free, as he is led by reason; because so far he is determined to action by such causes, as can be adequately understood by his unassisted nature, although by these causes he be necessarily determined to action." [36]

Thus right, for Spinoza, is no moral justification of things as they are, although it does imply an acquiescence of spirit. But the acquiescence of spirit which the free man possesses is a loyalty to an ideal which, as it were, transcends the environment in which he finds himself. He does not take advantage of the weaknesses of others, nor does he submit to the false valuations which chance to control the social environment in which he lives. He renders back "love or kindness for other men's hatred, anger, contempt." [37] Courteously and kindly he tries to lead others by reason,[38] and should he fail, he accepts the situation stoically, conscious of its eternal necessity, and "endeavors, as we said before, as far as in him lies, to do good and to go on his way rejoicing." [39]

We may conclude, then, that Spinoza and Hobbes speak quite differently regarding the function of the state, regarding the ideal social order, the nature and destiny of the individual, the place of reason in human life, and in the character and ultimate purpose of the impulse for self-preservation. Their relation is not that of master and disciple. If we may take an illustration from industrial life,

[36] *Political Treatise. Works*, Vol. I, p. 295.
[37] *Ethics*, IV, Prop. 46.
[38] *Ethics*, IV, Prop. 37.
[39] *Ethics*, IV, Prop. 73, note.

we might say that Spinoza's relation to Hobbes is that of a manufacturer to the producer of his raw materials. Hobbes supplies the raw produce, Spinoza makes it over into a new and original article.

Their disagreements find an explanation in the metaphysical backgrounds of the two men. Hobbes is a mechanical empiricist, Spinoza is a rationalist. Spinoza cannot admit that the individual is other than an expression of a deeper and more fundamental reality. Each individual, as he sees it, testifies in a unique way to the boundless and infinite possibilities of Substance; but Substance is an immanent Energy. Man is Substance and Substance is man. In God and in God alone man lives and moves and has his being. Consequently, in identifying his personal ends with the highest good of his fellows, man approaches to the supreme ethical ideal, "a knowledge of the union existing between the mind and the whole of nature."[40] Hence, the fragmentary and short-sighted character, as Spinoza must see it, of Hobbes' individualism. It is true only as a description of man's condition of bondage; and it has value only as it enables him to escape into the life of reason. Its truth is merely the truth of Imaginative Knowledge. Hence, it lacks ultimate validity both as a description of human relations as they *really* are, and as a program for attaining to a state of blessedness.

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[40] *Improvement of the Understanding. Works,* Vol. II, p. 6.

# THE LOGIC OF DISCOVERY

IT IS CONCEIVABLE that in the nature of things the mind is unable to comprehend its highest movements with clearness. If the act of discovery or invention, in its rarer and more profound phases, is, as it seems to be, one of the highest experiences of the mind, it would then be natural to expect that the mind itself could not explain such an act of invention or discovery. If any portion of the mental processes should thus lie beyond the reach of scientific analysis, one would have a domain in which precision of thought could not be attained. This looks too much like a mystery to be accepted without repugnance. But there seems to be a real difficulty in supposing that the mind can explain, or comprehend clearly, all of its acts. If so, it could explain also the act of explanation, and then this act in turn, and so on, apparently with an infinite regression. Whether such impossible infinite regression can or cannot be avoided, it is clearly a conceivable possibility that certain mental processes cannot be fully apprehended by the experiencing agent, so that one must not assume in advance that the mind can certainly comprehend clearly all of its own movements.

But this conceivable possibility need not at all affect the development of a logic of discovery in the sense of a logic by which one infers from the known to that unknown which hitherto has not been apprehended or suspected. It is with the laws of such inference that a logic of discovery

will be concerned. The data from which one starts and the conclusions reached through the use of these laws will be clearly apprehended in their relation to the process; and the completed act with its several steps can be held up, as it were, before the mind for inspection and analysis. The processes of thought in such discovery may be studied objectively after the act by means of the remembered steps of the inference; and the laws of such inference may be successfully investigated.

But it must not be assumed in advance that all the processes of discovery are carried forward by means of a logic—the inference from known to unknown. It is conceivable that any logic of discovery will necessarily leave out of account some of the most characteristic acts of discovery. One might prefer to avoid this conclusion, but one must be prepared to face the possibility. As J. A. Thomson has said: "It may be that the imaginative brooding suggests a solution in some way that we do not at present understand—life is essentially creative; it may be that there is a more or less unconscious cerebral experimenting; it is certain that letting the mind play among facts has often led to magnificent conclusions. It seems that the solution is often reached first and the proof supplied afterwards." There is the conceivable possibility of an actual creative activity of the intellect which is not suitably analyzed in terms of any logic of inference from the known to the unknown. It is clear that many of us wish to have our universe so tidy that nothing actually novel could happen in it, that every event should be a mere consequence of what had preceded it. But there are others who would not object to the surprises of true novelty, who indeed would be pleased with them rather than disconcerted by them. But the matter is controlled by the wishes of neither group. The question is one of fact, hard to be ascertained perhaps, and not one of opinion.

In this also, whatever the truth may be, it does not affect the development of a logic of discovery. For such creative acts, if they really take place, are outside of the category of inferences from the known to the unknown; and hence lie outside of the domain of a logic of discovery.

To our definition of the logic of discovery as the science of inference from the known to that unknown which hitherto has not been apprehended or suspected, we may add a few remarks as to what it is not by way of more clearly delimiting the meaning of the term. In the first place it is not necessarily a logic of demonstration. One may discover a truth by means of a definite process of inference which leads forward by well-defined steps to a clearly ascertainable proposition without carrying with it a demonstration of that proposition. This is often done by the mathematicians in important ranges of investigation. An actual demonstration of the result thus heuristically attained is then frequently given in a *de novo* argument. If some particular type of inference which is thus often successful should turn out to be always so, in the sense of never leading to false conclusions, one would suspect that a more careful analysis of it would reveal the fact that it could be put into the form of an actual demonstration. But some of these heuristic methods, which are successful in yielding true conclusions when they are sagaciously employed, sometimes lead one also to formulate propositions which turn out to be false. They give a method of discovery which must be checked by a subsequent demonstration of the results. Such a method affords an example of a logic of discovery which is not a logic of demonstration.

We have already considered the possible existence of a creative method of invention which cannot be treated by a logic of discovery, the latter being confined to inferences from the known to the unknown. Such a logic, then, does not take account of all things, even of all important things,

relative to discovery. Some of these may belong to the psychology or the physiology of the investigator. The logic of discovery has to do only with certain laws of inference. If we should think of the body of known truth in a given field as being inclosed in a region beyond the boundary of which lies what is unknown, it might be an important question whether one is most likely to be able to break over this boundary if he approaches it on a gradual spiral path taking him through a large part of what is known already or along a radial path quickly bringing him squarely against the boundary; such a question, whether important or not, would have nothing to do with a logic of discovery. It belongs rather to the psychology of investigation relative to the particular domain of truth. The logic of discovery has to do only with the laws of inference from the known to the unknown.

When we notice to what extent discoveries in science appear to be made in unforeseen ways and how often an accidental juxtaposition of thoughts leads to the detection of something new, we feel that it would be hazardous to miss one link from the chain of scientific progress or leave out of our thought even the least inkling as to what may be a successful logic of discovery. We have not yet learned how to systematically explore new territories of thought. We can only look over the field at random and hope to find here and there a pearl of great price. In the absence of a guiding logic of discovery we have no systematic method of procedure.

Francis Bacon has emphasized this matter, saying, "So it cannot be found strange if sciences be no farther discovered, if the art itself of invention and discovery hath been passed over. That this part of knowledge is wanting, to my judgment standeth plainly confessed; for first, logic doth not pretend to invent sciences, or the axioms of sciences, but passeth it over with a *'cuique in sua arte cre-*

*dendum.'"* He says further: "So it should seem, that men are hitherto rather beholden to a wild goat for surgery, or to a nightingale for music, or to the ibis for some part of physic, or to the pot lid that flew open for artillery, or generally to chance, or anything else, than to logic, for the invention of arts and sciences."

Descartes also speaks much to the same tenor; he says: "I found that, as for logic, its syllogisms and the majority of its other precepts are of avail rather in the communication of what we already know, or even . . . in speaking without judgment of things of which we are ignorant, than in the investigation of the unknown." As to places where he might find correct reasonings, Descartes writes: "For it occurred to me that I should find much more truth in the reasonings of each individual with reference to the affairs in which he is personally interested, and the issue of which must presently punish him if he has judged amiss, than in those conducted by a man of letters in his study, regarding speculative matters that are of no practical moment, and followed by no consequences to himself." But the primary source of that method which is so clearly described in Descartes' classic *Discourse* is in mathematics. He gives the following account: "The long chains of simple and easy reasonings by means of which geometers are accustomed to reach the conclusions of their most difficult demonstrations had led me to imagine that all things, to the knowledge of which man is competent, are mutually connected in the same way, and that there is nothing so far removed from us as to be beyond our reach, or so hidden that we cannot discover it, provided only that we abstain from accepting the false for the true, and always preserve in our thoughts the order necessary for the deduction of one truth from another."

Aristotle was almost entirely concerned with establishing what had been conceived already or of refuting

error, but not with solving the problem of the discovery of truth. Now and then, in reading his organon, one feels that he has almost sensed the nature of this problem, only to find that he lapses immediately into a discussion of the logic of demonstration. He thinks of confirming truth rather than of finding it.

The Renaissance gave birth to the demand for a new organon, "a scientific method which shall face the facts of experience and justify itself by its achievement in the reduction of them to control." Bacon called for the overthrow of the dominant system, for a new beginning, for recourse to nature, for induction in a safeguarded form, for experiment, for a logic of discovery. He objected to the syllogism as constraining assent where we want a control over things. But he did not succeed in his great object of founding a logic of discovery. "It has been pointed out, and with perfect justice, that science in its progress has not followed the Baconian method, that no one discovery can be pointed to which can be definitely ascribed to the use of his rules." Descartes' doctrine of clarity as the supreme criterion for a method of discovery does not come to grips with the real problem. His extension of the method of mathematics into a general method of reasoning and discovery is not adequate to the varied needs of the investigator.

So goes the story through the whole history of logic. The developed systematic logic is a logic of demonstration. Whewell saw that "science advances only in so far as the mind of the inquirer is able to suggest organizing ideas whereby our observations and experiments are colligated into intelligible system"; but he could give no direction for the capture of these organizing ideas. In the article on induction in the *Encyclopaedia Britannica* we read: "The most important faculty in scientific inquiry is the faculty of suggesting new and valuable hypotheses. But

no one has ever given any explanation how the hypotheses arise in the mind; we attribute it to 'genius,' which, of course, is no explanation at all. The logic of discovery, in the higher sense of the term, simply has no existence. Another important but neglected province of the subject is the relation of scientific induction to the inductions of everyday life. There are some who think that a study of this relation would quite transform the accepted view of induction. Consider such a piece of reasoning as may be heard any day in a court of justice, a detective who explains how in his opinion a certain burglary was effected. . . . What the detective does is to reconstruct a particular crime; he evolves no general principle. Such reasoning is used by every man in every hour of his life; by it we understand what people are doing around us, and what is the meaning of the sense-impressions we receive."

Two distinct causes may contribute to the failure to produce a logic of discovery, one having to do with the nature of the mind and the other with the assumed nature of the logic.

It is conceivable, as we have said, that the primary acts of discovery should be so largely or so thoroughly creative in their character that no science of inference from the known to the unknown can be developed; or, if it can be developed in part, that it cannot be adequate except in a very restricted range. If the process is a creative one then it would probably be agreed that it is a process whose movements cannot be predicted or analyzed into cause and effect, so that the ascription of the results solely to creative action would have the effect of closing inquiry into the nature of the process. On the other hand, if the process is truly creative in its nature it is clear that we come to no better understanding of it by shutting our eyes to that fact. But if there exists at all such a method of procedure lying beyond the reach of systematic analysis,

it appears to belong to the greater minds and to their rarer moments. In large part the method of discovery seems to belong to a logic of discovery. Leaving unsettled the question as to whether any discovery is truly creative in character, one may justly proceed to ascertain to what extent the methods of discovery can be described in terms of a science of inference. If in the nature of things we are kept away from the goal of an adequate logic of discovery, we shall nevertheless in this way get as near to it as is possible for us. We shall, however, not fail to remember the fact that we are investigating only one of two conceivable methods of discovery.

It is evident that Bacon conceived of the logic of discovery as a unit which is scarcely separable into parts. Descartes obviously held the same view in a different form. Such seems to have been the opinion of most of those who have sought to develop the subject. In several places, I have met the term logic of discovery but seldom or never the notion of logics of discovery. It is conceivable that the logic of discovery is not one in the sense of something indivisible, but that it is relative to the field of investigation or the point of view so that one should not speak of a logic of discovery in any absolute sense, but only of such a logic as relative to a given discipline or a given goal of investigation.

The usual failure to divide the problem into the parts thus suggested has, I believe, been a chief hindrance to the development of the logic of discovery. The fact that the logic of demonstration is a unit, being the same whatever the field of investigation, has led to a too ready acceptance of the view that a logic of discovery should also be a unit.

Discovery itself may be relative to the point of view of the investigator. Through the changes induced in the philosophy of science by recent advances in physics the

concept of explanation has undergone a considerable modification through the formation of a new list of basic fundamental elements in terms of which explanation has to be made. Of this change Rougier (*"Philosophy and the New Physics,"* pp. 146-147), has written as follows:

"In former days a physical phenomenon was explained by reducing it to the principles of classical mechanics, by giving to its laws the form impressed by Lagrange on the equations of dynamics. To explain a phenomenon today is to give it a statistical explanation, by regarding it as the resultant of a very large number of underlying phenomena governed by the laws of chance. . . .

"Thus not only do the most fundamental categories of our mind, those of space, of time, of causality, pass through an evolution with the progress of science but the same holds even for the concept of intelligibility. To explain a phenomenon is, for primitive man, to interpret it anthropomorphically by a supernatural agent endowed with psychological life in his own image; for a scholastic it is to explain it by ultimate causes; for Maxwell it is to deduce it from the principles of mechanics; for Gibbs and Boltzmann it is to account for it by the calculus of probabilities, by starting from a system of elements subject to given conditions. Human reason is not *'une et entière en chacun'* as Descartes taught. It varies with the abstract or concrete nature of our thought, and in proportion as, on contact with experimental facts, the adaptation of our mind to nature becomes progressively realized."

Now when one modifies the meaning of such a fundamental thing as intelligibility, or explanation, he changes his point of view so radically that he will, in his investigations, look for quite different things from those for which he would otherwise look. There is a great difference between the way of work of one who expects to find the inner secrets of phenomena and that of one who supposes that he

is only to get some convenient shorthand way of expressing the relations of phenomena without any approach to their ultimate explanation. One can set himself to find only those sorts of explanation which he deems to be possible. He may find other things by accident as it were. But he cannot seek them systematically. His logic of discovery, the way he infers in fact as opposed to the way in which he infers perfectly, varies profoundly with changes in his point of view and especially in his view as to what constitutes explanation.

When one conceives a definite law of progress from lower to higher methods of thought, as Comte did in connection with his law of the three states, he will carefully direct his own thought towards what he conceives to be the higher. Indeed, in a case so well marked as that of Comte, he will avoid entirely the methods which are conceived to be of the lower sort and will undertake to carry forward his investigations solely by means of what he conceives to be the higher method. In this way an abstract ideal of excellence, when formed in accordance with a classification of method as more primitive or more secure, necessarily dominates the order of procedure in demonstration. The logic of discovery is a function of the ideal of excellence in different sorts of truth; it depends on the point of view.

With the conception of scientific explanation which is prevalent in our times it would be quite impossible for one to proceed as Descartes proceeded. One of his editors says of him: "Refusing to let himself be hindered by lack of adequate information, he thought out what the constitution of the world and man must be if they were to be clearly understood." Descartes conceived of clearness of thought as a criterion for truth and was convinced that God had arranged things so that true knowledge is possible. Then if it were true that the ideal of clearness of thought could

be attained under only one conception of the nature of God and the world and man, then this must be the valid conception of these things. Out of the mind itself and the ideal of true thinking which was imposed upon it, he believed that some of the most momentous conclusions of science could be deduced without any experimental evidence. The necessary type of scientific explanation could be deduced by considerations having to do with the mind itself. He realized that experience is necessary for details; but the fundamental terms in which the explanation must be made he deduced by means of his ideal of clearness of thought.

The logic of discovery which is implicit in this type of argument makes no strong appeal to scientific thinkers of our day. It depends upon a conception of nature and thought too far removed from what is now current. But the example serves to enforce the fact that such a logic may be relative to the ideal elements in the point of view.

Before one can proceed to a detailed development of any logic of discovery it is important that he shall determine in what part of the thinking process is to be found the essential step of discovery. It is clear that it is not in the proof of a proposition once conceived, even though with uncertainty; the latter requires only the use of the logic of demonstration, whether the conjectured proposition is established or shown to be false. The essential step is in a much more original act. It is in the formation of the conjecture itself or goes back even farther to the formation of the hypothesis out of which comes the proposition to be tested, whether by experiment or by reasoning. It may even be found in a more remote place in the process of discovery than this, its chief element resting in a principle partaking somewhat of a metaphysical nature (as in the general principle of relativity), or in an ideal of a purely abstract character (as in Descartes' doctrine of clar-

ity). It is as if the mind were seeking to impose itself upon nature, insisting that whatever explanations we may finally adopt they shall be such as satisfy the requirements of a norm set up by the mind itself. Certain of these demands may be impossible of realization. One then constructs a norm of a modified sort. The essential step in discovery is in the construction of definite hypothesis in the form of a particular or a general law or proposition and in the formulation of a principle or norm lying back of the hypothesis and contributing effectively to giving it existence.

Galileo informs us that he discovered by reason the law of distance for falling bodies and that he afterwards verified it by experiment. The Copernican assertion of the motion of the earth is neither a deduction of the pure reason nor a datum of experience but an hypothesis which has been verified. Kepler can tell us the precise date on which he conceived correctly the relation of periodic times in planetary motion though it was more than a month afterwards before he succeeded in verifying the law by detailed computations. The law of gravitation offered itself clearly to Newton's thought in 1666, but was temporarily discarded from lack of agreement with recorded observations, to be revived and accepted later when more accurate observations were available for a better check. He was so agitated over the possibility that these new observations would verify his theory that he got one of his friends to undertake the necessary computations for him because in his emotional excitement he did not feel capable of doing it himself. The laws of nature, in the absence of sufficient experimental evidence to prove them, are often conceived through a happy combination of thoughts in the mind of the investigator. Innumerable useless combinations are passed over and the vital ones rise to consciousness to bring new truth to light.

An instructive failure to realize the importance of new hypothesis in physical science is brought out by the following two paragraphs taken from the "Register" of one of our leading American universities where they appeared regularly for a decade and a half overlapping the end of the last century and the beginning of this:

"While it is never safe to affirm that the future of Physical Science has no marvels in store even more marvelous than those of the past, it seems probable that most of the grand underlying principles have been firmly established and that further advances are to be sought chiefly in the rigorous application of these principles to all the phenomena which come under our notice.

"It is here that the science of measurement shows its importance—where quantitative results are more to be desired than qualitative work. An eminent physicist has remarked that the future truths of Physical Science are to be looked for in the sixth place of decimals."

This conception of the state of physical science seems to have had considerable currency in the earlier nineties. Since then a veritable revolution has taken place. New theories have sprung up and have manifested remarkable vitality. An eminent physicist has advised the young men to try all sorts of "fool experiments" on the ground that there is no way to anticipate what remarkable things may thus be brought to light. After a period in which successful hypotheses were seldom formed there has come one when new and even startling hypotheses have followed one after another with bewildering rapidity, and physical science has taken such a leap forward as has been witnessed only two or three times in its history.

This emphasizes the importance of the place of hypothesis in the process of discovery.

The logic of demonstration is by definition only that sort of logic which compels assent to the conclusion when

the premises are granted. It is this logic by which one is always to establish those results which are to be made secure in virtue of their logical dependence on results which are already known to be secure. It is the universal method of the mathematician when he sets forth for others the proofs of the truths discovered by him. In the natural sciences it is often true that one must start from principles which are only probably, or even only conjecturally, true. There is always the possibility that some new phenomenon will be brought to light not in agreement with the principles already accepted, so that one never establishes precise results with compelling logic. There is always the need for an experimental test. In a certain part of mathematics this is not so, namely, in that part in which the doctrine advanced gives rise admittedly to a body of results that follow from given postulates which are accepted.

This marked difference between mathematics and physical science is not altogether so universal as has sometimes been supposed. In some fields, as in that of the theory of numbers for instance, we are dealing with a set of objects which we assume ourselves to know so thoroughly that our basic propositions are not so much postulates as the statement of known properties, as of the positive integers in the field mentioned. It is conceivable that as a matter of fact we do not know the positive integers well enough for this; and that, on the basis of the initially accepted properties, we may be led to some result not holding for all positive integers. We should then be compelled to modify our statement regarding our theorems and say merely that they are true for those entities which satisfy our basic propositions. We have such confidence, through the result of previous experience, that such a breakdown is not going to ensue that we proceed without any systematic experimental verification of our results. We do, however, subject them often to the test of more or less random

numerical verification; and this is in many respects similar to the experimental test in the laboratory of some conclusion in natural science obtained from theoretical considerations.

If it is objected that we do not develop the theory of positive integers from our clear conception of their basic properties but from an assumed basis of postulates, the answer is that the latter is indeed the theoretically satisfying form, but that investigators and expositors in the theory of numbers have for the most part proceeded in the way we have indicated from propositions the truth of which they have granted without question, and have not thought of their work as giving the consequences of certain postulates so much as yielding veritable properties of clearly perceived existent entities. If it is objected that this is not a perfect procedure, it may be said in reply that it is the procedure which has actually been employed. The theory of numbers, as a matter of historical fact, has been developed from certain propositions concerning numbers the truth of which one seems to ascertain immediately from his acquaintance with integers either through experience of them or through the invention or creation of them by the human mind.

These basic truths are closely analogous to the laws of the physicist. Perhaps one has a right to accept them with greater confidence than is legitimate for the physicist in his more complex domain; but the ground of the confidence seems to be of essentially the same sort, the increased confidence being due to our fuller knowledge of positive integers than of electrons for instance. Whether this fuller initial knowledge is due to the fact (if it is a fact) that the human race created integers or is due to a longer and more intimate experimental acquaintance with integers does not seem to alter the essential character of this initial knowledge. If we accept the principles of an exact logic

and agree to a given set of basic propositions, then we necessarily accept the results which flow from these propositions by means of these logical principles. Thus we are accustomed to begin the development of the theory of positive integers, not from a body of assumptions but from a body of propositions which we agree are true of positive integers—for instance, the proposition that the larger number A of two positive integers can be written as a sum of two terms one of which is an integral multiple of the smaller number B and the other is zero or a positive integer less than B. These we take not so much for postulates as for true propositions from which we begin our argument.

There are two possibilities concerning the character of this knowledge. Either we have it by an immediate insight or intuition of its truth; or we have attained it on some sort of experimental basis. If it is by the former, then we have no suitable means of knowing the validity of our insight; if it is by the latter, then we cannot be said to have tested the matter fully until we have examined every aspect. But to examine every aspect of it we shall have to verify every logical consequence of the originally accepted propositions. Then we can never be said to know fully the truth of a proposition which we have derived logically unless we subject it to some sort of experimental test, provided that that truth is not merely one which asserts the logical connection of propositions.

Hence, either from the lack of complete certainty of our insight or of the full reach of our experience we are in the position of being short of absolute logical certainty even for our propositions about positive integers. But we have so frequently verified our results in the past that we have attained to an emphatic confidence that they will be verified in the future. Our experimental evidence is great enough to give us a strong feeling of security. And yet it may be observed that workers in number theory still seem

to feel a certain satisfaction in exhibiting numerical verifications of their more abstruse theorems.

This seems to me more like the experimental verification of the natural scientist than is usually supposed. The mathematician does not feel so keenly the need of it as the physicist; but is not this confidence, after all, due primarily to the mathematician's previous experience of almost constant success whereas the physicist has more often reached wrong conclusions, due presumably to the greater intrinsic difficulty of his subject matter? Even the physicist, as we have already seen, has passed through stages in which he was almost absolutely confident of his principles and was looking around only to find means to get the right figure in the sixth decimal place. In biology there appears to be almost the same feeling of absolute certainty that things have come to their present state through some process of evolution which is not merely one cycle in an unending sequence of repeated cycles.

In the progress of knowledge we are concerned both with the logic of demonstration for the firm establishment of truths once suspected and with the way or means by which one may come in the first place to formulate a proposition and to suspect its truth. The question also arises as to whether a sure process of inference from the known to the unknown exists—that is, whether there are well defined characteristic processes, imbued with full logical rigor, by which one may pass directly from the known to the unknown in such a way that in the very passage to the new truth there is inherent the forcible logical demonstration of that truth. Or, should we seek rather some sort of heuristic logic by which one comes first to formulate a proposition whose truth he suspects, while the demonstration of it is to be sought later by more secure processes?

It seems certain that the former alternative is not realized. There is no secure logic of discovery different from

the logic of demonstration. Whatever process of reasoning ends with a new truth, demonstrated as it is attained, is carried out only by a secure logic of demonstration. This does not mean that the latter is never a logic of discovery; in fact, it is often this. Many truths which assert merely the logical dependence of propositions are attained by a logic of demonstration—especially when the propositions are conceived in their abstract form. And not a few others are also derived in this way. Maxwell's prediction of the pressure of light resulted from a truth discovered by a logic which carried with it a demonstration of the fact that this truth is a consequence of accepted laws of physical phenomena. The most striking recent instance of this sort of discovery is that of the bending of a ray of light in a strong gravitational field, as predicted by Einstein in his general theory of relativity.

It appears to me that the use of a logic of demonstration for the purposes of discovery does not afford a typical instance of the logic of discovery. If there is any point to considering the latter at all it is because it has, in important instances at least, characteristic qualities which are worthy of investigation. Accordingly we turn now to a further consideration of the question of the existence of some sort of heuristic logic by which one comes first to formulate a proposition whose truth he suspects, while the demonstration of it is to be sought later by more secure processes. That such processes of inference exist has certainly been recognized since the time of Aristotle. Analogical reasoning is of just this sort and so is the conclusion from the particular to the general. But the problem which we have in mind is not so much that of the general principles of probable but insecure inference, as of that which arises in consideration of some such question as the following:

Have the particular sciences certain heuristic logics and do these vary, in whole or in part, as we pass from one particular science to another? This question forces upon our attention another, namely, the question as to whether all logic is one or whether logic is relative to the field or the subject matter to which it is to be applied. The foregoing separation of logic into two parts seems important here; and we should probably press a two-fold question: Is the logic of secure demonstration one, the same in all ranges to which it may be applied, or is it something relative to the subject matter under investigation? Is the logic of discovery, the guiding but nevertheless not absolutely trustworthy logic of the preliminary stages of an investigation, the same for all ranges of subject matter or is it relative to the subject matter of the different sciences? Without attempting to go into a full discussion of the question we may say that the secure logic of demonstration appears to us to be one and the same whatever the field of investigation. The forms of reasoning which in one science compel assent to its conclusions from accepted propositions are the same as those which in any other science have the same compelling power. One form, for instance, mathematical induction, may be rather frequently employed in one science and appear seldom or never in another; but it is valid wherever it applies and has the same compelling power.

But it seems not improbable that a certain heuristic logic in one science may have no conceivable place at all in another.

In some investigations which I have carried out in the past two or three years, I have had occasion to treat a great variety of related transcendental problems by means of methods to which I was led by certain fundamental algebraic guides to transcendental problems. Such guides appear to have been first employed by Sturm (in 1836) and by Cauchy (see Moigno's lectures, 1844). They were

brought into great prominence in more modern times through the initiative of Volterra whose work in this direction first became explicit in his publications in 1896. They appear to be of such a nature as to be useful only in mathematics; and in fact in only a certain well-defined region of mathematics, though their full value here seems not yet to have been realized in accomplished use. It is of interest to note the sort of results to which they give rise. One is led by them to a more definite and precise formulation of a variety of problems originating from certain applications of mathematics to physical phenomena, the formulation being so sharp and clear as to enable the mind to concentrate its thought upon the leading issues and to avoid the waste due to a distraction of attention by irrelevant matters. The central fundamental theorems around which the detailed theory of these problems gravitates are suggested so clearly by the heuristic process to which one is led in an unmistakable manner as to leave no room for a failure to discover these theorems, at least in a wide range of problems. The process does not directly and immediately afford us a proof of the theorems. But it does yield precise suggestions concerning the method of proof by which one may establish them through a rigorous logical procedure. This particular heuristic logic, then, serves the three-fold purpose of making the problems definite, of suggesting the central theorems, and of indicating suitable methods of proof.[1]

Perhaps this example represents the extreme of definiteness and serviceability in these heuristic logics. All gradations exist between this and that other in which the inference from the known to the unknown is through well-defined processes which are imbued with full logical rigor

[1] This heuristic logic afforded the principal subject of my retiring address as Chairman of the Chicago Section of the American Mathematical Society in December, 1921; a detailed account of it for mathematicians will be found in that address as published in the *Bulletin* of the Society for April-May, 1922, pp. 179-210.

and by which one may pass directly from the known to the unknown in such a way that in the very passage to the new truth there is inherent the compelling logical demonstration of that truth. Let us for a moment contrast this later sort of logic of discovery with the former.

We can get it before us best by taking an example where it would naturally be employed. Let us suppose that one has observed that the positive integers may be separated into two classes: in the one class are those positive integers each of which is a product of two smaller positive integers; in the other are all positive integers not in the first class. Let us call the integers of this second class prime numbers. Let us suppose now that one has already found out in some way that every integer of the first class contains as a factor some prime number greater than unity. Suppose then that he raises the question as to the number of integers in each class. In both classes together there is an infinitude of numbers, since these classes together contain all positive integers. That the first class contains an infinite number of integers is obvious, since it contains an infinitude of powers of each integer or since it contains the double of every positive integer. The question which remains and calls for answer is whether the number of primes is infinite. The answer, complete or in part, must evidently be one or the other of the following: the number of primes is finite; the number of primes is infinite. The order of procedure is obviously to assume one or the other of these alternatives and to test it; if we assume the wrong one we can expect to arrive at a contradiction. Let us try out first the simpler assertion that the number of primes is finite. Then let P be the product of all of them, and consider the number P+1. It is divisible by no one of the primes except unity, since we have a remainder of one on any such division owing to the fact that P is now supposed to be the product of all prime numbers. Hence we have a

number larger than any prime and without a prime factor, in contradiction with what we already knew. Hence, the number of primes is infinite. Our question is therefore answered with a precision which may be accepted as tentatively satisfying.

Here the process by which we arrive at the answer to our question contains the proof that the answer is correct. Here the logic of discovery is in no wise different from the logic of demonstration.

But there is something peculiar about this case which is not present in all cases. The question whose answer we sought has by its nature one of a finite number of answers which press themselves at once upon the attention as the logical possibilities, and this almost as soon as one has clearly conceived the question. Let us ask, on the other hand, what is the law of force among the atoms or parts of atoms in chemical combination. There is no such finite set of exhaustive and useful logical possibilities to arrest the attention. So far as the logical elements in the situation are involved there is an infinitude of logical possibilities of co-ordinate importance. What we know about the matter is far too little to compel as inference one or the other of any finite set of useful logical possibilities. We cannot proceed to the desired truth by means of logical processes compelling the conclusion and demanding confidence in the results attained. We need some logic of discovery different from that which is suitable in demonstration.

Such a heuristic logic will be necessary partly (and roughly) in proportion to the definiteness and completeness of the underlying truths already in hand and on which we proceed to build the theory. Such necessity will increase with the complexity of the problem to be investigated and the consequent difficulty of an orderly procedure from the known to the unknown. Hence there are two

stages in the development of a science when one will be in an especial need of a heuristic logic. The first of these is that necessary for a science in the nascent stage of its development when its underlying basic principles are being discovered and put in order. This need will subsist in greater or less measure for each experimental science as a whole, and especially in its infancy. The second of these necessities is that which arises in the remote and complex developments of some phase of a science when one wishes to branch off rather widely from the beaten trail and to develop a new chapter or section of the science.

It is not our purpose to consider the logic of discovery outside of the domain of the exact sciences nor indeed to discuss the variety of heuristic logics suited to the various sciences or their several parts except in so far as this may be convenient in analyzing the general character of such logics. From the example which we have exhibited from mathematics it must be clear that a logic of discovery may be special to a particular well-limited class of closely related problems and hardly have a point of contact with any other investigations whatever. Other examples with the same character can be found in mathematics, especially in those fields where the physical intuition can be brought to bear upon the mathematical problem, as in the theory of differential equations (ordinary and partial) with boundary conditions. It seems likely that this relativity of the logic of discovery to the particular subject matter of investigation will be found to be a characteristic of it in all divisions of science.

The tentative nature of the logic of discovery allows room for an error of a dangerous sort. In some cases the measure of sagacity required in the successful use of such a logic is not very great, and the investigator is therefore able to sense accurately a considerable class of results without a need for discarding any. His success for a time is

so uniform that he begins to lose sight of the tentative character of his processes of inference and to think of these as secure in the sense that they guarantee the validity of the conclusions attained. Then he gradually ceases to feel the need of a test of verification and is inclined to be satisfied without it, particularly if it is hard to devise such a test. He begins to have an undue confidence in his heuristic method. He has often found it successful. If it has ever led him astray he has seen clearly where he lacked in sagacity. In the new situation he seems to have avoided all extraneous sources of error. He concludes therefore that the result heuristically attained is valid even though he has not tested it independently.

It appears that an error of this sort is especially likely to arise in those domains of natural science in which one initially makes large abstraction of the actual complexity of the phenomena in order to bring them within the range of successful investigation.

Let us take an example of this, purposely put into extreme form in order to make the point clear.

Let us suppose that one is investigating the processes of thought. He examines all the observable circumstances connected with a process which yields a poem or an hypothesis in natural science or the consequence of some physical law or a theorem in mathematics. Waiving the question, for the moment, as to whether there is something hidden which he cannot see, he proceeds to make a complete catalog of all that he can find as he looks upon the thinker while in the process of thought. He varies the individual under investigation and the circumstances under which he is examined and the subject matter of his meditations. After a time he has a large number of facts of observation. Testing them for common properties, he finds that every one of them is of the nature of a physico-chemical fact. They are the sort of thing which the careless observer

might call the physico-chemical concomitants of the processes of thought. Our investigator is too careful to name them in this way; for that would already be to read into the observed facts a large measure of deep-lying hypothesis, and it is desirable to avoid this lest we read our prejudices into the facts. The subject whose processes of thought are being investigated will not be asked to give an account of his own experience; for, in doing so, his prepossessions will necessarily color his account. The matter must be made more thoroughly objective than would be possible under such a plan of procedure. The observed facts are before the investigator, shorn of everything which might be colored with prepossessions. They are all physico-chemical in their character. In how far may they account for the processes of thought? For each recorded movement of the thought process there is a physico-chemical phenomenon. What is the connection between them? In how far can one describe or explain the processes of thought in terms of these physico-chemical changes? These are natural and legitimate questions.

Let us suppose that a very considerable success has been attained in setting up a definite one-to-one correspondence between isolated items of thought and particular physico-chemical changes, so that one may measure certain movements of the physical frame and tell the subject truly at least a part of what he was thinking at the moment. As the work proceeds the investigator becomes more successful in recording the thoughts by means of the observed reactions of the physical frame. He begins to raise definitely the question as to how far he may go in accounting for thought in terms of physico-chemical changes and (let us say) he begins to incline to the view that a complete success is possible. In his meditations he begins to say to himself that if there is anything in the thought process besides these physico-chemical changes then that part of it

cannot be subjected to scientific investigation. A long experience with the matter inclines him more and more to identify these physico-chemical changes with the thought process which he started out to investigate; and he is led finally to assert the identification.

This case I have purposely made extreme; but I find it difficult to tell how far our psychologists have gone in this direction. Some of them have gone quite far enough to leave me bewildered. They seem to have laid aside some of the fundamental elements in the problem and to have neglected the fact that they have done so. Or have they merely delimited the field of "psychology" and left the study of the mind, in the older sense, to philosophy or to some science not yet created?

If I seem to have departed from my subject of the logic of discovery I wish now to come back to it and to say that it is legitimate to make any tentative abstraction of elements whatever that may seem desirable in a particular investigation, provided that it is always remembered that such abstraction has been made. It is desirable to know in how far the processes of thought can be described and explained in physico-chemical terms. But it is undesirable to allow the success in establishing the correspondences between the two things to obscure the fact that the style of the investigation necessarily leaves out of account entirely a certain type of phenomenon, and that it therefore throws no light on the question of the existence or non-existence of this type of phenomenon.

It has been said that "the greatest discovery ever made in philosophy was that the way to discover whether a thing is present is to look and see." In ancient times the exponent of this doctrine was Aristotle, while Francis Bacon brought it to clear notice in the modern world long after Telesio and Roger Bacon had unsuccessfully insisted upon it. But, as applied to external nature, the doctrine was

not in good repute with Plato who considered it erroneous and upheld as true that which agreed with his sentiments of propriety and beauty. Since the latter were supposed to have been ascertained by looking inward upon the mind this process of introspection finally came into disrepute because it led to contradictions with what was found by looking upon the external world to see. This disrepute of the practice of looking inward has been so great that some psychologists seem to have become afraid to use in their science as a method of discovery the simple one of looking and seeing. Their logic of discovery, so some of them seem to insist, must be one of induction from the observation of physico-chemical phenomena. To an outsider they seem to be in need of finding out again that the way to discover whether a thing is present is to look and see, at least in matters pertaining to the experience of concentrated thought. Their logic of discovery seems to have been made too narrow.

This narrowing of the range of the logic of discovery is not peculiar to any one science. It seems to me to be an ever-present danger in the necessary form of the process of discovery in any natural science. We cannot deal at once with the whole complexity of phenomena. We choose a certain part of them and try to find our explanations in terms of that part. It is always admitted that complete explanations have not been found; but that failure is accounted for by an insidious error common in what I would like to name the proof by ignorance. It is likely to arise when great abstraction has been made and this fact has been ignored. A good example of it in a general situation is afforded in condensed form by the following quotation:

"If, then, it is impossible, through deduction beginning from the transformist hypothesis, to build a theory of morphological evolution verified by experience, this is not be-

cause the hypothesis is false; it is because it is incomplete and corresponds solely to certain factors of evolution. In order to foresee the results it is necessary to know all the factors. The apparent indetermination arises solely from the insufficiency of our knowledge."

"The apparent indetermination arises solely from the insufficiency of our knowledge." Here in a single sentence is the essence of the proof by ignorance which in one form or another is often advanced in scientific discussions and sometimes treated in a dogmatic way widely variant from the spirit of true science. If one is to maintain the method at all he will probably have to do it merely by dogmatic assertion; for there does not appear to be any argument in its favor. It should be apparent to every one, on reflection, that we can never prove any positive proposition, any significant truth, by means of our ignorance of the facts in the case—except the one fact of our ignorance. When it is said in the foregoing statement that "the apparent indetermination arises solely from the insufficiency of our knowledge" the truth of this cannot be known from our knowledge of the facts, for we are confessedly ignorant of them. Three possibilities arise then: either the truth of the statement is not known at all; or it is known by some transcendental insight into external phenomena; or it is known through our ignorance of the facts. There is no room to doubt what the scientific conclusion is in the matter: the statement is not known to be true. It is neither demonstrated logically nor verified experimentally.

Perhaps we should dwell still longer upon the absurdity of this proof by ignorance, for it contains the essence of frequent error which vitiates many conclusions. W. K. Brooks in his *Foundations of Zoology,* truly says: "The hardest of intellectual virtues is philosophic doubt, and the mental vice to which are most prone is our tendency to believe that lack of evidence for an opinion is a reason for

believing something else." In the absence of any evidence that the indetermination is not due to our ignorance we are inclined to conclude that that is the ground of it. The second law of thermodynamics rests on just such insecure foundations. We know no facts to dispute it; we are far from having established it. The present situation warrants our taking it as an hypothesis to see what we can get out of it; but it does not justify any uneasiness of mind as to what it may say about the future history of the universe. The principles of the conservation of energy and of the conservation of mass, or their modern combination into a single principle, may well serve as a working hypothesis. But we must remember that no conclusion based on an hypothesis of such a character carries with it its own validity. Such a logic of discovery requires to be supplemented by an independent test of the result; and the latter may be accepted only provisionally in the absence of such a test.

There is something finer in a possible logic of discovery than anything which we have so far made explicit. One may arrive at truth not only by induction and deduction but also under the impulse to realize directly an ideal as to the form of the truth to be attained. The most striking recent instance of this is found in the general theory of relativity as developed by A. Einstein. In order to bring out clearly its character in this case we shall have to present certain elementary considerations associated with one aspect of the theory of relativity.[2]

Let us approach the matter by thinking of a geometrical curve fixed in the space interior to a given room of four walls meeting at right angles. If we take the floor and two adjacent walls to be a system of reference by means of which to locate the positions of points in the room,

[2] By taking the technical mathematical terms in their usual non-technical sense the non-mathematical reader will have a sufficiently clear idea to make the argument intelligible.

then we can uniquely define the position of a point on our curve by giving its distance from the floor and from each of the two walls selected. If the point moves along the given curve then the numbers expressing these distances will vary and will be related according to a law determined by the shape and position of the curve; these three variable numbers will satisfy certain equations of condition. Now suppose that we modify our procedure by using the ceiling and the other two walls as a system of reference. Since the relation of the curve to this system of reference is in general different from that to the former we obtain in general different equations of condition for the same curve. These conditions for the same curve would be modified still further if we should choose some other set of three mutually perpendicular planes for the system of reference, and especially so if these planes should be oriented in some new directions.

It is clear that the properties of the curve itself have in no wise been affected by these changes in the system of reference, even though we have several times modified the mathematical expressions by means of which these properties may be most compactly and most completely described. Let us for a moment forget these systems of reference and consider the curve itself by passing along it from point to point. Two characteristics will force themselves upon our attention: The amount of bending of the curve as we pass along it, its curvature; the amount of twisting of the curve, its torsion. These are intrinsic properties of the curve itself, capable of representation at each point on it by definite numerical values. These numerical values can be expressed in terms of the three distances pertaining to any given one of the systems of reference mentioned above; it turns out that definite rather simple formulae exist for expressing the curvature and torsion in terms of the named measurements. Since these describe

intrinsic properties of the curve their values must be unaltered by the transformations of variables due to the changes in the system of reference; that is, they must be invariants of the transformation.

Thus it is seen that the analytic expressions for the curvature and torsion are unchanged in form and in value as we pass from one of our systems of reference to another. It can be shown that they completely determine the intrinsic properties of the curve. Then we have in them a complete mathematical description of the intrinsic properties of the curve in a form from which we have abstracted those peculiarities which belong to the special system of reference by means of which we described the curve and its position in the first place. This sort of abstraction is of frequent and important use in mathematical investigations. It affords one of our methods of excluding from consideration those things which are irrelevant to the central purpose of the investigation and of fixing attention upon those things alone which are unaltered by, or are invariant under, the transformations permissible among the elements in consideration. A similar but extended use of invariants gives substance to the ideal which guided the development of the general theory of relativity.

Two considerable extensions are necessary before we can realize precisely the situation in the development of the Einstein theory. The first has to do with a generalization of the system of reference. We must replace the three mutually perpendicular planes of our system of reference by three warped surfaces, perhaps twisted and corrugated and irregular in shape and restricted only enough to allow us to utilize them successfully for the unique location of points in space. By means of these we are to describe the space configurations with which we have to deal. The other extension consists of the introduction of time into our system. We cannot well develop the mechanics of

three dimensions by means of what is merely geometric in three dimensions; but if we introduce time and think of our space-time continuum as affording a four-dimensional world, then our mechanics in three dimensions is replaced by a geometry in four dimensions. In the Einstein theory one no longer tries to maintain the separation of measured space and time; they are not independent; they are indissolubly united into a four-fold space-time extension. In this space-time of four dimensions we are to choose as a system of reference four warped three-dimensional spaces by means of which the location of points in this four-dimensional space-time shall be defined. We can go from one such system of reference to another by a change of fundamental variables. The totality of these changes is said to form a group, and certain subsets of them are called subgroups.

With these conceptions in mind, it is easy to make clear the nature of the ideal upon which Einstein insists as to the character of the laws of nature. He wishes to have them expressed in such form with respect to this four-dimensional continuum that there shall be no change in the form of these laws when we pass from one of these systems of reference to another, the mathematical relations expressing the laws are to be invariant when all quantities involved are changed in accordance with a transformation from one system of reference to another; let us say for convenience that the laws are to be stated in covariant form. When we have put them into such form we have abstracted from the statement of them everything which pertains to the particular system of reference employed.

It is a grave question whether the laws of nature are capable of formulation in accordance with the requirements of such an ideal; and an affirmative answer can be maintained only after a searching examination. All precise

evidence which exists up to the present time is in favor of the conclusion that such an ideal may be realized.

It is not necessary to our present purpose to go into a further analysis of the question as to whether this ideal may be realized in practice. We wish to look upon it as affording an example of a logic of discovery. One here sets up a certain ideal as to the form of the laws of nature. He then takes those known laws which agree closely with experimental facts and enquires whether their statement meets this ideal. This affords the best way to make trial of the validity of the character of the law which his ideal would impose upon nature. If the law as previously conceived meets this ideal there is a certain satisfaction, but there is nothing further to be done with this law. The investigator passes to another in order to discover, if possible, one which is not yet subject to this ideal. Suppose that he finds one, as Einstein did in the case of the Newtonian law of gravitation. It turned out that this law does not accord with the ideal of covariance of the laws of nature. Shall one then give up the ideal on the ground that the Newtonian law is so well established that any deviation from it required by a new ideal shows that this ideal is not realizable? No, not on this evidence alone; it may be, after all, that the law of Newton is not exact and that some modification of it will bring it into covariant form without disturbing its agreement with observed facts. If so, there is likely to be some range of facts, perhaps not previously observed, in which the two laws will give measurably different results; and one will then have a crucial experiment by means of which to discard one in favor of the other.

As a matter of fact, the Einstein theory appears to have triumphed over the Newtonian theory in precisely this way. Whether it has or not is not essential to our present purpose. We are concerned with the heuristic logic in-

volved in the process. In accordance with the Einstein demand it is desirable to enforce upon nature, if possible, a certain ideal as to the mathematical form of the statement of the laws of nature. This ideal guides one's investigations and leads to conclusions as to laws of experimental phenomena. It does not prove these laws; for we have not yet any means of knowing that the laws of nature are capable of expression in covariant form. But it does give us certain new laws, or modified forms of old laws, which we would probably not have reached except under the guidance of such an ideal. The ideal thus gives rise to a logic of discovery. The supposed law once attained is subjected to a searching test. If it survives under this test, then we have a veritable advance brought about by the guidance of an ideal which affords a heuristic means of inferring the unknown from the previously known.

According to this aesthetically satisfying ideal of Einstein, then, we are to have in the mathematical form of the laws of nature a complete covariance under the general group of transformations of coordinates in the four-dimensional space-time continuum. But in the complete realization of this ideal there are certain difficulties of the nature of mathematical complication the avoidance of which would be welcome. The question, then, arises naturally as to whether we might not take certain subgroups of the general group of transformations and apply these to the separate fields for the purpose of obtaining approximate laws—laws which are covariant under the subgroup applicable to a given field or under several subgroups of a given type. Perhaps one may not have as strong expectancy of the validity of such a law as of one which is wholly

covariant; nevertheless one may naturally expect in this way to make closer approximations than he would make without this aid.

This is a new sort of approximation in theoretical physics of a much more profound character than numerical approximation. It is an approximation in the sense demanded by covariance under a subgroup instead of under the whole group. As the subgroup is enlarged, and the statement of the law undergoes consequent modification, the approximation will presumably become closer and may even become as close as is needed for agreement with experiment even though the entire group of transformations is not employed.

The group of the Lorentz-Einstein transformations of the special theory of relativity is precisely such a subgroup as we have just described; and it is well known how it has led to more satisfactory theories of certain phenomena than those which had preceded them. Under this group the Maxwell-Hertz electrodynamic equations and the wave equation deduced from them are invariant. Now, if all the phenomena in a certain field are invariant under this transformation group then every mathematical formulation of law in this field should be in the form of an equation which is covariant under this group. This would afford a precious guide as to the necessary form of such an equation. If an equation is obtained in an empirical way, then one considers it only as a rough approximation to the true equation; and one seeks a better form of it which shall have the two properties of being invariant under the named group and of being represented to a suitable approximation by the empirical equation first obtained.

It seems not unlikely that each large and well-defined class of phenomena may have associated with it a certain group of transformations—a subgroup of the general group of transformations of axes in the four-dimensional space-time continuum—of such sort that the phenomena in question (or at least the most of them) may be represented to an approximation quite within the range of experimental error by requiring merely that modification of approximate empirical laws which is required to bring them into a form that shall be covariant under the given subgroup of the total Einstein group.

This procedure might well have the advantage that it may be carried out much more readily than the corresponding one based on the original group, at least if the subgroup is of simple character, say analogous to the group of all multiplicative linear homogeneous transformations.

From certain mathematical considerations it seems probable that in many important cases one would actually obtain in this way precise laws the covariance of whose statement under the general Einstein group could be readily established. The number of invariants relative to a given subgroup and agreeing with a given rough empirical law to a suitable approximation may often be small or be even unity. In the latter case it must be the law which is covariant under the general Einstein group, if there is such a law. In the former case, one would probably have little difficulty in choosing the appropriate one for general covariance (if such a one exists).

Thus it seems likely that a systematic study of the covariance of laws under subgroups of the general Einstein group will lead to useful means of discovering the

laws of phenomena. Thus for each of these subgroups we appear to have the possibility of a logic of discovery of the same general sort as that employed by Einstein in his general theory and differing from the latter only by virtue of its being relative to a subgroup of the Einstein group instead of to the whole group. Thus it seems probable that we may have in physical science a considerable variety of logics of discovery based on transformation groups as the ultimate ground under the ideal as to the form of statement of the laws. Of course, every law obtained in this way (as well as in any other) must be subjected to experimental test before it can be accepted. The method would profess only to help in discovery. Some of the results to which it would lead would turn out to be valid (if the general theory of relativity is valid) and others would need further modification under the guidance of a more comprehensive group.

Let us give a brief summary of the foregoing discussion:

It is conceivable that in the nature of things the mind is unable to comprehend its highest movements with clearness and also that certain of its acts of discovery are essentially creative in character; but neither of these possibilities need affect the development of a logic of discovery in the sense of a logic by which one infers from the known to that unknown which hitherto has not been apprehended or suspected. Such a logic may lead forward by well-defined steps to clearly ascertainable propositions without carrying with it a demonstration of those propositions. The already developed systematic logic is a logic of demonstration. The logic of discovery, in the higher sense of

the term, has no existence as an actually developed science. This was insisted upon in effect by Descartes and Francis Bacon and remains as true in our day as in theirs. Bacon's supreme effort to found a logic of discovery ended in failure. No single discovery can be pointed to which can be definitely ascribed to the use of his rules.

There are two causes which may have contributed to this failure. It is conceivable that the primary acts of discovery are so essentially creative in their character that no science of inference from the known to the unknown can be developed; or, if it can be developed in part, that it cannot be adequate except in a very restricted range. This does not seem to have been the main difficulty. The conception of the logic of discovery as a unit, comparable in unitary character to the logic of demonstration and scarcely separable into parts, has, I believe, been a chief hindrance to its development. We can have not so much a logic of discovery as logics of discovery each relative to some field or subject matter of investigation. Discovery is relative to the point of view. We can no longer proceed as Descartes did when he "thought out what the constitution of the world and man must be if they were to be clearly understood."

The essential step of discovery is in the formation of a conjectured proposition to be tested or of an hypothesis out of which the proposition comes or of an ideal which guides in the formation of both hypothesis and proposition. It is possible that truth may be discovered by means of a logic which carries with it the demonstration of the truth; but the more characteristic logics of discovery are merely suggestive in character. The former is often illus-

strated in mathematics, particularly in the theory of numbers and the theory of finite groups. In physical science it is the latter which is generally in evidence. This latter also has a wide usefulness in mathematics. Every result of a heuristic logic must be subjected to some suitable adequate test after it is obtained. In mathematics it is often demonstrated by a *de novo* argument. Such a heuristic logic serves the three-fold purpose of making the problems definite, of suggesting the central theorems of an investigation, and of indicating suitable methods of proof.

The tentative nature of logics of discovery allows room for an error of a dangerous sort. When the measure of sagacity required for the use of such a logic is not great one may so uniformly succeed with it for a time as to lose his sense of the need of an independent test of the result. By hidden gradations of error one may then pass step by step to the condition of being satisfied with the unsound "proof by ignorance" so that he is able to conclude an argument with the absurd climax: "The apparent indetermination arises solely from the insufficiency of our knowledge."

The finest thing associated with a logic of discovery is that one may arrive at truth under the impulse to realize directly an ideal of far-reaching importance as to the form of the truth to be attained. The most recent striking instance of this is to be found in the general theory of relativity. In accordance with Einstein's demand it is desirable to enforce upon nature, if possible, a certain ideal as to the mathematical form of the statement of the laws of nature. This ideal guides investigation and leads to new laws. It affords a heuristic logic associated with a general

group of transformations in space of four dimensions. It seems probable that certain related but more special logics of discovery of distinct usefulness may be associated similarly with certain subgroups of this general group and that we may thus have in physical science a considerable variety of logics of discovery each based on a transformation group as the ultimate substance of a related ideal as to the form of statement of the laws.

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## CRITICISMS AND DISCUSSION

# SPENGLER'S THEORY OF THE HISTORICAL PROCESS

OF MAKING many books there is no end, but of selling and even of reading them the end is often speedy, if indeed there was any beginning. Only now and then there appears a fateful volume, that seems to fall from the sky, from "the chill bosom of the desert air," which an age, a generation, a people at once recognizes as its own, as the large utterance of its inmost soul, and proceeds to appropriate, to assimilate, to embody in its own life and aspirations and destiny. Such a work, "sky-descended" like the Artemis-image at Ephesus, has now for over a year possessed the consciousness of Central Europe and filled it with amazement and awe. Whether it will finally establish its possession and mould the mind of the people into its own likeness, it is of course too early to say; but not too early to take the measure of the work itself, to set forth its central contentions, and to appraise their scientific and critical value.

The book in question is Oswald Spengler's, *Der Untergang des Abendlandes*, and in the beginning it must be admitted that only the first volume has reached the present writer. The tabulated contents of the second volume, however, do not promise any notable expansion of the wide horizon of thought already disclosed in the first, unless perhaps in the closing chapter, on "Russia and the Future," to which one must look forward with the keenest interest.

The title of the work, *Downfall of the Western World*, is certainly inadequate, nor does it so much as hint the essence either of the matter or the method of these pages; it merely indicates the alleged trend and issue of the historic process that Spengler claims to have discovered and identified and has certainly illuminated with

extraordinary splendor of scientific and philosophic, mathematical and historic, political and socio-economic learning, while at the same time interpreting with almost demonic skill in combination and depth of insight. Such, indeed, is the loftiness and aloofness of the author's spirit, that it seems strange beyond measure that his work should have made any popular appeal whatever, and it is a perpetual wonder, who buys the book? and still more, who reads it? For Spengler does not stoop to his readers; from beginning to end he seems to hold communion with himself on the Andean summits of the most recent mathematical, philosophical, archeological thought; like Nietzsche he might boast, and with far better reason, to "have sought the heights where blows the keenest air, and few there be find breathing easy there." Continental, in truth, is the range of his vision, reaching from Minkowski to G. B. Shaw, from the Entropy of Clausius to the counterpoint of Bach and Beethoven. The work may indeed have tendencies, but it seems singularly free from sympathies or human feelings. Apparently it might have been written by a Russian, or a Frenchman or Turk or Prussian, or even an Italian, nay even by an Englishman; if the author inclines at all from the vertical of impartiality, it would appear to be toward Britain or Egypt and away from Athens and Rome. More than all, however, he seems to resemble some piercing intelligence, from Mars perhaps, who has visited many planets and reports upon them all without hate and without love, unmoved as the "breast of some stone Dian at thirteen." If he has any passion at all, it is apparently for the Infinite Space of the Nordic Culture, for the intricate harmonies of Bachian music, for the stony simplicity of Egyptian art, for the character-drama of Shakespeare, above all for the poetic-philosophic mind of Goethe and its profound morphologic interpretation of Nature. If he has any pet aversion, it is seemingly Darwin and Haeckel and the jealous finity of the classical soul. Often he complains of the hopeless hardness, petrifaction, and death of the "world-city," of the cosmopolitan spirit, and one is tempted to wonder if the modern malady has not infected himself.

What then is the plan and aim of his volume? It is an interpretation, one might almost say a philosophy, of history. At this term the forms of Hegel and Buckle and Guizot and our own Draper and a host of others start from their slumber in the unconscious and float forward into light, but Spengler is not in line with any of these. Not one of them, significant men though they were, approached his

task with the rigorous technical equipment and mastery of Spengler. In particular, they rather depreciated or neglected mathematics, to which Spengler is by vocation devoted and if not the lord of all its subtleties—as who can be?—he is at least at home in its highest regions and possessed of its daring spirit. Neither did any of his elders follow either the idea or the method of Spengler, both shadowed forth in the subtitle—"Outlines of a Morphology of World-history." He might indeed have inserted an adjective and called it Comparative Morphology; for he seems to contend that there is really no unital history, no career of Culture, but only many histories of many individual cultures. And here indeed by over-accent he is tempted to fall into unfortunate Pluralism and to lose hold of the Oneness of the world, which his adored master Goethe not only recognized and expressed, but would seem also to have felt even keenly. A Culture may indeed undergo fission like a protozoön, and some developments of the individual may outrun others in time, and so present the aspect of several cultures, but surely this does not overcome the fundamental unity of culture as a whole. That there is some One called Man, at least the Antique, the Egyptian, the Nordic Man, Spengler himself attests in speaking of their cultures as units, as expressions of different Souls, as living definite lives and moving forward to definite ends. But these expressions were each through countless millions of men; if this multiplicity did not attaint the unity of the cultures, why should still further multiplicity, or any cleavage of men into Races, attaint it? Indeed, the comparative morphology of cultures is implied in the whole process of our author's thought. In every chapter, in almost every section, he is comparing some "stadium" of one culture with some "stadium" of another, identifying or contrasting the two, and this has no meaning unless there be some deeper unity of the cultures themselves; just as it signifies nothing to discover homologous organs in fish and birds, unless this points back to a primitive unity of pattern which the fish has realized along one line of growth and the bird along another. Of course, our author has not forgotten this, he indeed dwells upon the distinction between homologous and analogous, but he has not duly weighed all the implications, and has stressed unduly the severalty of cultures—a want of proportion that avenges itself by vitiating measurably his final conclusions.

To return, what then is the author's main conception of Culture? The answer is that Culture for him is the realization of Soul-

possibility. In fact, Soul is for him "that which may or can be." In this realization he distinguishes anxiously if not always clearly between the process and the result, the Becoming and the Become. It is the former that constitutes Culture proper; the latter is civilization rather, which is really the consummation and death of Culture, the arterio-sclerosis of history. Much of this seems to hearken back to Goethe. The author conceives of culture as a growth, an organism, a plant that springs up from the soil,[1] and lifts and spreads its leaves and fronds and branches, and buds and blooms and waves all its splendor in the wind, and then fades and withers and falls back to earth. But the analogy does not hold throughout; for the plant scatters not only its leaves but its fruit, its seeds, upon the earth and renews itself in the next generation:

> "Leaves now sheddeth the wind on the earth, now others the forest
> Buddeth anew in its bloom, when the spring-tide season appeareth."

But there is no recurrent spring for Spengler's Culture; once petrified or moribund in Civilization, its career is accomplished. However, men are actually like leaves:

> "So generations of men: one passeth, and cometh another."

Possibly it were more just to Spengler to say that he regards a Culture as the budding, fruiting of a single branch on the great tree of Humanity, and Civilization as the fading and fall, the tree remaining to weave anew its songs of spring—but only on another branch. Again the image is imperfect. Any satisfactory theory of history should certainly take into consideration that men continue to inhabit this planet long after their culture has become rigidified and (according to Spengler) dead in decadent civilization. But for him they have no interest, he passes them by without notice; yet, interesting or no, they actually are, and they must in some way be fitted into the general scheme of history and historical theory. It is a serious delinquency of Spengler's that he makes no place for these multitudes. The continuity of history suffers violence at his hands.

But we must come closer to Spengler's notion of the Culture-Growth. Strive as we will, we cannot escape philosophy or even

[1] A Culture effloresces on the soil (Boden) of an exactly definable region (Landschaft) on which it remains bound, like a plant." "The classic soul (Scelentum) was born about 1000 B. C. of the region of the Aegean Sea." . . . "The Arabic Culture springs wholly from the bosom of the region between the Nile and the Euphrates, Cairo and Bagdad." . . . "The trend to the Infinite (and so to the Faustian) slumbered deep in the Northern region, long before the first Christian trod it." But Spengler makes little or no attempt to relate the characteristics of the Culture to the peculiarities of the parental "Landschaft."

metaphysic. Hartmann is right in avowing (*Kategorienlehre*, xiii.), in defiance of prevalent prejudice, that for him at least the centre of interest still remains in metaphysic; and our author commends his work by his frequent implication of a thoroughgoing Idealism (some might say Relativism rather). From no other viewpoint is interpretation of human history possible; from none other can a discussion of Values, of the great achievements of Man, of Art and Science and Literature, be even attempted. For Spengler, then, the active element in history is the human Soul or Mind or Spirit (there is no strife about words), which grows and struggles to express or objectify itself continually in all manner of forms, in Space and Time, in Percepts and Concepts, in Numbers and Diagrams, in Algebra and Geometry, in Analysis and Logic, in Physics and Metaphysics, in Architecture and Sculpture, in Painting and Music, in Literature, in Commerce, in Religion. One and all these are regarded as creations, as outputs of Culture-Soul.

If now we ask more closely what is the typical career of a Culture, the answer is that Spengler has given no formal and satisfactory statement, but on comparing a number of detached sayings he appears to conceive of a Culture as implicit in the racial soul inhabiting a certain definite region and bodying forth at birth a formless half-conscious mysticism, a cloud-land of dream experience, for which our sophisticated tongues have few or no symbols; as the Soul lives and grows it passes into the child-stage of myth-making, projecting its colossal creations upon the screen of folk-lore and poesy; advancing through youth into maturity it evolves its forms of philosophy and monotheistic religion and unrolls the rich tapestry of its art: its architecture, its sculpture, its monuments, its painting, and its music; in the days of its full strength it perfects and even begins to conventionalize all these, it develops a comprehensive and aggressive science, it systematizes and rationalizes both philosophy and religion; later it veers toward the arid regions of Materialism, it begins to lose the elasticity, the exultant bound, the joyous note of youth and early manhood, it adopts the steady stiffening step and the sobering hues of Age; now at length it has done its work, it has wrought out its Culture, it settles down into the rigidity and formalism of accomplished Civilization, it closes the cycle of its strange eventful history. Meantime it has elaborated many abstract Ideas, such as Time, Space, Number and others less mathematical, and it is on such that Spengler has delighted to expa-

tiate in contrasting the various Cultures. Thus he finds that the Egyptian Soul has fairly reveled in developing depth, the third dimension of Space. Its symbol is the Vista ("der Weg"). It ranges its figures in endless processions—on, on forever march its corridors of kings and gods and men and sphinxes. One is led to ask whether the Nile has not done its part in bringing to birth this child of the Egyptian Soul? The Greek or classical Soul realized its space-striving in the bounded Body, the definite form, whether of statue or temple, of drama or of state, of poetic measure or of Euclidean geometry,—and perhaps no other realization in history has been so nearly perfect. We naturally inquire, has the dominance of the Boundary in classic culture any connection with the narrow circumscription of the Isles of Greece?

The Nordic or modern Soul has burst the classic bars; it is possessed by restless yearning for the Infinite; its Space, like that of the opium-eater, swells to unimaginable dimensions, it projects titanic systems of mathematics swinging like a pendulum between the infinitely great and the infinitely small, it opens up ever widening perspectives in painting, it sounds unfathomable depths in astronomy, it dissolves the universe into limitless oceans of harmony in the polyphories of Beethoven and especially of Bach.

On all these and many other related themes, Spengler is intensely interesting and often illuminating, though not always convincing. In particular, he tells us nothing about the pre-natal pre-mystical phase of the Culture-Soul,—and yet such there must have been, if not a Soul performed, at least the preformative elements of a Soul, gathering on the "Landscape," as that wisp of cloud now gathers on the blue of the sky. Here, indeed, we stand at the parting of the ways, and it seems regrettable that Spengler has not more formally ranged himself in the ranks of positivistic Idealism, where he certainly belongs, as many dicta scattered through his volume attest (e. g., pp. 222ff., as "to be sure, man is an atom in the universe, but the universe at the same time is the product of his reason." . . . "This soul, and indeed the soul of each individual that experiences in itself the whole world of historic event and *therefore creates* it, etc."). But a Materialist, or at least a Realist, might grant many of his contentions and still think of the Culture-Soul not as a creator but only as an explorer amid a wholly material objective and independent world, as discovering a variety of relations among a "variety of things," things and relations that were

in full force before his own arrival on the scene, and are inappreciably affected by his presence, and will endure with perfect composure his early departure. He would say that such a mere observer and his race would also develop a culture of this kind or that according to the nature of the man and especially according to the nature of the milieu, of the object-world in which he finds himself immersed. Such a Realist would relate the peculiarities of the Greek culture in great measure to the geography of Hellas, to its pellucid air, its myriad-smiling seas, its rugged mountains, its mysterious glens, its marble quarries, its sparkling streams. He would try to state Homer and the Iliad, Plato and the Republic, Phidias and the Parthenon in thermo-barohygro-metric terms, even as Taine correlates Shakespeare, Milton and the rest with the snow and foam and tempest of the low-stretched North-Sea shore, and its low-hung clouds swart under heaven, its starless skies, its short fierce summer, and its winter without end. In the hands of a Buckle such an explanation may attain a momentary plausibility, and we are not able to deny that such or indeed any environment may modify more or less, may shape and tinge the outward projection of the inmost Soul. But any profounder influence is unthinkable, and the materialistic interpretation of history leaves it in the main uninterpreted. Burns may have sung of field mice and Highland Mary and chill November and Saturday night rather than of olive groves and tournaments and April skies and cathedral aisles, because he was a peasant of Scotland and not of Italy or Provence, but no amount of environment will ever explain why he sang or felt at all.

But has Spengler anything better to offer? And here it must be confessed that logical rigor is not the *pièce de résistance* in this author's work. His thought is amazingly abundant. Throw open his volume anywhere, and ideas seem to fly forth like birds from a magician's basket; but he is at no great pains to order them aright in firm irresistible phalanx; he lets them loose to our delight and amazement, but he lets them wander as they will if only their general direction seems not away from the lines of his thought. Spengler hardly suggests that climatic or other external influences have moulded in any measure the cultures of which he speaks.[2] Of these there are two, the Nordic or Faustian and the Antique, classic, or Apollinian (a borrowed Nietzschean term) that interest him most as polar opposites; in less degree the Early Arabic or Magic (which includes the Hebrew and early Christian) and the Egyptian com-

[2] See note, p. 7.

mand his attention, while the Indian and the Chinese receive only occasional mention. The Renaissance is elaborately treated but not as a single original impulse, rather as a hybrid resultant of Antique and Magic and Nordic confusion. Such is the group of Cultures whose birth, growth, consummation and final mummification constitute the history of the circum-mediterranean world from Thebes to London, from Poland to Spain. Each of these Cultures is the Striving of a Soul for the most part unconscious, that incorporates itself in countless individuals simultaneously and successively and bodies itself forth in Symbols on Symbols in every art, every science, every institution, every activity of man. What a shallow philosophy is prone to regard as the deepest realities of the outer world, the invariable verities of the universe, are only the elaborate symbols of this age-long spiritual unrest and life-urge shaping the symbols of itself into forms of various beauty and terror and awe. But these Cultures realize themselves independently. It is false and misleading to speak of ancient and middle and modern age. The last is not a continuation of the second nor the second of the first. The torch falls and is quenched; it is not handed on. The antique completed itself and filled its span and ossified in death. The early Arabic (or Magic) irrupted as early Christianity upon the stage but in a measure was hemmed by the antique that lingered superfluous; then in the seventh century its high-mounting wave suddenly overflowed and surged with unparalleled speed even to the walls of Paris, where it dashed into foam; the Nordic or Faustian, Belgian-born, has flowered from the Vistula to the Tagus and now having reached its climacteric in Shakespeare, Napoleon, Bach, Gauss and their kin, it nods to its end in the men of machines, in Cecil Rhodes, Journalism, Socialism, skyscrapers and all the dead or dying Civilization of the World-City of today.

What reason has Spengler for this last diagnosis? If you observe the development of one organism, as a lily, from its sprouting to its fading and its fall, and of another very similar, and of still another, and then if a fourth one be watched carefully through various stages, you would doubtless declare with confidence at a certain point: *The sprouting, the budding, the blossoming have come and gone; the time of seeding and decline and death is near.* Such is the movement of our author's thought. From the examples of Greek, Egyptian, and Arab he discovers the life-process of a Culture; he then turns to the Faustian or Nordic and finds all the signs

that point to a sad senescence. After all, then, it is only history teaching by example. A very impressive and yet it would seem a rather unprofitable method of instruction, if, as Hegel tells us, the great lesson that history teaches is this: that we never learn what history teaches. How, indeed, should we, if the analogy of the plant must really hold good? In that case the scheme is all made out beforehand, it is all enshrined in the primal germ of the Culture-Soul, in what Spengler calls *Schicksal*,[3] and Crile the hereditary "pattern." Circumstances (he would seem to concede) may indeed modify slightly but not significantly. France instead of Spain might have fitted out Columbus and have initiated the grand colonization. But are we quite sure the modification would have been slight? Who knows what might have happened? We cannot appeal to the plant-analogy to prove more than it really proves. Analogies are valuable—stimulating and highly suggestive—but their logical worth is not great; from resemblance between some relations we may suspect but cannot infer a resemblance between others. Besides, the inductive base in Spengler's reasoning is very narrow. The plants and other organisms whose life-careers have been observed are countless; not so the cultures; even a crow can count three. Moreover, the conditions have undergone profound variation. The Greek and Egyptian developed comparatively freely, in almost complete isolation; the Arabic was balked at the start, but finally burst forth with prodigious urgence, only to meet with restraint and repression; the Nordic alone has gone on conquering and to conquer, absorbing energy from without while expending it from within, striking ever wider and deeper roots into the mold of centuries, populating new continents and assimilating old-world forms outworn. If the elder cultures were annual plants, is it not barely possible that the Nordic may prove to be a perennial,

> . . . . . ein starker Baum
> Der ein Sommertausend lebt,
> Nach verträumten Winterstraum
> Neue Lenzgedichte webt?

This would not offend against our author's just and central idea that a Culture is a growth, a realization of Soul-possibility. But it would recognize another idea that he has unduly neglected, the idea of the Communal Soul. He indeed tells us clearly enough, though it will bear exceeding emphasis and repetition, that each individual

[3] Compare the similar pronouncement of Raymond Pearl in *Harper's* for May, 1921, p. 713: "Whatever the ultimate destiny of the universe it will unswervingly be carried out."

spirit makes its own world of Space and Time, that it builds up its own universe about it, which titanic Symbol has no existence independent of the Self that constructs it. Such indeed is the sure result of psychologic analysis and philosophic thinking, if there be any sure result at all. But it is not the whole story. It leaves quite unexplained the Time-and-Space uniformities of the symbolic world, which we call the Laws of Nature, a rock on which all crafts of pure Solipsism seem to wreck. The fact is that the great Symbol is social as well as Individual. The Constructive Souls are fundamentally one both at any given moment of time and through the long stretches of human and even planetary history. Each is a wavelet of the One universal wave. The individual human consciousness is not the final form to which Consciousness may attain. The Communal Consciousness Divine lies far ahead on the path that we are all stumbling along. It is the goal of history, if there be any goal, if we are not whirled on forever in an endless, unmeaning circle. It seems hard to look abroad upon the world of Mathematics and Painting and Music,—upon which Spengler has fixed such a penetrating gaze, discerning more clearly than any before him the all-pervasive urge to the Infinite,—or even upon the humbler worlds of Commerce, Industry, Politics and Society, and not behold how "the thousand-folded vault of Being with might combines itself in one." Is not such indeed the sense of Goethe's impressive lines, which form the motto to Spengler's book?

Wenn im Unendlichen dasselbe
Sich wiederholend ewig fliesst,
Das tausendfältige Gewölbe
Sich kräftig in einander schliesst;
Strömt Lebenslust aus allen Dingen,
Dem kleinsten wie dem grössten Stern,
Und alles Drängen, alles Ringen
Ist ewige Ruh in Gott dem Herrn.

Only in this Communal Consciousness, germinal as yet, lies the eternity even of mathematical truth, the meaning of morality and sympathy and love, as well as the promise and potency of "the parliament of man, the federation of the world."

Undoubtedly the undulation of history, the rise and fall of the wave of life throughout the world, is the most solemn and awful of all spectacles. Well may it fill the beholder with dismay if not with despair. If there is any refuge, any "asylum from age unto age," it must be found in this concept (which is also the logical

necessity) of the Eternal Unity, so wonderfully shadowed forth in Goethe's verses:

> And all the wide world's wild commotion
> Is endless rest in God the Lord.

It cannot be that Spengler disclaims or discredits this notion, without which indeed all history would seem to remain forever unintelligible, but he has certainly not weighted it properly in the *Untergang*, else the general outlook of the work would have been quite another. It may be that we have reached or passed a crest of the great Nordic wave of Culture, but it does not follow that there will never be another great mathematician, or painter, or musician, or poet, or even sculptor. A trough may follow the crest, but another and even a higher crest may follow the trough. There is nothing in Spengler's masterly work to certify that the Nordic Soul has been exhausted.

The "world-city" is not the world. Capitalism, and Imperialism, and Socialism, may all be very unpromising, but they do not embody the sum total of the efforts, tendencies, and aspirations of the modern Soul. There may be, there are many others, many that we do not now recognize and cannot even name, germinal impulses that will gather strength from the years and effloresce at last in forms of truth and beauty as alien from Gauss and Bach and Shakespeare as they are from Archimedes and Phidias and Homer. "The world is deep, and deeper than the day can sound." Not even the thought of Spengler has plumbed its depths. Even if ennui or slumber overtake and overpower our present mathematics and philosophy, who knows when some new interest shall suddenly awake and arouse them like strong men to run a race? Though poetry and plastic and music may fall into triviality, who knows where the gods shall again pour out the sacred oil upon the altar, and lo! it shall leap into flame? The variety of Nature still surpasses the imagination of man. Nay, not even the all-dreaded Entropy, not even the "heat-death" of Clausius, need rob us of our trust and peace. If the steady degeneration of energy were doomed to end the world in uniformly distributed heat some day, why has it not done so already? Surely it has had time enough, it has had eterity—*ab ante*. Can it accomplish in eternity from now forward what it has failed to accomplish in eternity from now backward? Such chilling vaticinations as Spengler's,—perhaps not quite so chilling, but at least proclaiming *non plus ultra*, with awful solemnity,—have sounded forth at every sharp turn in the ascending path of human-

ity; but all the alleged demonstrations of the impossibility of further progress have been disproved by one and the same argument—by progressing further.

It is interesting and important to compare our author's notions with those of Flinders Petrie as set forth in his Revolutions of Civilization," published in 1912, the year in which the "Untergang" was begun. Petrie is, of course, far less ambitious; he is chiefly concerned with constating and arranging the facts in the case; of the great body of Spengler—interpretations Petrie has never dreamed. Yet he agrees in the main idea of a natural life of a "Civilization," and in the further contention that we are approaching the last stages of such a life. Petrie's profound historic-archeologic investigations have revealed to him eight successive waves of civilization (culture) that have swept over the circum-mediterranean world. Of these the first two were prehistoric, and perhaps the less said of them, in our present ignorance, the better. The next two rose and subsided in the great Nile valley, but the second (the pyramid-building culture) overflowed into Crete, giving us the early Cretan civilization of the fifth millenium B. C. The fifth wave swelled up high in Egyptian and mid-Cretan culture, then sank in sudden ruin, all in the fourth prechristian millennium. In the sixth Great Year the Egyptian wave again lifted its crest, in the third and second millennia, while the Late Cretan shot up to towering heights, not surpassed if indeed since matched, at least in sculpture, and overflowed to Mycene on the continent of Europe until its dazzling splendor was totally eclipsed in the Dorian Invasion. The seventh wave rose feebly if at all in Egypt, but towered in broad, unexampled and many-crested glory over Greece and in less degree over Italy and other circum-mediterranean lands, as the well-known classic civilization culminating in Athens, 450 B. C. Thence, it sank by slow degrees for six hundred and fifty years, thence more swiftly to its deepest trough, A. D. 800. The eighth (or modern) wave rose slowly from the dark profound and broke into a number of successive crests, the first of which (in the Bamberg sculptures and the Salisbury Cathedral) it reached about 1250; the others have followed at unequal pace. Petrie finds that these successive crests of the same wave observe a soldierly order and multiply as the ages revolve. This order he finds to be Sculpture, Painting, Literature, Mechanics, Science, Wealth, and the lag (or hysteresis, as the mechanician would say) may reach nearly a thousand years. Thus of the classic wave the successive partial crests were reached

after lags, from the sculpture-crest, of 100, 200, 450, 600, 650 years, while the corresponding tops of the modern wave were attained about 1240, 1400, 1600, 1790, 1890, 1920—? But surely in no proper sense can Mechanics be said to have culminated in 1790 or even yet—as witness the airplane and wireless telegraphy and what not; neither will Planck or Einstein or Michelson admit that Science has ceased to mount since 1890. And as to Wealth, in spite of the vast destruction of recent years, it seems likely that in another decade the losses may all be made good and the average of human comforts be steadily increasing. While then there may be much that is just and illuminating in the Classification of Petrie and Evans, it is none the less clear that the facts of the modern wave will not fit into the scheme without violence and distortion. Petrie's work has many other very interesting *aperçus*, and it is distinctly cheering that he recognizes "the widening of the outlook in the summer of each period, and the amelioration of the collapse in the winter," whether or no "this is the real nature of human progress."

Spengler has also the notion of the "Great Year," with its Spring, Summer, Autumn and Winter, but not of a succession of such years wrapped in a spiral continuously round the axis of Time, in Petrie's striking but fantastic fashion; his scheme of history is arranged in "parallel series" of four chief cultures: Indian (since 1500 B. C.), Antique (since 1100 B. C.), Arabic (since A. D. 0), Occidental (since 900 A. D.). These four begin respectively with the Veda-Myth, the Olympic Myth.

Protochristianity (elsewhere called the Early-Arabic Myth), and Germanic Catholicism each reaching through three hundred years;—a period of splendid energy, of new-born sense of God, expressing itself in majestic myth and symbol, of *Weltangst* and World-longing, the age of the Aryan Hero—Saga, of Homer, of the Gospels and Apocalyse, of the Edda and the Nibelungen. A bold and impressive correlation, but it is certainly bewildering to find the "Gnostics" in the second half, between the "Neo-Platonists" and "Church fathers," when the "Gnosis" is now well known to have been proto-and even pre-christian; Spengler is considering only its degenerate and excommunicated forms, he has forgotten the Naassenes. The Summer is the glorious season of "Ripening Consciousness"; it begins everywhere with "Reformation" in Religion, with popular insurrection against the great forms of the earlier time; it passes over into the philosophic form of the World-feeling, into the Upanishads, the Pre-Socratics, the nameless heroes of

Syrian, Coptic, Neo-Persian thought (of sixth and seventh centuries), whose greatness only the Twentieth Century has begun to teach us, into Galilei, Descartes, Bruno, Bacon, Boehme, Leibniz; it is continued in the "New Mathematics" (*spurlos versenkt* in India!), in the brilliant Geometry of the Greeks, realizing the notion of limited magnitude and number as its measure in the Arabic conception of Algebra and unlimited (unknown) number, in the Occidental conception of number as Function, issuing in the Infinitesimal Analysis; it closes in "Puritanism," a rationalistic—Mystic impoverishment of Religion, an intellectual fanaticism traceable in the Upanishads, in the Pythagorean League, in Muhammad, in the Puritans and Jansenists. The faint suggestion is in the air, of Approaching Autumn, the season of "Metropolitan Intelligence," attaining the apex of "purely spiritual formative power," opening in "Illumination," with Faith in the Omnipotence of Reason, with the worship of Nature, with "National Religion,"—the era of the Sutras (Sankhya), of the Sophist, and Socrates and Demokritos, of Nazzam, Alkindi, Alkabi, of Locke and Rousseau and Voltaire (and why not Diderot and D'Alembert?), marking then the culmination of mathematical thought in the Indian conception of zero and place-value and angular functions, in Plato and his mates, in unexplored Arabic researches in number-theory and spherical trigonometry, in Euler, Lagrange, Laplace; and closing in the great definitive philosophic systems of India, of Plato and Aristotle, of Alkarabi, Allaf, Avicenna, of Goethe, Kant and their continuators. Herewith Culture passes over into Civilization, centering its life in overgrown "world-cities," quenching the formative power of the Soul, turning life into a problem, exalting the practical-ethical tendencies of an unreligious and unmetaphysical cosmopolitanism. It is veritably a polar winter that settles down upon Spengler's world, and we shall not pursue it through its dreary stages of "materialistic world-view" and "philosophy without mathematics" (!) and "inner completion of the mathematical world of forms" (in Gauss, Cauchy, Riemann), and declining philosophy reclining in "chairs" logical and psychological, and of "ethical Socialism" spreading itself from 1900 on like ice and snow descending from the pole. Such, we are told, is "the End: Expansion of the final cosmic mood"—Buddhism in India (since 500 B. C.), Stoicism in the classic world (since 200 B. C.), Fatalism in Islam (since 1000 A. D.), Socialism in the Occident (since 1900). The spiritual moods that agree in their distinctive features are classified as "contemporary," though thousands of

years apart in time, as the youth and prime and age of Bach might correspond to those of Phidias.

In equally ingenious and impressive fashion has Spengler arranged his second table, of "contemporary" epochs in art, but the Twentieth Century brings the Occidental column only to the "end of music" (Wagner), the "Episode of Impressionism" (Constable, Corot to Manet and Leibl) and the Pre-Raphaelites. Spengler spares us any but a general forecast of the two ages of decrepitude to come.

A Third Table ranges "contemporary" political epochs also side by side in parallel vertical rows. Here we find ourselves again in the first Stadium of "Civilization," the dissolution of nations into the great Fourth Estate, the People, into anorganic cosmopolitical international masses interested in bread-and-butter, under Parliamentarism, from 1800 to 1900, under Socialism and Imperialism from 1900 to 2000, the Stadium of Money, during which economic complexes absorb the form of the State. We are now "isochronous" with Scipio and Marius in Rome (200 to 100 B. C.); what awaits us from 2000 to 2200 will be something akin to the Golden Age of Rome (200 B. C. to 100 A. D., Sulla, Caesar, Tiberius), and in the third Stadium (2200—) something like the Silver Age from Trajan to Aurelius (100 to 300 A. D.), a deepening twilight, brightened by the Evening Star of Marcus Aurelius. For us then, in the present and approaching stages there is little to hope.

It can hardly be denied that these Tables of Isochronism present an imposing aspect and furnish much food for thought. In many cases it is not easy to deny the parallelism claimed, and the interpretations of Religion, Art, Science, Philosophy, in less degree of Politics, are often profound and plausible to a degree. In discussing Number, Space and Time, the significance of the third dimension, in refuting the favorite dogma sanctioned by Kant, and even by Sir William Rowan Hamilton, that number-theory is rooted in the intuition of time, that Algebra is the science of pure time, Spengler appears at his best and his book is an excellent tonic. Hardly less arousing his contrast so often enforced between the ancient and the modern mathematics in relation to the notions of the Infinite and the Irrational. The grave objection seems to be that Spengler hold his parallel but asynchronous cultures in unnatural isolation so that each shall develop independently unaffected by any other, though it seems out of question that the cross-currents of influence have been numerous and important and especially the

classic culture has propagated itself in the occidental and even in the Arabic along countless and interminable lines. It seems strange that such a broad-browed intelligence as Spengler should allow himself, in the interest of a theory, to do the Greek spirit such a sad injustice.

But the most serious fault in the schemes of both Petrie and Spengler is the overweighting of the artistic and intellectual and the underweighting or almost total omission of the moral elements of Culture or Civilization. "Forms of government are left to the last, as the regulation of daily affairs, and the repression of wrong, is of little meaning in civilization, when compared with the great formative interests of man's mind whose phases we have studied." We may agree with Petrie as to the rest, but not as to "the repression of wrong," if this be extended to denote the gradual evolution of the idea of Justice and its realization in the organization of Society and the conduct of Life. We may even contend that this is a matter of supreme "import" as well as "concern." As almost the very last in its apearance in man's history, it seems almost like the sixth day's work of Creation in comparison with its forerunners, whether these be Art or Science or Wealth. That Justice should prevail throughout the land, that Right should reign over all men and over all the world, seems quite as important as that temples and statues should be beautiful, epics majestic, oratorios entrancing, eclipse calculations accurate, and mathematical-philosophic theories profound. Moreover, the metaphysical freightage of the idea of the Just is not inferior to any other; for it implies a single most highly organized consciousness of Each in its identity with All, a Communal Consciousness Divine. If now we try our present day civilization by this standard, we shall find it indeed very far from approvable but very far from hopeless or decadent. In spite of the mounting wave of crime, in spite of numberless wrongs unredressed and injuries unavenged, in spite of inequity everywhere rampant and misery widespread and appalling, in spite of an horizon temporarily lowering all around, it is nevertheless true that the Dignity and Rights of Man are now affirmed more widely and effectively than ever before. From sea to sea, from pole to pole, the urgent and persistent demand for the rectification of age-long inequity is heard, and it awakens echoes in millions on millions of hearts. The "lamentation and the ancient tale of wrong" "steams up" no longer unavailing. We are beholding in fact the travail of humanity in bringing to light the prodigious birth of Socio-economic Justice. To be

sure, the old Dragon waits to devour it—but we do not fear, it will be saved in the Wilderness. Not for an instant would we undervalue or disparage the great formative powers and interests of mind that Petrie and Spengler have glorified in their tabulations. They are much, they are very much, but they are not all. The Himalayan peaks are not the whole mountain range, even the table land and the lowest valleys count in the total, and they must be regarded if we are to understand the whole formation aright. It is easy to deride Democracy and to present a strong case against it; but what better have you to substitute therefor? It is only Man that can save Man, and his salvation is a process of Growth. This growth is slow and often whimsical and even disappointing, but nevertheless it actually takes place, as the schemes of Petrie and especially Spengler abundantly show. Our present civilization has yet two or perhaps four hundred years in which to die. Perhaps in that time America may make herself heard in the choir of cultures. Neither Petrie nor Spengler has yet caught the tone of her voice, for them she has no spiritual significance as yet. Be it so. But Petrie thinks that every culture-wave swells up from a blend of bloods, a mixture eight centuries old, and then rejoices in its energy for five hundred years. Here then we have this alleged primal condition of the culture-producing urge, but clearly we have yet long to wait before the blend is quite complete and yields its maximum of power. However, the process of amalgamation may not wait on precedent, but may quicken its pace in an age of speed, and long before the year 2400 the crest of a culture whose slogan is Justice and whose flying goal is a Communal Consciousness Divine may lift itself on high over all America of the North.

There are many collateral matters in the *Untergang* that deserve and even call for mention, but one seems to be of special importance. It is the attitude of the author with respect to the historical fact of Christianity and its general cultural significance. Of course, we do not look in a History of Civilization for any discussion of critical questions, but we might expect some indication of the place assigned to such a dominant historical phenomenon in the general list of culture-factors, or at least culture-products. Petrie disappoints any such a natural expectation; he has nothing to say on the subject in his small but exceedingly compact and pithy volume. H. G. Wells in his ambitious *Outline* gives a chapter on the "Beginnings of Christianity." His treatment of the Origins is feeble, flighty, sketchy,

without critical warrant, a hotch-pot of errors. For him the Proto-christian movement was a social revolt, its "seed rather than founder" was an unparalleled preacher of righteousness, whose followers believed he had been raised from the dead after judicial crucifixion! To them the scholarly Paul supplied a theology, and they proceeded to convert the world to their semi-communistic doctrine of universal Brotherhood! Of the many impossibilities that confront such an easy-going Naturalism Wells seems to have no inkling. His only answer to objections would seem to be simply to ignore them. Nevertheless, in discussing the Deutero-christianity of the following centuries, he does emphasize with truth and justice the cultural significance of the church in supplying the connective tissue of society, a spiritual center and a moral authority, a guiding thought—the unity of Man—a code of conduct and a theory, however imperfect, of history and the government and destiny of the universe—all matters (as Petrie might say) "of great concern, but little import." One may ask of "little import" to what or whom? And the answer would show that Art and Science and Literature are themselves only Symbols of Man's activity, of his soul-struggle towards realizing his possibilities, and that the age-long will to Justice and the perfect socio-economic organization of humanity is not second in importance to the will to Beauty and even to Truth. In recognizing this cultural service of the medieval church, Mr. Wells has done altogether well.

On turning to Spengler we find that his attitude is highly enlightened on this as on almost all other questions. His classification is indeed different from any other we have examined, but it seems to disclose a far clearer apprehension and profounder penetration of the historical-cultural situation itself. Spengler coördinates the Christian or monotheistic movement (under the name "Arabic") with and between the classic and occidental Cultures, and traces it through all its "contemporary" epochs side by side with the other two. It may be startling to many to find "Urchristentum" in the same column with "Muhammad" and the like, still more to find the Edda, Dante, Thomas Aquinas, Galilei, Luther, Rousseau, Voltaire, Marx, Schopenhauer, Nietzsche, and many other such—all in one happy family, dwelling together like brethren in unity in the great occidental column. But there is the severest logical method in his madness. Spengler has done well to recognize fully the Arabic Culture and its world-significance (as Petrie also in less measure).

He might have done still better to call it Semitic and to assign the Jew his fitting place of honor amid the Makers of the Present as well as the Past. But one sadly suspects even Spengler, magnanimous as he is, of being infected with the anti-Semitism that has scattered its germs from Moscow to Dearborn and beyond. He awards ample credit to the Arab, but the Hebrew and the Jew he rarely mentions and never in terms of just appreciation. Protochristianity (*Urchristentum*) he dates from the year 0 B. C., not unnaturally but yet erroneously, for the movement was in reality prechristian. Simon Magus, the patristic father of heresy, had been preaching the "Great Power of God" in Samaria a "long time" before the death of Stephen in the early dawn of the Christian day, and Hippolytus shows that even he was several steps down in the list of Gnostic heresies. Spengler treats the matter with considerable reserve, in utterances where more is meant than meets the ear. Apparently he regards this "Urchristentum" as the symbol of the early mythopoetic soul, as the "birth of a Myth of the Grand Style, as an expression of a new Sense-of-God" (Geburt eines Mythus grossen Stils als Ausdruck eines neuen Gottgefühls),—at least, as such he classifies it side by side with the *Mythologie des Veda* and *Olympischer Mythus*.

The reader may be interested in this paragraph (p. 576): "In the world-historical word, 'Render unto Ceasar what are Caesar's and unto God what is God's,' which is laid on the lips of the Christ of the Gospels, the classic and the Arabic God-consciousness appear in the sharpest antagonism and necessarily in mutual misunderstanding. Any reconciliation of the strictly Euclidean almost posthumous "Divus-cult" (of the deified Caesar) with the primitive (*ganz jungem*) magic-monotheistic Christianity was made impossible by the culture-stadia that both pre-supposed, the first an end, the second a beginning." It seems doubtful whether the term "magic" be justified, but there can be no question about "monotheistic." Only as such a monotheistic crusade is "Urchristentum" intelligible —not at all in Wellsian fashion as a socialistic insurrection—and only as such has it the unsurpassable significance that Spengler's classification with justice assigns it.

Profound and exact in scholarship as our author is, he is not quite inerrant. On page 48 the "Jew-king Herod" should perhaps be Herod Atticus, who built the Odeon at Athens and otherwise beautified the city. In spite of an inflated Josephine question, and

a vague uncertain inscription (No. 550), we do not know of anything King Herod did for Athens. Again, the Law of Least Action was first proposed by Maupertuis, exactly formulated by LaGrange. On page 588 the honor is assigned to D'Alembert, who does not need it, to whom it does not belong. But it is an ungrateful and ungracious task to pick out spots on the sun.

In conclusion, this volume of Spengler's takes a long step forward in the interpretation of human history. Not all his individual judgments will approve themselves, and his final result may have gone far astray. But his whole work is grandly conceived and his philosophic postulates and method, in spite of the most prevalent and passionate contradiction, must win their way to wider and wider and more unreserved acceptance. Spengler has not attained the goal,—by no means! but he has blazed a path that will surely be followed by such as cannot accept a fortuitous concourse of atoms as the ultimate content and meaning of the history of the World.

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# A SYNTHETIC LANGUAGE FOR INTERNATIONAL USE

(In English and Ido)

A WELL-KNOWN journalist has said: "The world has always laughed at the things that have come true. In every age there are those who think and those who laugh. When that age has passed, there comes another, and always it is true—there has never been an exception to it—that when the world looks back it builds statues to those who thought and laughs at those who laughed."

Bone konocata jurnalisto dicis: "La mondo sempre ridis pri la kozi qui divenis vera. Dum omna epoko esas ti qui pensas e ti qui ridas. Kande ta epoko pasis, altra venas, ed esas sempre vera—nultempe esis ecepto—ke kande la mondo regardas retrospektive ol konstruktas statui a ti qui pensis e ridas pri ti qui ridis."

One can easily foresee the renaissance of that glorious time when the learned were able to intercommunicate by means of a language common to all.

On povas facile pre-vidar la ri-nasko di ta glorioza tempo kande l'eruditi povis interkomunikar per linguo komuna ad omni.

The League of Nations and many scientific societies and other organizations are considering the problem seriously and officially.

La Ligo dil Nacioni e multa ciencala societi ed altra organizuri konsideras la problemo serioze ed oficale.

Professor Gilbert Murray, vice-chairman of the executive committee of the League of Nations, said that the necessity of an international language was forcibly borne upon him when attending the meetings of the League-Council at Geneva. He added that Ido was easier than Esperanto for most Europeans.

Profesoro Gilbert Murray, vice-prezidanto dil exekutiva komitato di la Ligo dil Nacioni, dicis, ke la neceseso di ula internaciona linguo forte impresis il, kande il asistis la kunveni di la konsilantaro di la Ligo en Geneve. Il adjuntis, ke Ido esas plu facila kam Esperanto por la maxim multa Europani.

The common language of the future will not be, cannot be, the Latin of antiquity, nor the Latin of Newton and Halley or the other philosophers and mathematicians of their epoch.

La komuna linguo di la futuro ne esos, ne povas esar, la Latina antiqua, nek la Latina di Newton e Halley e l'altra filozofi e matematikisti di lia epoko.

Furthermore, the common language of the future will not be, cannot be, restricted to the learned or even to "intellectuals" in any sense whatever.

Pluse, la komuna linguo di la futuro ne esos, ne povas esar, restriktata a l'eruditi o mem a l'inteligenta personi, en irga senco.

The world moves, customs change, civilization becomes modified, and the ordinary man seeks self-expression, in person. He trusts no longer, or not exclusively, to his guides. He insists that he have the right and the chance to speak for himself.

La mondo movas, kustumi chanjas, civilizado divenas modifikita, e l'ordinara homo serchas su-expreseso, persone. Lu ne fidas pluse o ne exkluzive, a sua guideri. Lu insistas, ke lu havez la yuro e la chanco por porolar por su ipsa.

Therefore, it is evident that the common language of the future ought to be able to respond to the requirements of the educated and the uneducated world.

Do esas evidenta, ke la komuna linguo di la futuro devas povar respondor a la postuli di l'erudita e la ne-erudita mondo.

But the Latinists have nothing to fear. They certainly would not wish to see the mutilation of Latin as such.

Ma la Latinisti ne bezonas timar. Li certe ne volus vidar la mutilado di la Latina kom tala.

Consequently, the true solution of the whole problem is in some form of Neo-Latin.

Konseque, la vera solvo di la tota problemo esas en ula formo Neo-Latina.

The world requires a common language which is founded upon the scientific principle of maximum internationality as governed by regularity and facility.

La mondo postulas komuna linguo qua esas fondita sur la ciencala principo di maxima internacioneso segun regulozeso e komodeso.

Of course, such a language should be as nearly "natural" and "readable at first sight," as possible, without the loss of regularity, precision and facility. These qualities are much more important than "naturalness."

Komprenende, tala linguo devas esar maxim "natural" e "lektebla ye l'unesma vido," posible, sen perdar regulozeso, precizeso e komodeso. Ica qualesi esas multe plu importanta kam "naturaleso."

The supporters of the idea of an "invented language" seem to be divided into two groups:

(a) Those who opine, that in order of importance the desiderata are: (1) regularity, (2) precision, (3) naturalness.

(b) Those who opine, that in order of importance, the desiderata are: (1) naturalness, (2) regularity, (3) precision.

L'adheranti di l'ideo di ula "inventita linguo" semblas esar divi-dita en du grupi:

(a) Ti qui opinionas ke, segun ordino di importanteso, la dezirindaji esas: (1) regulozeso, (2) precizeso, (3) naturaleso.

(b) Ti qui opinionas ke, segun ordino di importanteso, la dezirindaji esas: (1) naturaleso, (2) regulozeso, (3) precizeso.

Ido is very regular and precise and seems to be sufficiently natural.

Ido esas tre reguloza e preciza, e semblas esar suficante naturala

Professor A. L. Guerard says that in point of immediate intelligibility, Ido could almost compare with the purely Neo-Latin schemes—Neutral, Panroman and Latino.[1]

Profesoro A. L. Guerard dicas ke, del vid-punto di quika komprenebleso, Ido povus preske komparesar kun la pure Neo-Latina sistemi—Neutral, Panroman e Latino.

The purely Neo-Latin schemes, for example, Latino sine flexione, Inter-lingua, are too natural and irregular. Such a language is easy to read but difficult to write or to speak.

La pure Neo-Latina sistemi, exemple Latino sine flexione, Inter-lingua, esas tro natural e ne-reguloza. Tala linguo esas facile lektebla, ma desfacile skribebla o parolebla.

[1] *A Short History of the International Language Movement*, A. L. Guerard, page 155; London, 1922.

Among other projects are Romanal and Occidental which are very natural but not so regular as Ido.

Inter altra projeti esas Romanal ed Occidental qui esas tre naturala, ma ne esas tam reguloza kam Ido.

Guerard declares that "the sound of Esperanto-Ido is more pleasing than that of Romanal." [2]

Guerard deklaras ke "la sono di Esperanto-Ido esas plu plezanta kam ta di Romanal."

A writer in the London *Times* says that Ido in appearance and sound is more attractive than Esperanto.[3]

Skribanto en la London *Times* dicas, ke Ido relate aparo e sono esas plu atraktiva kam Esperanto.

Other impartial critics say that "between Esperanto and Ido, the latter seems to be preferable." [4]

Altra senpartisa kritikanti dicas, ke "kompare Esperanto ed Ido, Ido semblas esar preferebla."

Brander Matthews acknowledges the "undeniable merits" of Ido.[5]

Brander Matthews agnoskas la "nerefutebla meriti" di Ido.

Guerard says that the final solution seems to lie between the dialect of Zamenhof (Esperanto), too hybrid and arbitrary, and that of Peano (Latino, Inter-lingua), too irregular in its "naturalness": more precisely, between Ido and Romanal.[6]

Guerard dicas, ke la finala solvo semblas restar inter la dialekto di Zamenhof (Esperanto), tro mixita ed arbitrala, e ta di Peano (Latino, Inter-lingua), tro ne-reguloza pro olua "naturaleso"; plu precize, inter Ido e Romanal.

[2] *Ibid.*, page 235.

[3] Quoted in *Science* (U. S.), Vol. LV., No. 1426, April 28, 1922, page 458.

[4] *The American Journal of Pharmacy*, June, 1922, published by the Philadelphia College of Pharmacy and Science, 145 North Tenth Street, Philadelphia.

[5] *Essays on English*, Brander Matthews, page 277.

[6] *A Short History of the International Language Movement*, A. L. Guerard, page 193; London, 1922.

Dr. F. G. Donnan, F. R. S., professor in the University of London, said, in a discourse before the Royal Institution, that if the final selection were to be either Esperanto or Ido, he would prefer Ido, but he predicted that the ultimate solution would be some form of Neo-Latin similar to Romanal, with various modifications to secure regularity and precision.[7]

Doktoro F. G. Donnan, F. R. S., profesoro en l'universitato di London, dicis, en diskurso koram la Rejal Institucuro, ke, se la finala selekto esos sive Esperanto sive Ido, il preferos Ido, ma il predicis ke la definitiva solvo esos ula formo Neo-latina, simila a Romanal, kun diversa modifiki por obtenar regulozeso e precizeso.

One of the most charming of the recent contributions to the discussion of the subject is an article on "Babel and Geneva," by Prof. A. L. Guerard, published in *The Texas Review.*

Un de la maxim atraktiva di la recenta kontributi a la diskuto di la temo, esas artiklo "Babel e Geneve," da prof. A. L. Guerard, editita en *"The Texas Review."*

There exists a large collection of books in Ido. Among them is a pamphlet entitled, "The Notion of Time," by Bergson, translated from the original French text into Ido by Paulo Dienes, doctor of mathematical sciences at the Sorbonne. The Ido text, published at Buda-Pest, is perfectly clear, although the subject is philosophical and sufficiently difficult.

Existas granda kolekto di libri en Ido. Inter oli esas broshuro titulizita "La Nociono di la Tempo," da Bergson, tradukita de l'originala Franca texto aden Ido, da Paulo Dienes, doktoro di la matematikala cienci, de la Sorbonne. L'Ido texto, editita en Buda-Pest, esas perfekte klara, quankam la temo esas filozofiala e pasable desfacila.

It is evident that the world should not be required to adopt, even provisionally, any international language which is not so good as Ido.

Esas evidenta, ke on ne devez postular, ke la mondo adoptez, mem provizore, ul internaciona linguo qua ne esas tam bona kam Ido.

[7] *Auxiliary Languages*, F. G. Donnan, in *Nature* (London), April 15, 1922, pp. 491-495. To be reprinted in the *Proceedings* of the Royal Institution.

The gift of a common language to the peoples of the world would be exceedingly useful to them in all international relations. Let us hope that a greater number of philologists and linguists will interest themselves in the problem.

La donaco di ula komuna linguo a la populi di la mondo esus tre utila a li, en omna internaciona relati. Ni esperez ke plu granda nombro di filologi e linguisti interesos su pri la problemo.

EUGENE F. MCPIKE.

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NOTE.—Information concerning Ido is obtainable from The International Language (Ido) Society of Great Britain, 57 Limes Grove, Lewisham, London, S. E., England, and from The American Ido Society, 7616 Tioga Street, Pittsburgh, Pennsylvania, U. S. A.

## NEW ALTARS.

BY ETHEL TALBOT SCHEFFAUER.

She with her iron hands
To whom the peoples bowed,
Throned above all the lands,
Once called aloud:

Bring unto me the young men,
With flowers and with mirth,
Bold songs shall be sung then
In all the earth.

Honor and fame will I buy them,
They that are young and brave,
After, I will deny them
Even a grave.

They shall be flung like rain
Over the wailing ground—
None of these many slain
Shall more be found.

And men came to her altars,
Young men and old,
And women with fiery psalters
And flowers and gold.

Fools, caught by her wonder,
Throning over the lands,
Saw not her claws of plunder,
Nor her iron hands.

The blood-wave heavy and tidal,
Swept over many a race.
Would it had taken the Idol
And rolled her from her place!

That the repentant nations,
Slowly, each one alone,
Might seek in forgotten patience,
Stone by stone.

Slabs for the new altar
Where the new god shall reign,
Before whom the old gods falter,
Hallowing his fane.

Whose words are pity and sorrow,
Whose words can build
The temple of to-morrow
For freedom's guild.

With no mistrust of a neighbor,
Nor hate, nor envy, nor fear—
A white altar of labor,
A gold altar of cheer—

An altar of freedom and peace,
Glowing out of the sand,
And bidding the tumults cease
In every land.

This is the new fane,
With tears of longing wet—
But the peoples hope in vain,
For none is building yet.

# BOOK REVIEWS AND NOTES

THE MANHOOD OF HUMANITY: THE SCIENCE AND ART OF HUMAN ENGINEERING. *By Alfred Korzybski.* E. P. Dutton & Co., 1921. Price, $3.00.

"In the name of all you hold dear, you must read this book; and then you must re-read it, and after that read it again and again, for it is not brewed in the vat of the soft best-sellers to be gulped down and forgotten, but it is hewn out of the granite, for the building of new eras."

It must not be supposed that those powerful words are an irresponsible utterance of an excited enthusiast. Far from it. They were written by no less a person than Mr. H. L. Haywood, the sober-minded editor of *The Builder*, and may be found in the August number of that official organ of The National Masonic Research Society.

Indeed Haywood's estimate of the book does but confirm the judgment of many other competent critics including educators, engineers, logicians, mathematicians, biologists, psychologists, political philosophers, publicists, and other thinkers.

Let us hear a word from some of them.

"It is," writes Alleyne Ireland, "a contribution of the highest importance to the study of every problem in which human life is one of the factors."

In *The Freeman*, Ordway Tead says: "It is a forthright, earnest book by one who has seen a vision and would share it with his fellows."

Dr. Eric T. Bell, an eminent mathematician, says that "it took a genuine flight of genius" to make "Korzybski's main discovery that plants, animals and men are respectively energy-binders, space-binders, and time-binders." And Dr. Bell adds that "Anyone but a congenital idiot will get out of this book as much entertainment of a lasting kind as is contained in a whole library of romance."

"I consider Count Korzybski's discovery of man's place in the great life movement," writes Robert B. Wolf, Vice-president of the American Society of Mechanical Engineers, "as even more epoch-making than Newton's discovery of the law of gravitation."

Writing in *The Journal of Applied Psychology*, Max Meenes states that "The Manhood of Humanity is a truly remarkable contribution toward a scientific study of humanity and should command the attention of all interested in humanity's problems."

Dr. Petrunkevitch, Professor of Zoology at Yale, thinks its "main principles are so important that the book should be carefully studied by all men of science."

Dr. L. O. Howard, eminent entomologist, in his presidential address at the annual meeting of the American Association for the Advancement of Science, Toronto, 1921, said: "Count Korzybski in his remarkable book, Manhood of Humanity, gives a new definition of man, . . . and concludes that humanity is set apart from other things that exist on this globe by its time-binding faculty, or power, or capacity." Dr. Howard adds: "It is, indeed, this *time-binding* capacity which is the principal asset of humanity."

And Dr. Walter N. Polakov, well-known engineering counselor and distinguished author of *Mastering Power Production*, says: Korzybski's book "is bound to become our new Organum, interpreting Humanity to itself, and ushering in a new epoch."

It would be easy to swell the chorus of similar testimony to vast proportions, for abundant material is at hand, but it would be superfluous to do so. What has been submitted is enough to arrest the attention of even the dullest minds. For it is perfectly evident that a book that calls forth such words from such men, representing as they do almost every great field of scientific scholarship, is a book that you and I must read, and re-read till we understand, if we are not to be dumbly ignorant of the most helpful and hopeful thought of our troubled time.

Lest any one reading these words might suspect that my own estimate of the book is but an echo of the opinions above quoted, I may be permitted to say that more than a year ago and shortly after the book came from the press I wrote as follows in *The New York Evening Post:* "We have here a book that is worthy of the times. Physically it is not large, but spiritually it is great and mighty—great in its enterprise, in its achievement, in the implications of its central thought, and mighty in its significance for the future welfare of men, women and children everywhere throughout the world."

What, pray, is that enterprise? What does the book aim at? It aims at turning the world's thought towards establishing the greatest of all conceivable things—the science and art of *human* engineering—the science and art of an engineering statesmanship magnanimous enough to embrace the entire world.

But what, pray, is human engineering? Human engineering—engineering statesmanship—is to be the science and art of coordinating the civilizing energies of the world and directing them to the advancement of the welfare of all mankind including posterity. Nothing conceivable could be nobler than that. In that great good are embraced all possible goods.

We are at once confronted with a great question. What is the science and art of human engineering to be *based* upon? It goes without saying that the basis must be a scientific basis—some kind of scientific knowledge. And the question is: scientific knowledge of *what?* The answer is: scientific knowledge of human nature—scientific understanding of the essential nature of Man.

Here we encounter the most important question that can be asked: What *is* Man? What is that quality or capacity in virtue of which human beings are *human?* What is the *distinctive* place of mankind in the hierarchy of the world's life?

In connection with that question Korzybski has rendered the world an immeasurable service. He has indeed propounded the question to himself but that is not what I mean. He has made it perfectly plain that the question is

at once supreme and fundamental but neither is that what I mean. What I mean is that he has given the great question the *best* answer it has received in the history of thought—an answer which, because it is true, is infinitely superior to all its rivals. What is the answer, It is an answer defining our humankind in terms of Man's peculiar relation to what we call *Time*. The words are these (p. 60): "Humanity is the time-binding class of life."

What do the words mean? It is evident that the burden of the meaning is borne by the term *Time-binding*. For the significance of this really mighty term the reader must be referred to the book itself where, says the mathematician and poet, Professor Bell, "the ideas are stated with such admirable clearness in so many different and illuminating ways that any person of average intelligence can grasp the essential meaning at one reading." Should any one desire to examine my own attempt to lay bare in a few words the great term's central nerve, I may refer him to pages 428-431 of my *Mathematical Philosophy* where I have dealt with Korzybski's conception of man in the light of modern advances in logical theory.

Just as soon as readers grasp the meaning of the term *time-binding* and come thus to understand the author's concept of Humanity, then and not before they will understand both why he denies the ages-old mythical idea that humans are hybrids of natural and supernatural and why he also denies and denounces the zoological conception that humans are a *species* of *animals*.

It is instructive to compare the logic of Korzybski's great work with that of Professor Robinson's interesting book, *The Mind in the Making*. The aim of the authors is the same—the welfare of mankind. They are both of them evolutionists. They both believe that man is sprung from Simian stock. Korzybski nevertheless maintained that humans are not animals for animals, says he, are merely *space*-binders while man is a *time*-binder. Robinson, on the other hand, contends that humans are animals and endeavors again and again to rub that belief indelibly into the minds of his readers. *Why* do such thinkers as Robinson regard man as a species of animal? Is it because man has been evolved out of animal ancestry? If $A$ has been evolved from $B$, do they really think that $A$ is therefore necessarily a species of $B$? If heat be applied to ice there is evolved first water and then steam. Is steam to be rightly regarded as a species of ice? Man has been evolved from Simian mammals, mammals from reptiles, reptiles from fishes, and these probably, through a long course, from "microscopic globules of living matter, not unlike the simplest bacteria of today." Are thinkers like Professor Robinson prepared to follow their own "logic" and say that our humankind may be helpfully regarded as a species of ape, as a species of reptile, as a species of fish, as a species of ancient microscopic globule of living matter, not unlike the simplest bacteria of today? If it should be discovered in Professor Robinson's time that the organic and the living have been evolved from the non-living and inorganic, would the learned historian then argue that the living is a species of the non-living and that the organic is a species of the inorganic? The evolution of the *Novel* is an indubitable fact but it is a ridiculous contention that whatever is new must be a species of all the things from which it has sprung.

Is it contended that humans are species of animal *because* humans have certain animals organs, functions, and propensities? One would be not less

foolish to contend that animals are plants because they have many organs and functions that plants have or to contend that solids are surfaces because solids have some properties that surfaces have or to contend that fractions are whole numbers because they have some properties that whole numbers have.

The philosophy of many a historian and many a zoologist would be greatly improved by a solid course in freshman logic.

Korzybski's concept of Man is the core of his book and the organic center of his philosophy. If you will master that concept you will find that it is related to the other ideas in the work as the sun is related to the planets and planetoids of our solar system. And as you continue your meditation you will discover much more.

If you are a historian you will find that the new concept of man demands a new philosophy of history—a philosophy that shall study the evil rôles which false concepts of human nature have played from time immemorial.

If you are a student of ethics, you will find that the new concept affords a scientific basis for a moral system infinitely superior alike to the ethics of magic and myth and to the zoological ethics of the righeousness of might—the ethics of tooth and claw, competition, combat, and war.

If you are an educator you will find that the highest obligation of home, school, and press is to teach boys and girls and men and women everywhere to understand and to feel what they as humans really are—not animals nor hybrids of angel and beast but time-binders, civilizers, inheritors of the achievements of the dead, charged to use the inheritance justly and to transmit it with increase to he yet unborn.

If you are an engineer—and we are all of us engineers in some respect—you will find that Korzybski's conception of man is the solid basis for that science and art of *human* engineering—that science and art of engineering statesmanship—whose function it is to study the time-binding energies of the world, the civilization-producing energies of our kind, to coordinate them and to direct them to the welfare of all mankind including posterity.

I will close by repeating what I said elsewhere. "Not to read this book is to miss the best thought of these troubled years."

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